Marking Sheet Assignment 3 CS 370 Winter 2018

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Section: 1 ✓ (8:30pm) 2 (11:30pm) 3 (1:30)

1. (a) Fourier	(5)	_________	
(b) Inverse Fourier	(5)	_________	
			......../10
2. (a) F_k	(7)	_________	
(b) F_{2k}	(8)	_________	
			......../15
3. (a) g, h	(5)	_________	
(b) G, H	(5)	_________	
(c) F	(5)	_________	
(d) F (butterfly)	(5)	_________	
			......../20
4. Edge Sharpening			
			......../15
4. train / bird			
			......../20
5. Image compression			
			......../20

Total: (100) _____________

CS370

Assignment 3

Yandong Zhu

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1. (a)

$$
\begin{aligned}
F_k &= \frac{1}{N}\sum_{n=0}^{N-1} f_n W^{-nk} \\
F_0 &= \frac{1}{4}(1+2+3+2) = 2 \\
F_1 &= \frac{1}{4}(1+2\times W^{-1}+3\times W^{-2}+2\times W^{-3}) \\
&= \frac{1}{4}(1+2\times -i+3\times -1+2\times i) \\
&= -0.5 \\
F_2 &= \frac{1}{4}(1+2\times W^{-2}+3\times W^{-4}+2\times W^{-6}) \\
&= \frac{1}{4}(1+2\times -1+3\times 1+2\times -1) \\
&= 0 \\
F_3 &= \frac{1}{4}(1+2\times W^{-3}+3\times W^{-6}+2\times W^{-9}) \\
&= \frac{1}{4}(1+2\times i+3\times -1+2\times -i) \\
&= -0.5
\end{aligned}
$$

Thus

$$F = \{2, -0.5, 0, -0.5\}$$

(b)

$$
\begin{aligned}
f_k &= \sum_{j=0}^{N-1} F_j W^{nj} \\
f_0 &= (4 + -1 + 0 + -1) \\
&= 2 \\
f_1 &= (4 + (-1) \times W^1 + 0 \times W^2 + (-1) \times W^3) \\
&= (4 + -i + 0 + i) \\
&= 4 \\
f_2 &= (4 + (-1) \times W^2 + 0 \times W^4 + (-1) \times W^6) \\
&= (4 + 1 + 0 + 1) \\
&= 6 \\
f_3 &= (4 + (-1) \times W^3 + 0 \times W^6 + (-1) \times W^9) \\
&= (4 + i + 0 + -i) \\
&= 4
\end{aligned}
$$

Thus

$$f = \{2, 4, 6, 4\}$$

2. (a) When $f_n = (-1)^n$

$$F_k = 1 - W^{-k} + W^{-2k} - W^{-3k} + \cdots + (-1)^n \times W^{-nk}$$

When N is even

$$F_k = 1 - W^{-k} + W^{-2k} - W^{-3k} + \cdots - W^{nk}$$

let

$$
\begin{aligned}
S &= 1 - W^{-k} + W^{-2k} - W^{-3k} + \cdots - W^{(N-1)k} && (1) \\
W^{-k} \times S &= W^{-k} - W^{-2k} + W^{-3k} - W^{-4k} + \cdots - W^{-Nk} && (2)
\end{aligned}
$$

We use (2) + (1) and we got

$$(1 + W^{-k})S = 1 - W^{-Nk}$$

Since $W^{Nk} = 1$, $S = -W^{-k}S$, So

$$S = \begin{cases} N & W^{-k} = -1 \\ 0 & otherwise \end{cases}$$

(b)

$$\begin{aligned}
F_{2k} &= \frac{1}{N}(f_0 + f_1W^{-2k} + f_2W^{-4k} + \cdots + f_{\frac{N}{2}}W^{-Nk} + f_{\frac{N}{2}+1}W^{-(N+2)k} + \cdots + f_{N-1}W^{-2(N-1)k}) \\
&= \frac{1}{N}(-1 - W^{-2k} - W^{-4k} - \cdots + W^{-Nk} + W^{-(N+2)k} + \cdots + W^{-2(N-1)k}) \\
&= \frac{1}{N}(-1 - W^{-2k} - W^{-4k} - \cdots - W^{-(N-2)k} + 1 + W^{-2k} + W^{-4k} + \cdots + W^{-(N-2)k}) \\
&= \frac{1}{N} \times 0 \\
&= 0
\end{aligned}$$

