Marking Sheet   Assignment 2   CS370 Winter 2018

Name: ......Yandong Zhu...........   ID:....20588720.....

Section: 1 ✓ (8:30am)   2 (11:30am)   3 (1:30pm)

1. (a)	first-order system	(6)	_________
(b)	system dynamics function	(9)	_________
			......../15
2. (a)	Solve (Euler)	(3)	_________
(b)	Modified Euler function	(3)	_________
(c)	Solve (Modified Euler)	(3)	_________
(d)	Numerical evidence (Euler)	(3)	_________
(e)	Numerical evidence (Mod Euler)	(3)	_________
			......../15
3.	local truncation error		......../10
4.	stability analysis		......../10
5.	Pursuit Problem code (10) & tests (15)		......../25

Total: (75) _____________

CS370

Assignment 2

Yandong Zhu

20588720

1. (a)

   Let $z_1 = u$, $z_2 = v$, $z_3 = u'$, $z_4 = v'$
   Then we have the system

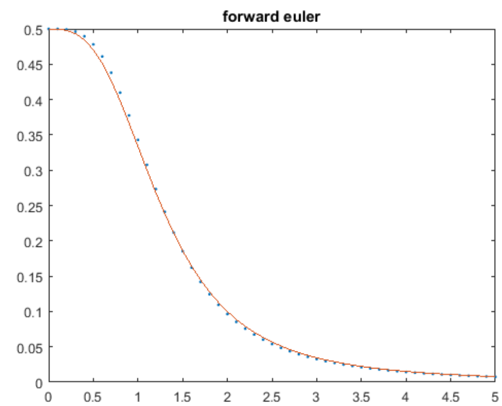
   $$z_1'(t) = z_3(t)$$
   $$z_2'(t) = z_4(t)$$
   $$z_3'(t) = sin(t) - c_1(z_3(t) - z_4(t)) - k_1(z_1(t) - z_2(t)) - k_2 z_1(t)$$
   $$z_4'(t) = c_1(z_3(t) - z_4(t)) - c_2(z_1(t) - z_2(t))$$

   $$z_1(0) = 1, z_2(0) = 2, z_3(0) = 0, z_4(0) = 0$$

   (b)

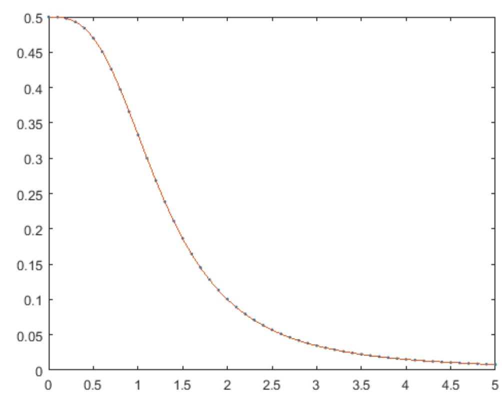
```
function Z = Dynamic (t, z, c_1,c_2,k_1,k_2)
    Z = zeros(4, 1);
    Z(1) = z(3);
    Z(2) = z(4);
    Z(3) = sin(t) - c_1*(z(3)-z(4)) - k_1*(z(1) - z(2)) - k_2*z(
    Z(4) = c_1*(z(3)-z(4)) - c_2*(z(1)-z(2));
end
```

2. (a)



(b)

(c)



(d)

0.8802	0.9509	0.9779	0.9895	0.9949
0.9975	0.9988	0.9994	0.9997	0.9998

Thus $p \approx 1$

(e)

2.5944	2.2502	2.1049	2.0481	2.0231
2.0113	2.0056	2.0028	2.0014	2.0007

Thus $p \approx 2$

3.

For the question, we have, $y_{n+1} = y_{n-1} + 2hf(t_n, y_n)$

First we replace the RHS with exact approximate value

$$y_{n+1} = y(t_{n-1}) + 2h \cdot y'(t_n)$$

Then we do the Taylor Expand for $y(t_{n-1})$ at time t_n

$$y(t_{n-1}) = y(t_n) - hy'(t_n) + \frac{h^2}{2}y''(t_n) - \frac{h^3}{6}y'''(t_n) + O(h^4)$$

plug the value back to y_{n+1}

$$\begin{aligned}
y_{n+1} &= 2hy'(t_n) + y(t_n) - hy'(t_n) + \frac{h^2}{2}y''(t_n) - \frac{h^3}{6}y'''(t_n) + O(h^4) \\
&= y(t_n) + hy'(t_n) + \frac{h^2}{2}y''(t_n) - \frac{h^3}{6}y'''(t_n) + O(h^4)
\end{aligned}$$

