Marking Sheet Assignment 4 CS370 Winter 2018

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Section: 1 ✓ (8:30pm) 2 (11:30pm) 3 (1:30)

1. (a) Google matrix	(5) _________		
(b) PageRank algorithm	(10) _________		
(c) verification	(5) _________		
			......../20
2. positive Markov matrix			......../10
3. quadratic solve algorithm			......../10
4. PA = LU factorization	(5) _________		
solution	(5) _________		......../10
5. (a) recurrence equations	(10) _________		
(b) cost	(5) _________		
(c) cost	(5) _________		
			......../20
6. (a) PageRank function	(10) _________		
(b) small web	(5) _________		
(b) math_uwaterloo.mat	(8) _________		
(d) math_uwaterloo.mat	(7) _________		
			......../30

Total: (100) _____________

CS370

Assignment 4

Yandong Zhu

20588720

1. (a)

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 1 \\ 0 & \frac{1}{2} & 1 & \frac{1}{2} & 0 & 0 & 0 \end{bmatrix}$$

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$Q = P + \frac{1}{7} e d^T$$

In this case we have $Q = P$

$$M = \alpha Q + \frac{1-\alpha}{7} e e^T$$

$$M = \begin{bmatrix} \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-6\alpha}{7} & \frac{1-\alpha}{7} \\ \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{2-5\alpha}{14} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} \\ \frac{2-5\alpha}{14} & \frac{2-5\alpha}{14} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} \\ \frac{2-5\alpha}{14} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} \\ \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{2-5\alpha}{14} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} \\ \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{2-5\alpha}{14} & \frac{1-\alpha}{7} & \frac{1-6\alpha}{7} \\ \frac{1-\alpha}{7} & \frac{2-5\alpha}{14} & \frac{1-6\alpha}{7} & \frac{2-5\alpha}{14} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} & \frac{1-\alpha}{7} \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0.2147 & 0.0505 & 0.1326 & 0.1108 & 0.068 & 0.2223 & 0.2011 \end{bmatrix}^T$$

(c) When we do 30 iterations we get

$$\vec{x} = \begin{bmatrix} 0.2110 & 0.0506 & 0.1326 & 0.1111 & 0.0686 & 0.2232 & 0.2028 \end{bmatrix}^T$$

$$M \cdot \vec{x} = \begin{bmatrix} 0.2112 & 0.0506 & 0.1326 & 0.1111 & 0.0686 & 0.2230 & 0.2028 \end{bmatrix}^T$$

2.

$$P_{ij} = \begin{cases} \frac{1}{deg(j)} & \exists j \to i \\ 0 & otherwise \end{cases}$$

thus

$$\sum iP_{ij} = \begin{cases} \sum i \frac{1}{deg(j)} = deg(j) \cdot \frac{1}{deg(j)} = 1 & deg(j) \neq 0 \\ 0 & deg(j) = 0 \end{cases}$$

$$0 \leq P_{ij} \leq 1$$

$$\frac{1}{R} ed^T = S$$

$$S_{ij} = \begin{cases} \frac{1}{R} & deg(j) = 0 \\ 0 & otherwise \end{cases} (R \geq 1)$$

$$P + S = Q$$

$$\sum iQ_{ij} = \sum iP_{ij} + \sum iS_{ij} = \begin{cases} 1 + 0 & deg(j) \neq 0 \\ 0 + 1 & deg(j) = 0 \end{cases} = 1$$

$$(1 - \alpha)\frac{1}{R} ee^T = X$$

$$X_{ij} = \frac{1}{R} - \frac{1}{R}\alpha$$

$$\sum iX_{ij} = R \cdot (\frac{1}{R} - \frac{1}{R}\alpha) = 1 - \alpha$$

$$\begin{aligned} \alpha \sum iQ_{ij} + \sum iX_{ij} &= \alpha \cdot 1 + (1 - \alpha) \\ &= \alpha + 1 - \alpha \\ &= 1 \end{aligned}$$

3.

$$\begin{aligned} A &= LU \\ A^T &= (LU)^T \\ &= U^T L^T \end{aligned}$$

$$\begin{aligned} A \cdot A^T \vec{x} &= \vec{b} \\ A \cdot U^T \cdot L^T \vec{x} &= \vec{b} \\ L \cdot U \cdot U^T \cdot L^T \vec{x} &= \vec{b} \end{aligned}$$

Since we can compute
1. compute $L \cdot U$ in $O(n^2)$
2. compute $L \cdot U \cdot U^T$ in $O(n^2)$
3. compute $L \cdot U \cdot U^T \cdot L^T$ in $O(n^2)$
4. compute $L \cdot U \cdot U^T \cdot L^T \vec{x} = \vec{b}$ in $O(n^2)$

4.