3. (a)

$$\begin{aligned}
g &= \frac{f_i + f_{\frac{N}{2}+i}}{2} \\
g_0 &= \frac{f_0 + f_{\frac{N}{2}+0}}{2} \\
&= 0 \\
g_1 &= \frac{f_1 + f_{\frac{N}{2}+1}}{2} \\
&= 0 \\
g_2 &= \frac{f_2 + f_{\frac{N}{2}+2}}{2} \\
&= 0 \\
g_3 &= \frac{f_3 + f_{\frac{N}{2}+3}}{2} \\
&= 0
\end{aligned}$$

Thus

$$g = \{0, 0, 0, 0\}$$

$$
\begin{aligned}
h &= \frac{f_i - f_{\frac{N}{2}+i}}{2} \times W^{-i} \\
h_0 &= \frac{f_0 - f_{\frac{N}{2}+0}}{2} \times W^{-0} \\
&= \frac{1-(-1)}{2} \\
&= 1 \\
h_1 &= \frac{f_1 - f_{\frac{N}{2}+1}}{2} \times W^{-1} \\
&= 0 \\
h_2 &= \frac{f_2 - f_{\frac{N}{2}+2}}{2} \times W^{-2} \\
&= \frac{4}{2} \times W^{-2} \\
&= 2 \times W^{-2} \\
h_3 &= \frac{f_3 - f_{\frac{N}{2}+3}}{2} \times W^{-2} \\
&= 0
\end{aligned}
$$

Thus

$$h = \{1, 0, 2 \times W^{-2}, 0\}$$

(b) since $g_i = 0$ thus

$$G = \{0, 0, 0, 0\}$$

$$
\begin{aligned}
H_0 &= \frac{1}{4}(1 + 0 + 2 \times W^{-2} + 0) = 2 \\
&= \frac{1-2i}{4} \\
H_1 &= \frac{1}{4}(1 + 0 + 2 \times W^{-2} \times W^{-4} + 0) \\
&= \frac{1+2i}{4} \\
H_2 &= \frac{1}{4}(1 + 0 + 2 \times W^{-2} \times W^{-8} + 0) \\
&= \frac{1-2i}{4} \\
H_3 &= \frac{1}{4}(1 + 0 + 2 \times W^{-2} \times W^{-12} + 0) \\
&= \frac{1+2i}{4}
\end{aligned}
$$

thus

$$H = \{\frac{1-2i}{4}, \frac{1+2i}{4}, \frac{1-2i}{4}, \frac{1+2i}{4}\}$$

(c) So

$$F = \{0, \frac{1-2i}{4}, 0, \frac{1+2i}{4}, 0, \frac{1-2i}{4}, 0, \frac{1+2i}{4}\}$$

(d)

