Marking Sheet   Assignment 1   CS370 Winter 2018

Name: Yandong Zhu   ID: 20588720

Section: 1 ✓ (8:30am)   2 (11:30am)   3 (1:30pm)

1.	(a) roots by quadratic formula	(4)	________	
	(b) alternate quadratic formula	(3)	________	
	(c) roots by alternate formula	(3)	________	
				......../10
2.	(a) Matlab function	(4)	________	
	(b) stability analysis	(6)	________	
	(c) number of steps	(5)	________	
				......../15
3.	(a) Conditions and values	(6)	________	
	(b) Lagrange	(4)	________	
				......../10
4.	(a) Linear system	(8)	________	
	(b) Spline coefficients	(8)	________	
	(b) Graphic	(4)	________	
				........./20
5.	(a) points	(4)	________	
	(b) lines	(4)	________	
	(c) splines	(12)	________	
				......../20

Total: (75) ____________

CS370

Assignment 1

Yandong Zhu

20588720

1. (a) In my calculation, $x_1 = -21.75227$, $x_2 = -0.0050146$ So the relative error will comes to $Err_{rel1} = 0.0012413\%$, $Err_{rel2} = 0.045887\%$

   (b) suppose another root can be compute by $\frac{2c}{ax_1}$, we have

$$x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{2c}{ax_1}$$

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{2c}{-b - \sqrt{b^2 - 4ac}}$$

$$(-b + \sqrt{b^2 - 4ac}) * (-b - \sqrt{b^2 - 4ac}) = 4ac$$

$$b^2 - b^2 - 4ac = 4ac$$

$$ac = 0$$

   if the condition holds, $|b| \approx \sqrt{b^2 - 4ac}$, then

$$b^2 \approx b^2 - 4ac$$

$$ac \approx 0$$

   Thus another root can be represented as $\frac{2c}{ax_1}$

   (c) By applying the new formula $x_{2\_new} = -0.0050123$, and the $Err_{rel2\_new} = 0$

2. (a)

```
f(1) = 1;
f(2) = 0.5714285;
for i = 2:40
    f(i+1) = -41/14 * f(i) + 2 * f(i-1)
end
```

2	3	4	5	6
0.3265	0.1866	0.1066	0.0609	0.0348
7	8	9	10	11
0.0198	0.0118	0.0051	0.0086	-0.0148
12	13	14	15	16
0.0605	-0.2068	0.7266	-2.5416	8.8969
17	18	19	20	21
-31.1373	108.9808	-381.4327	1.3350e+3	-4.6726e+3
22	23	24	25	26
1.3654e+4	-5.7239e+4	2.0034e+5	-7.0117e+5	2.4541e+6
27	28	29	30	31
-8.5894e+6	3.0063e+7	-1.0552e+8	3.6827e+8	-1.2889e+9
32	33	34	35	36
4.5113e+9	-1.5790e+10	5.5264e+10	-1.9342e+11	6.7698e+11
37	38	39	40	
-2.3694e+12	8.2930e+12	-2.9025e+13	1.0159e+14	

(b)

$$p_n^A = -\frac{41}{14}p_{n-1}^A + 2p_{n-2}^A$$

$$p_n^E = -\frac{41}{14}p_{n-1}^E + 2p_{n-2}^E$$

$$\epsilon_n = |p_n^A - p_n^E|$$

$$\epsilon_n = -\frac{41}{14}\epsilon_{n-1} + 2\epsilon_{n-2}$$

By sovling this equation, we get $x_1 = -\frac{4}{7}$ $x_2 = \frac{7}{2}$ so

$$\epsilon_n = c_1(\frac{4}{7})^n + c_2(-\frac{7}{2})^n$$

Since the c_1 and c_2 is the combination of ϵ_0 and ϵ_1 Thus when $n \to \infty$, $\epsilon \to \infty$

(c) assume that $p_0^A = p_0^E$ and $p_1^A = p_1^E(1+\epsilon)$ Sine $\epsilon < \epsilon_1$
So we have $\epsilon_1 = p_1^E\epsilon$
solving for c_1 and c_2

$$\frac{4}{7}c_1 - \frac{7}{2}c_2 = \frac{4}{7}\epsilon$$

$$c_1 + c_2 = 0$$

we get

$$c_1 = \frac{8}{57}\epsilon, c_2 = -\frac{8}{57}\epsilon$$

and we want

$$|\frac{8}{57}\epsilon \times (\frac{4}{7})^n| + |-\frac{8}{57}\epsilon \times (-\frac{7}{2})^n| > 1$$

Thus, by calculation, when $n > 15$, it greater than 1

3. (a) Because it is a natural spline, so we have $S''_{n-1}(x_n) = 0$, so:

$$S''_2(3) = 6 + 6a_4x$$

$$6 + 18a_4 = 0$$

$$a_4 = -\frac{1}{3}$$

Then we have

$$S''_1(0) = S''_2(0)$$

$$2a_3 = 6$$

$$a_3 = 3$$

$$S'_1(-1) = S'_2(-1)$$

$$a_1 - 18 + 3 = 19 - 3$$

$$a_1 = 31$$

$$S_1(-1) = S_2(-1)$$

$$28 - 31 + 9 - 1 = a_2 - 19 + 1$$

$$a_2 = 23$$

Thus

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 25 \\ 26 \\ 3 \\ -\frac{1}{3} \end{bmatrix}$$

