1

we can set

$s = 52$

$T = 3/12$

$k = 50$

$r = np.log(100/99.005)/3 \times 12$

$q = 0$

```
def bs(K, S, sigma, r, q, T):
    d1 = (np.log(S/K)+(r-q+sigma**2/2)*T)/(sigma*T**0.5)
    d2 = d1- sigma*T**0.5
    return(norm.cdf(d1)*S-norm.cdf(d2)*K*np.exp(-r*T)),norm.cdf(d1)

call,delta = bs(k,s,sigma,r,q,T)
print(call,delta)
```

use black-scholes formula to calculate the price of the call, equal to 3.693

and $\delta = 0.6923 = N(d1)$

Then we calculate the difference between the stock and the call, then borrow the difference

$\delta \times s - call = 32.307$

after one week, for rebalancing:

we updates the sets and

$s = 53.5$

$T = 3/12 - 1/52$

recalculate the delta and call to buy and borrow the difference:

$\delta 2 = 0.7824$ and $call2 = 4.7$

we need to buy 0.0900 more shares and borrow $37.1586 - 32.3073 = 4.8513$ more bonds.

The payoff for the first week would be

$(s2 - s) \times \delta - bonds \times (\exp(r \times \frac{1}{52}) - 1) + call \times 1.1 = 5.0767$

after another week, repeat again for rebalancing:

$s3 = 51.125$

$T = 3/12 - 1/52 \times 2$

we have $call3 = 2.897$ and $\delta 3 = 0.6383$

and the payoff will be $(s3 - s2) \times \delta_2 - bond_2 \times (\exp(r \times \frac{1}{52}) - 1) = -1.8868$

Thus, total payoff would be $5.0767 - 1.8868 = 3.18989$

2

2. In the money state 5 periods:

∴ we got $100 \times 1.05^5 - 120 = 7.63$

use risk neutral probability: $p^* = \frac{1+r-d}{u-d} = 0.625$

$\therefore 1-p^* = 0.375$

$c = \frac{1}{(1+r)^4} \times p^{*4} \times 7.63 = 1.075$

we use replicating portfolio to calculate:

$$\begin{cases} 105\Delta + 1.02B = 1.075 \\ 97\Delta + 1.02B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \Delta = 0.1344 \\ B = -12.783 \end{cases}$$

Cost < price

$\Rightarrow$ Bank can keep a margin profit with hedge