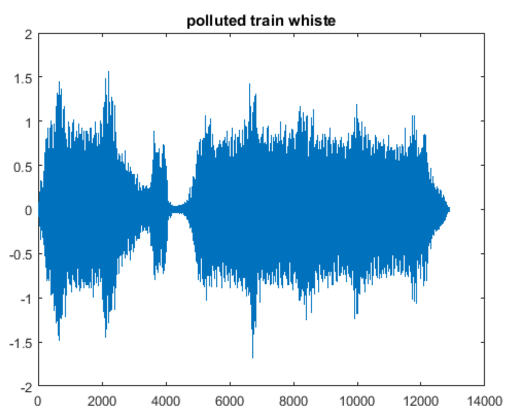
$$\begin{bmatrix}
f_0 & 1 & 0 & & & X_{000} & F_0 = X_{000} \\
f_1 & 0 & 0 & & & X_{001} & F_4 = X_{001} \\
f_2 & 2 & 0 & & & X_{010} & F_2 = X_{010} \\
f_3 & 0 & 0 & & & X_{011} & F_6 = X_{011} \\
- & - & --- & ---- & --- & --- & ------- \\
f_4 & -1 & 1 & \frac{1+2W^{-2}}{2} & \frac{1-2i}{4} & X_{100} & F_1 = X_{100} \\
f_5 & 0 & 0 & 0 & \frac{1-2i}{4} & X_{101} & F_5 = X_{101} \\
f_6 & -2 & 2W^{-2} & \frac{1-2W^{-2}}{2} & \frac{1+2i}{4} & X_{110} & F_3 = X_{110} \\
f_7 & 0 & 0 & 0 & \frac{1+2i}{4} & X_{111} & F_7 = X_{111}
\end{bmatrix}$$

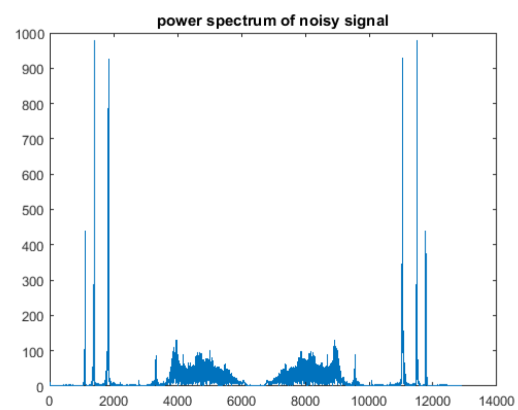
Thus the DFT of F is

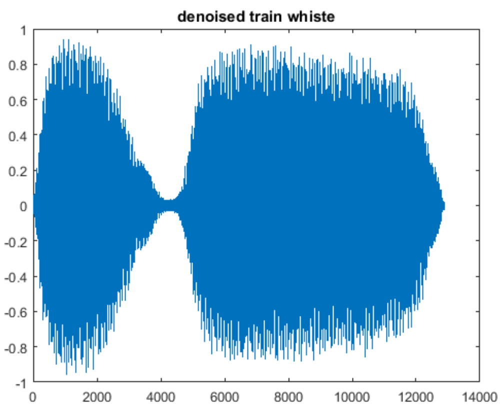
$$F = \{0, \frac{1-2i}{4}, 0, \frac{1+2i}{4}, 0, \frac{1-2i}{4}, 0, \frac{1+2i}{4}\}$$

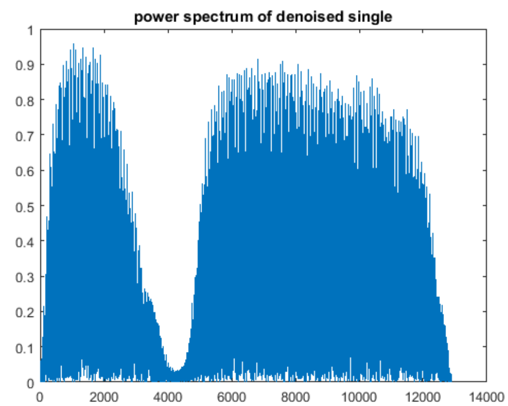
4.

5.









i) the code first calculate the spectrum of the original noise. On the graph we can clearly see the there are two area: bird noise at the beginning and last and train signal in the middle.
Then I set the bird noise to zero on its field to isolate the train signal to create the denoised train signal
ii) the frequency threshold i used to filter the signal is from 3000 to 10000.
iii) By denoising the bird noise, we can clearly notify that the train signal distributed more evenly on the graph

6.

by increasing the value of tolerance the error become greater and the drop ration become greater. And the image becomes vaguer. Since there are more points in the picture can be compressed which leads to the result.