(b)

$$L_1(x) = \frac{(x+1)(x-0)(x-3)}{(-2)(-3)(-6)} = \frac{(x+1)(x)(x-3)}{-36}$$

$$L_2(x) = \frac{(x+3)(x-0)(x-3)}{(2)(-1)(-4)} = \frac{(x+3)(x)(x-3)}{8}$$

$$L_3(x) = \frac{(x+3)(x+1)(x-3)}{(3)(1)(-3)} = \frac{(x+3)(x+1)(x-3)}{-9}$$

$$L_4(x) = \frac{(x+3)(x+1)(x-0)}{(6)(4)(3)} = \frac{(x+3)(x+1)(x)}{72}$$

from the prvious part, we can compute that

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \\ 26 \\ 101 \end{bmatrix}$$

with $p(x) = 7L_1(x) + 11L_2(x) + 26L_3(x) + 101L_4(x)$

4. (a)

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 & 0 \\ 1 & 6 & 1 & 0 & 0 \\ 0 & 3 & 8 & 3 & 0 \\ 0 & 0 & 2 & 10 & 2 \\ 0 & 0 & 0 & \frac{1}{2} & 1 \end{bmatrix} * \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} \\ 16.5 \\ 26 \\ -20 \\ -3 \end{bmatrix}$$

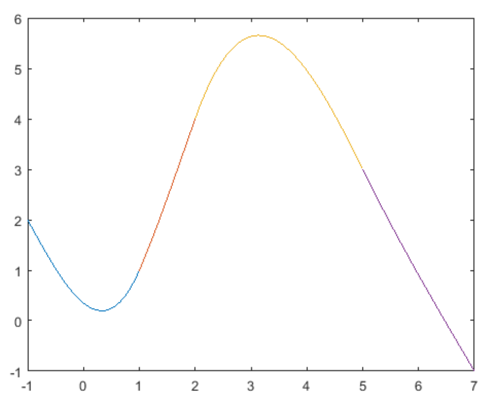
(b) For the previous part, the result is

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{bmatrix} = \begin{bmatrix} -2.0335 \\ 2.5670 \\ 3.1317 \\ -2.2515 \\ -1.8743 \end{bmatrix}$$

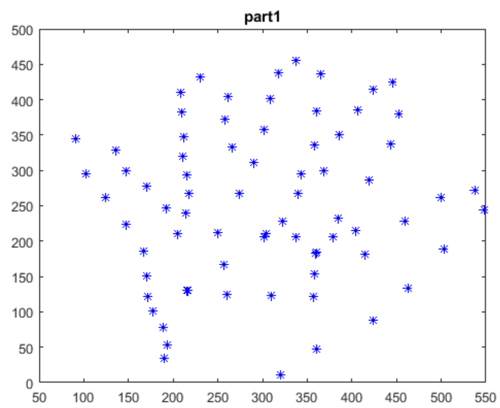
So we can compute that

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix} = \begin{bmatrix} 2 & -2.0335 & 0 & 0.3834 \\ 1 & 2.5670 & 0.7343 & -0.3013 \\ 4 & 3.1317 & -1.6706 & 0.1719 \\ 3 & -2.2515 & 0.18865 & -0.03145 \end{bmatrix}$$

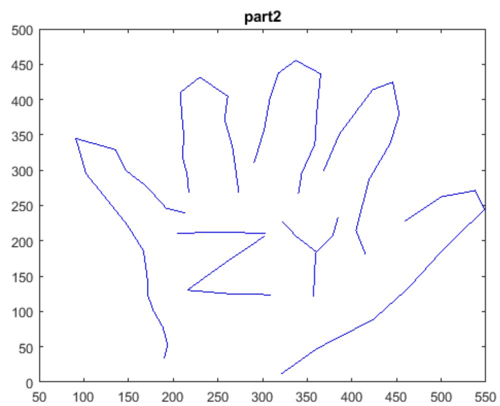
(c)



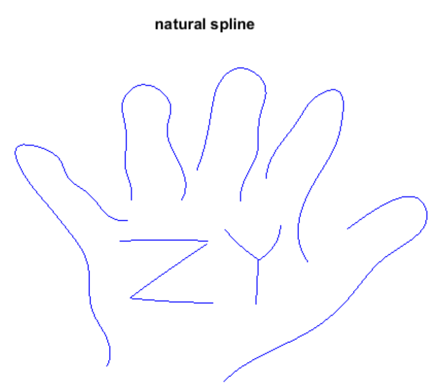
5. (a)


part1
500
450
400
350
300
250
200
150
100
50
0
50
100
150
200
250
300
350
400
450
500
550

(b)


part2
500
450
400
350
300
250
200
150
100
50
0
50
100
150
200
250
300
350
400
450
500
550

(c)


natural spline

not a knot