While the approximate value we get for $y(t_{n+1})$ is

$$y(t_{n+1}) = y(t_n) + hy'(t_n) + \frac{h^2}{2}y''(t_n) + \frac{h^3}{6}y'''(t_n) + O(h^4)$$

thus the difference is

$$y(t_{n+1}) - y_{n+1} = (\frac{h^3}{6} + \frac{h^3}{6})y'''(t_n) + O(h^4) = \frac{h^3}{3}y'''(t_n) + O(h^4) = O(h^3)$$

4.

plug in the test formula $y'(t) = -\lambda y(t)$ we can get

$$y^*_{n+1} = (1 - \frac{3h}{4}\lambda)y_n$$

$$\begin{aligned}
y_{n+1} &= y_n + \frac{h}{3}(-\lambda y_n - 2\lambda y^*_{n+1}) \\
&= y_n - \frac{h}{3}\lambda y_n - \frac{2h}{3}\lambda(1 - \frac{3h}{4}\lambda)y_n \\
&= y_n(1 - h\lambda + \frac{(h\lambda)^2}{2})
\end{aligned}$$

Thus

$$\begin{aligned}
e_{n+1} &= e_n \cdot (1 - h\lambda + \frac{(h\lambda)^2}{2}) \\
&= e_0 \cdot (1 - h\lambda + \frac{(h\lambda)^2}{2})^{n+1}
\end{aligned}$$

So we want $|1 - h\lambda + \frac{(h\lambda)^2}{2}| < 1$, when $n \to \infty$, $(1 - h\lambda + \frac{(h\lambda)^2}{2})^{n+1} \to 0$
Thus, $1 - h\lambda + \frac{(h\lambda)^2}{2} > -1$ and $1 - h\lambda + \frac{(h\lambda)^2}{2} < 1$

$$-1 < 1 - h\lambda + \frac{(h\lambda)^2}{2}$$
$$0 < 4 - 2\lambda h + (\lambda h)^2$$
$$0 < 3 + (\lambda h - 1)^2$$

which is always true; next is

$$1 > 1 - h\lambda + \frac{(h\lambda)^2}{2}$$
$$0 > -h\lambda + \frac{(h\lambda)^2}{2}$$
$$h < \frac{2}{\lambda}$$

So h is conditional stable

5. (a)
   target.m:

```
function T = target(t, P)

global SP ST RT D

    dt = direction(P(4), P(5), P(1), P(2), P(3));
    T = zeros(6,1);
    T(1) = SP*cos(P(3));
    T(2) = SP*sin(P(3));
    T(3) = SP*(dt-P(3))/(abs(dt-P(3))+D);
    T(4) = ST*cos(P(6));
    T(5) = ST*sin(P(6));
    T(6) = ST/(10*RT);
end
```

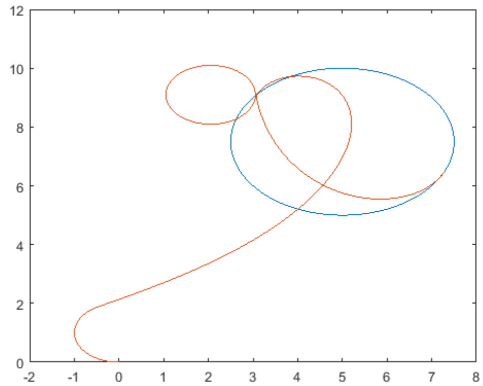
   rocket.m:

```
function [dP,halt,direction] = rocket(t,P)

global DMIN
    d = sqrt((P(4)-P(1))^2+(P(5)-P(2))^2);

    dP=d-DMIN;
```

```
        halt=1;
        direction=0;
    end
```

pursuit.m:

```
figure
global ST SP RT D DMIN

y0     = [0;0;-pi;5;5;0];
SP     = 3;
ST     = 2;
D      = 0.001;
RT     = 0.25;
DMIN   = 0.01;
tol    = 0.000001;

t0 = 0; tfinal = 20;

options=odeset('AbsTol',tol,'RelTol',tol,'MaxOrder',5,'Stats','o
options=odeset(options,'Events',@rocket,'Refine',4);

[t,y] = ode15s(@target,[t0,tfinal],y0,options);

plot (y(:,4),y(:,5));
hold on
plot (y(:,1),y(:,2));
```



the hitting time is 9.3174s

(b)

tol	Hitting Time	Number of Function Evaluations
10ˆ-3	9.1691	572
10ˆ-4	9.3174	719
10ˆ-5	9.3132	764
10ˆ-6	9.3126	908
10ˆ-7	9.3126	1124
10ˆ-8	9.3126	1550
10ˆ-9	9.3126	2057

As the Error Tolerance decreasing, the number of function evaluations increase and the Hitting Time becomes accurate

(c)

tol	Hitting Time	Number of Function Evaluations
10ˆ-4	9.3109	14215
10ˆ-5	9.3124	24391
10ˆ-6	9.3125	19441
10ˆ-7	9.3126	22459

By using ode45, the number of function evaluations greater than using ode15s, So in each step, the hitting time is more accurate than using ode15s