Homework 8

1 use Merton model

we can assume the
equity is a long call option on V with strike F and
debt is a short put on V and long risk free F

```
d1 = (np.log(v0/F) + (r + 0.5 * sigma ** 2) * T)/(sigma * T ** 0.5)
d2 = d1 - sigma * T ** 0.5
D0 = np.exp(-r * T) * F * norm.cdf(d2) + v0 * norm.cdf(-d1);D0
E0 = v0 * norm.cdf(d1) - np.exp(- r * T)* F * norm.cdf(d2);E0
```

use above parameter and black schole model calculate the

$D0 = 70.8797$ and $E0 = 29.1202$

2 Calculate the vega

take derivative and calculate the $vega = \frac{\delta C}{\delta \sigma} = \frac{\delta p}{\delta \sigma}$
vega of equity = vega of call option = 24.744357919569442
vega of debt= vega of F - vega of put option = -24.744357919569442
together the vega = -1033.3131
Since the vega is negative at this point, the minize the sigma will increase the value of assets and the value of the company. Thus, it is very essential to exercise the risk management.

2

(a) we can see the debt as

$D(T)$ = long call at K and short call at F and long K call binary at F
=$max(A-k,0) - max(A-F,0) + K \times 1_{A<F}$
The bankruptcy has nothing to do with equity holders in theory, thus the cost will not affect them anymore.

```
k = 30
d1_k =  (np.log(v0/k) + (r + 0.5 * sigma ** 2) * T)/(sigma * T ** 0.5)
d2_k = d1_k - sigma * T ** 0.5
d1_f = (np.log(v0/F) + (r + 0.5 * sigma ** 2) * T)/(sigma * T ** 0.5)
d2_f = d1 - sigma * T ** 0.5
```

```
# call at K use d1_k and d2_k
call_k = v0 * norm.cdf(d1_k) - np.exp(- r * T)* k * norm.cdf(d2_k)
# call at F use d1_f and d2_f
call_f = v0 * norm.cdf(d1_f) - np.exp(- r * T)* F * norm.cdf(d2_f)
# call binary at F use d2_f
call_binary = k * np.exp(- r * T)* norm.cdf(d2_f)
```

D0 = call_k - call_f + call_binary;

use above parameter and black schole model calculate the $D0 = 66.580045$ and $E0 = 29.1202$

(b) Calculate the vega of the debt first

since D0 = long call at K and short call at F and long K call binary at F

vega of the debt = vega of call at K - vega of call at F + vega of call binary at F

```
# vega of call at K use d1_k and d2_k
v_k = v0 * norm.pdf(d1_k) * T **0.5
# vega of call at F use d1_f and d2_f
v_f = v0 * norm.pdf(d1_f) * T **0.5
# vega of call binary at F use d2_f
v_binary = k *np.exp(- r * T)* (norm.pdf(d2_f) * d2_f_prime)
```

vd = v_k - v_f + v_binary = -27.82150
vega of equity = vega of call option = 24.744357919569442
vega of debt= vega of call at K - vega of call at F + vega of call binary at F = -27.82150
together the vega = -1131.79571
Interesting to notice that the vega is larger