$$\begin{bmatrix} 2 & 2 & 2 \\ -4 & 2 & 6 \\ 2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 2 \\ -4 & 2 & 6 \\ 2 & -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 2 & 6 \\ 2 & 2 & 2 \\ 2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 & 6 \\ 2 & 2 & 2 \\ 2 & -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 2 & 6 \\ 2 & 2 & 2 \\ 2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 & 6 \\ 0 & 1 & 5 \\ 2 & -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 2 & 6 \\ 2 & 2 & 2 \\ 2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 & 6 \\ 0 & 1 & 5 \\ 0 & -2 & 7 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 2 & 6 \\ 2 & -1 & 4 \\ 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 & 6 \\ 0 & -2 & 7 \\ 0 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 2 & 6 \\ 2 & -1 & 4 \\ 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 & 6 \\ 0 & -2 & 7 \\ 0 & 0 & 8.5 \end{bmatrix}$$

So we got

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} U = \begin{bmatrix} -4 & 2 & 6 \\ 0 & -2 & 7 \\ 0 & 0 & 8.5 \end{bmatrix}$$

then we can calculate $\vec{y}$ by $L\vec{y} = \vec{b}$

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} \cdot \vec{y} = \begin{bmatrix} 4 \\ 7 \\ 2 \end{bmatrix}$$

$$\vec{y} = \begin{bmatrix} 4 \\ 9 \\ 8.5 \end{bmatrix}$$

then we can calculate $\vec{x}$ by $U\vec{x} = \vec{y}$

$$\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

5. (a) by calculate we got

$$l_i = \frac{1}{d_{i-1}}$$

$$d_i = \begin{cases} 2 & i = 1 \\ 4 - l_i & i \in [2, n-1] \\ 2 - l_i & i = n \end{cases}$$

   (b) from the previous question we know that we can compute d_i by l_i and we can compute l_i by d_{i-1}
   And we are going to find $A = LU$, so we need to find d_n and l_n. Thus, the cost is $O(n)$ since we have n previous terms while each term takes constant time to compute

   (c) first we compute $\vec{y}$

$$\begin{aligned} y_1 &= b_1 \\ y_2 &= b_2 - l_2 y_1 \\ y_3 &= b_3 - l_3 y_2 \\ &\vdots \\ y_n &= b_n - l_n y_{n-1} \end{aligned}$$

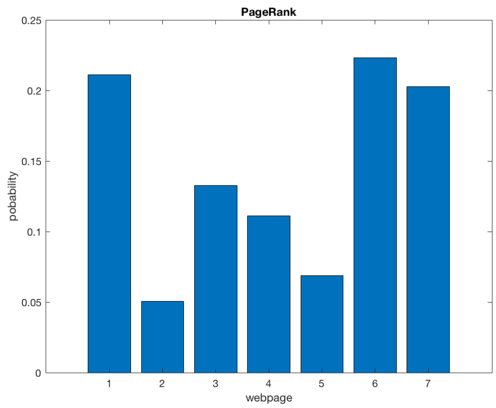
   Then we can compute $\vec{x}$

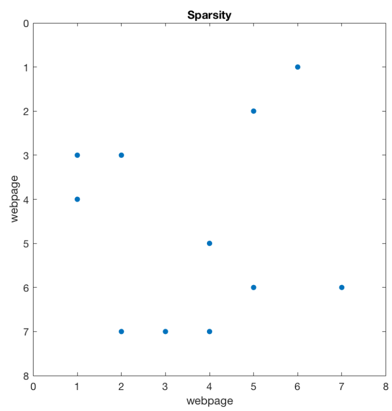
$$\begin{aligned} x_n &= \frac{y_n}{d_n} \\ x_{n-1} &= \frac{y_{n-1} - x_n}{d_{n-1}} \\ x_{n-2} &= \frac{y_{n-2} - x_{n-1}}{d_{n-1}} \\ &\vdots \\ x_1 &= \frac{y_1 - x_2}{d_1} \end{aligned}$$

   we can compute these two steps by O(n), So

$$T(n) = O(n) + O(n) + O(n) = O(n)$$

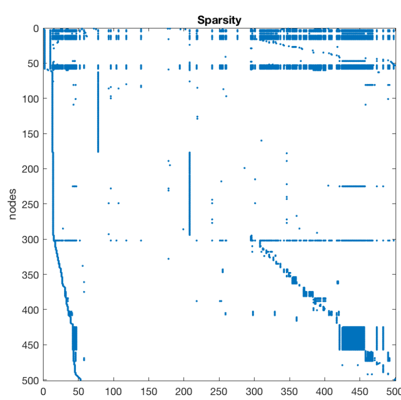
6. (a)
   (b)





the order of importance is $6 > 1 > 7 > 3 > 4 > 5 > 2$

(c)



(d) the iteration times are
7 12 20 41 225

when α grows, the iteration time grows the reason is α stands for the probability of a user starts a new webpage which is not connected to the previous one. So when the α grows, it is more possible to stay in the current cycle, which will cause more time one iterating in the same loop and increase the iteration times.