Sensitivity Analysis (Homework 5)

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There are a number of strong assumptions in our model. To check the sensitivity, ideally, we should try to weaken these assumptions and use nonparametric methods, but we do not have sufficient data for that. Instead, we check sensitivity by various perturbations:

1. Unmeasured Confounding:

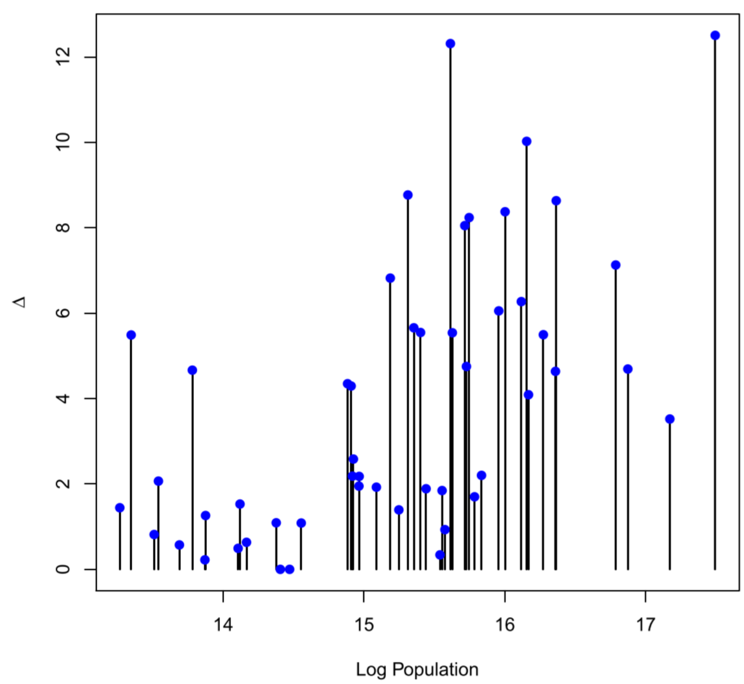
In our model, we assumed conditional independence: there is no unmeasured confounding. At time t, we treated $(A_1, Y_1), \cdots, (A_{t-1}, Y_{t-1})$ as confounders.

Now add a perturbation: consider there is an unmeasured confounder U. We want to assess $|\widehat{\beta}_U - \widehat{beta}|$ where $\widehat{\beta}_U$ is the estimate after adding U. To aviod add extra assumptions, consider the following quantity:

$$\Delta = \frac{|\widehat{\beta}_U - \widehat{\beta}|}{\mathrm{se}(\widehat{\beta})}$$

which represents the effect of the unmeasured confounding on the standard error scale. If $\Delta = 0$, then there is no effect of unmeasured confounding; if $\Delta = 1$, then the effect of unmeasured confounding is the same size as of the standard error.

For each estimate of $\widehat{\beta}$, we enlarge the confidence interval of $\widehat{\beta}$ by $\Delta \cdot \mathrm{se}(\widehat{\beta})$. Then we can get the critical value of Δ: how large would Δ have to be so that the enlarged confidence interval would contain 0? If Δ is larger than this value, our results would be invalidated by an unmeasured confounding of size $\Delta \cdot \mathrm{se}(\widehat{\beta})$. This critical value is given by the following plot (directly taken from the paper):



There are 50 data points in this plot, each represents one of the 50 states of the US. As we can see, the critical value of Δ increases as the population (of the state) increases, which is expected. And in most cases, it taks a relatively large value of Δ to break the statistical significance (so that the confidence interval of $\widehat{\beta}$ contains 0). This shows our model is robust to unmeasured confounding.

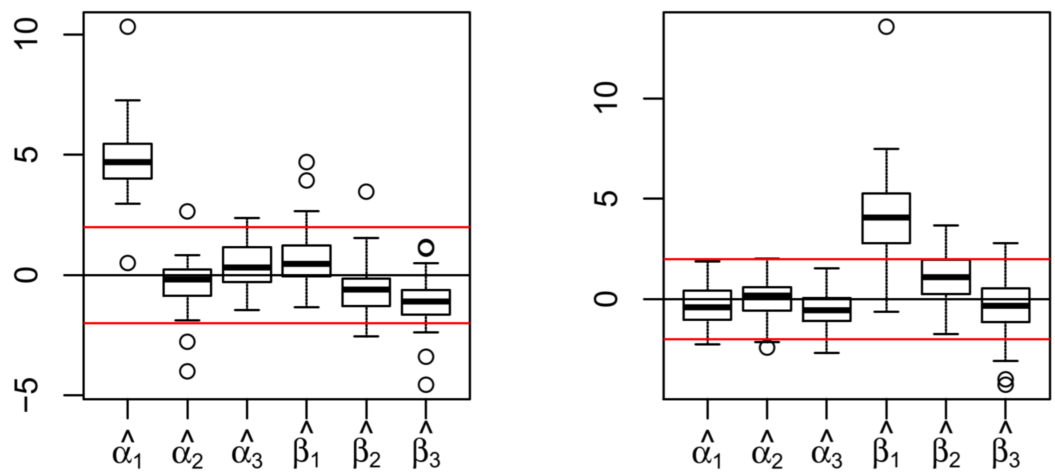
2. The Markov Assumption:

Our model is $L_t = \beta \sum_{i=1}^{t-\delta} A_s + \nu(t) + \epsilon_t$ with weights $W_1, \cdots, W_n$. In the estimation process, in order to solve for the weights in the moment constraints: $E[h_1(A_t)h_2(Y_{t-1})W_t] = 0$, we made a Markov assumption such that $A_{t-\delta}$ is conditional indpeendent of the past given $(A_{t-1-\delta}, L_{t-1-\delta})$ and L_t is conditional indpeendent of the past given $(A_{t-1-\delta}, L_{t-1})$.

Now consider a perturbation such that $A_{t-\delta}$ and L_t are dependent on the past three events, not just one:

$$A_{t-\delta} = \alpha_0 + \alpha_1 A_{t-1-\delta} + \alpha_2 A_{t-2-\delta} + \alpha_3 A_{t-3-\delta} + \beta_1 L_{t-1-\delta} + \beta_2 L_{t-2-\delta} + \beta_3 L_{t-3-\delta} + \epsilon_t$$
$$L_t = \alpha_0 + \alpha_1 A_{t-\delta} + \alpha_2 A_{t-1-\delta} + \alpha_3 A_{t-2-\delta} + \beta_1 L_{t-1} + \beta_2 L_{t-2} + \beta_3 L_{t-3} + \delta_t$$

The t-statistics for the parameters is given by the following plot (directly taken from the paper):



The left one is for A_t, the right one is for Y_t. As we can see, only $\widehat{\alpha}_1$ is statistically significant for A_t, and only $\widehat{\beta}_1$ is statistically significant for Y_t. This implies both are memory one processes, which checks the Markov assumption.

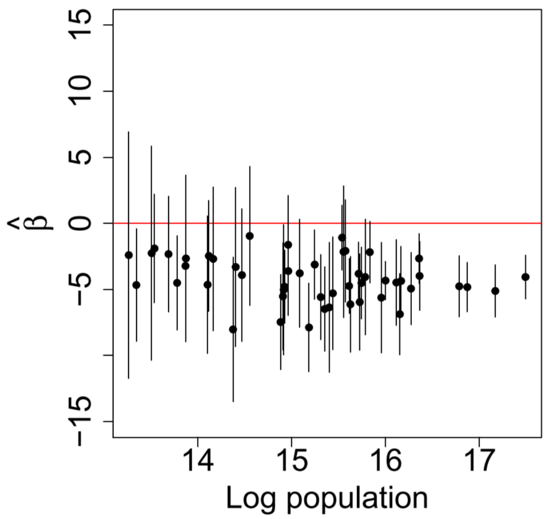
3. An Alternative Model to MSM:

We want to know if switching to a different model would change the result. Our model is the MSM model ($L_t = \beta \sum_{i=1}^{t-\delta} A_s + \nu(t) + \epsilon_t$). Consider switching to the following time series AR(1) model:

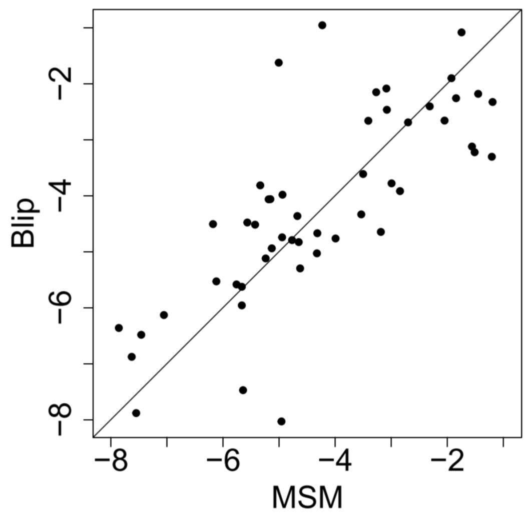
$$L_t = L_{t-1} + \beta A_{t-\delta} + r(t) + \epsilon_t$$

where $r(t)$ is a polynomial of degree $k-1$. This model is based on Robins' blip models. The result of the new model is given by the following plots (next page).

The left plot gives the estimates of $\widehat{\beta}$, the right plot compares the results of the two models (each data points represents one of the 50 states of the US). As we can see, for most states, the confidence interval still does not cover 0, which does not violate our original result. The regression shown in the right plot is also relatively reasonable. This shows the result is robust to the model selection.



15
10
5
0
−5
−15
$\hat{\beta}$
14
15
16
17
Log population



−2
−4
−6
−8
Blip
−8
−6
−4
−2
MSM