$D: \{(x_1, y_1), (x_2, y_2), \cdots (x_n, y_n)\}$

$y: w^T x + e$

(1) $w \sim N(0, b^{-1} I)$

(2) $p(D|w) = N(D | \phi w, \sigma^2)$

$e \sim N(0, a)$

$$p(w|D) = \frac{p(D|w) \cdot p(w)}{p(D)}$$

$$\propto e^{\frac{-1}{2a}(\phi w - y)^T(\phi w - y)} \cdot e^{\frac{-b}{2}(w-0)^T(w-0)}$$

取 log

$$\propto \frac{-1}{2a}(\phi w - y)^T(\phi w - y) + \frac{-b}{2} w^T w$$

係數會 normaliz 掉

$(w^T\phi^T y)^T = y^T \phi w$ and $w^T \phi y$ = value

$$\Rightarrow \frac{1}{a}[w^T\phi^T\phi w - w^T\phi^T y - y^T\phi w + y^T y] + b w^T w$$

$$= \frac{1}{a}[w^T\phi^T\phi w - 2w^T\phi^T y + y^T y] + b w^T w$$

$$= w^T\left(\frac{1}{a}\phi^T\phi + b\right) w - 2\frac{1}{a} w^T \phi^T y + \frac{1}{a} y^T y$$

$\Longleftrightarrow$ 對應 multivariate Guassian 的形式

$$(w-u)^T \Lambda (w-u) = w^T \Lambda w - 2 w^T \Lambda u + u^T u$$

因此 $\Lambda = \frac{1}{a}\phi^T\phi + b$

$u = \frac{1}{a}\Lambda^{-1}\phi^T y$

$\Rightarrow p(w|D) \sim N(u, \Lambda^{-1})$

如果要跑 online learning 把上面的 $b \Rightarrow \Lambda$, $0 \Rightarrow u$ 再推一次

得到 $\Lambda' = \frac{1}{a}\phi^T\phi + \Lambda$

$u' = \Lambda'^{-1}\left(\frac{1}{a}\phi^T y + \Lambda u\right)$