Bryan Torres Calle Tarea #2 E.D.

1) $f(t) = 1 + t + \frac{8}{3}\int_0^t \underbrace{(t-\tau)^3 f(\tau)}_{t^3 * f(t)} d\tau$

$f(t) = 1 + t + \frac{8}{3}\left(t^3 * f(t)\right)$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{1\} + \mathcal{L}\{t\} + \mathcal{L}\{8/3\,(t^3 * f(t))\}$

$F(s) = \frac{1}{s} + \frac{1}{s^2} + \frac{16}{s^4}F(s)$

$\left(1 - \frac{16}{s^4}\right)F(s) = \frac{1}{s} + \frac{1}{s^2}$

$\left(\frac{s^4-16}{s^4}\right)F(s) = \frac{1}{s} + \frac{1}{s^2}$

$F(s) = \frac{s^3+s^2}{(s^4-16)}$

$F(s) = \frac{s^3+s^2}{(s^2-4)(s^2+4)}$

* Fracciones parciales *

$\frac{s^3+s^2}{(s-2)(s+2)(s^2+4)} = \frac{A}{(s-2)} + \frac{B}{s+2} + \frac{Cs+D}{(s^2+4)} \Rightarrow s^3+s^2 = A(s+2)(s^2+4) + B(s-2)(s^2+4) + (Cs+D)(s-2)(s+2)$

$s^3+s^2 = As^3 + 4As + 2As^2 + 8A + Bs^3 + 4Bs - 2Bs^2 - 8B + Cs^3 - 4Cs + Ds^2 - 4D$

$s^3+s^2 = (A+B+C)s^3 + (2A-2B+D)s^2 + (4A+4B-4C) + 8A - 8B - 4D$

$\Rightarrow \left.\begin{aligned} 1 &= A+B+C \\ 1 &= 2A-2B+D \\ 0 &= 4A+4B-4C \\ 0 &= 8A-8B-4D \end{aligned}\right\} \Rightarrow \begin{aligned} A &= 3/8 \\ B &= 1/8 \\ C &= 1/2 \\ D &= 1/2 \end{aligned}$

$\Rightarrow F(s) = \frac{3}{8(s-2)} + \frac{1}{8(s+2)} + \frac{s}{2(s^2+4)} + \frac{1}{2(s^2+4)}$

$F(s) = \frac{3}{8} \cdot \frac{1}{s-2} + \frac{1}{8} \cdot \frac{1}{s+2} + \frac{1}{2} \cdot \frac{s}{s^2+4} + \frac{1}{4} \cdot \frac{2}{s^2+4}$

$\mathcal{L}^{-1}\{F(s)\} = \frac{3}{8}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + \frac{1}{8}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} + \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$

$f(t) = \frac{3}{8}e^{2t} + \frac{1}{8}e^{-2t} + \frac{1}{2}\cos 2t + \frac{1}{4}\operatorname{Sen} 2t$

2) $y'' + y' - 4y - 4\underbrace{\int_0^t y(\tau)\,d\tau}_{1 * y(t)} = 6e^t - 4t - 6$, $y(0) = 0$, $y'(0) = 0$

Aplicando Laplace a toda la ecuación:

$s^2 Y(s) + sY(s) - 4Y(s) - \frac{4}{s}Y(s) = \frac{6}{s-1} - \frac{4}{s^2} - \frac{6}{s}$

$\left(s^2 + s - 4 - \frac{4}{s}\right)Y(s) = \frac{2(s+2)}{(s-1)s^2}$

$\left((s^2-4)\left(\frac{s+1}{s}\right)\right)Y(s) = \frac{2(s+2)}{(s-1)s^2}$

$Y(s) = \frac{2}{s(s-1)(s+1)(s-2)}$

Aplicando fracciones parciales:

$Y(s) = \frac{1}{s} + \frac{1}{3} \cdot \frac{1}{(s-2)} - \frac{1}{3} \cdot \frac{1}{s+1} - \frac{1}{s-1}$

Aplicando transformada inversa:

$y(t) = 1 + \frac{1}{3}e^{2t} - \frac{1}{3}e^{-t} - e^{t}$

3/ $y'' + 4y' + 13y = \delta(t-\pi) + \delta(t-3\pi)$; $y(0)=1$, $y'(0)=0$

$s^2Y(s) - s + 4sY(s) - 4 + 13Y(s) = e^{-\pi s} + e^{-3\pi s}$

$(s^2 - 4s + 13)Y(s) = e^{-\pi s} + e^{-3\pi s} + 4 + s$

$(s^2 - 4s + 4 + 9)Y(s) = e^{-\pi s} + e^{-3\pi s} + 4 + s$

$((s+2)^2 + 9)Y(s) = e^{-\pi s} + e^{-3\pi s} + 4 + s$

$$Y(s) = \frac{e^{-\pi s}}{(s+2)^2+9} + \frac{e^{-3\pi s}}{(s+2)^2+9} + \frac{s+2}{(s+2)^2+9} + \frac{2}{(s+2)^2+9}$$

$$Y(s) = \frac{1}{3}\cdot\frac{3}{(s+2)^2+9}e^{-\pi s} + \frac{1}{3}\frac{3}{(s+2)^2+9}e^{-3\pi s} + \frac{s+2}{(s+2)^2+9} + \frac{2}{3}\frac{3}{(s+2)^2+9}$$

Aplicando transformada inversa de Laplace

$$y(s) = \frac{1}{3}e^{-2(t-\pi)}\operatorname{Sen}3(t-\pi)\,u(t-\pi) + \frac{1}{3}e^{-2(t-3\pi)}\operatorname{Sen}3(t-3\pi)\,u(t-3\pi)$$

$$+ e^{-2t}\operatorname{Cos}3t + \frac{2}{3}e^{-2t}\operatorname{Sen}3t$$