312554021 鄭丞洋 Hw4

$X: \{HHH, HHT, TTT\}$

$Z: \{\ ? \quad\quad\quad\quad \} \Rightarrow w$

may be C_0 or C_1

$k = 0.5 \Rightarrow$ the chance of choosing C_0

$P_0 = 0.6 \Rightarrow$ the chance of C_0 showing H

$P_1 = 0.1 \Rightarrow$ the chance of C_1 showing H

E step:

計算 w_i

係數省略，因為會抵消

$P(z_i=C_0, x_i=HHH \mid \theta) = 0.5 \times 0.6^3 = 0.108$

$P(z_i=C_1, x_i=HHH \mid \theta) = (1-0.5) \times 0.1^3 = 0.0005$

出現 HHH 時，是 C_0 的機率為

$\frac{0.108}{0.108+0.0005} = 0.995$

$P(z_i=C_0, x_i=HHT \mid \theta) = 0.5 \times 0.6^2 \times 0.4 = 0.072$

$P(z_i=C_1, x_i=HHT \mid \theta) = (1-0.5) \times 0.1^2 \times 0.9 = 0.0045$

$P(z_i=C_0, x_i=HTT \mid \theta) = 0.5 \times 0.6 \times 0.4^2 = 0.048$

$P(z_i=C_1, x_i=HTT \mid \theta) = (1-0.5) \times 0.1 \times 0.9^2 = 0.0405$

$P(z_i=C_0, x_i=TTT \mid \theta) = 0.5 \times 0.4^3 = 0.032$

$P(z_i=C_1, x_i=TTT \mid \theta) = (1-0.5) \times 0.9^3 = 0.3645$

HHT 時，是 C_0 ....

$\frac{0.072}{0.072+0.0045} = 0.94$

HTT ... C_0 ...

$\frac{0.048}{0.048+0.0405} = 0.54$

TTT .... C_0 ....

$\frac{0.032}{0.032+0.3645} = 0.08$

M step:

先列出式子: $\prod_{i=1}^{n} k\left[\binom{3}{x_i} p_0^{x_i}\cdot(1-p_0)^{(3-x_i)}\right]^{w_i}\cdot(1-k)\left[\binom{3}{x_i} p_1^{x_i}\cdot(1-p_1)^{(3-x_i)}\right]^{(1-w_i)}$

H的次數

取 log

$$J=\sum_{i=1}^{n} w_i\left[\log k+\log\binom{3}{x_i}+x_i\log p_0+(3-x_i)\log(1-p_0)\right]+$$
$$\sum_{i=1}^{n}(1-w_i)\left[\log(1-k)+\log\binom{3}{x_i}+x_i\log p_1+(3-x_i)\log(1-p_1)\right]$$

求 k 的 MLE:

$$\frac{\partial J}{\partial k}=0 \Rightarrow \sum_{i=1}^{n} w_i\cdot\frac{1}{k}-\sum_{i=1}^{n}(1-w_i)\cdot\frac{1}{1-k}=0$$
$$\Rightarrow (1-k)\cdot\sum_{i=1}^{n} w_i = k\sum_{i=1}^{n}(1-w_i)$$
$$\Rightarrow \sum_{i=1}^{n} w_i-\sum_{i=1}^{n} k w_i = kn-\sum_{i=1}^{n} k w_i$$
$$\Rightarrow k=\frac{\sum w_i x_i}{n}$$

代入 E step 的結果為 $k'=\frac{0.995+0.94+0.08}{3}=0.67$

求 p_0 的 MLE:

$$\frac{\partial J}{\partial p_0}=0 \Rightarrow \sum_{i=1}^{n} w_i\left[x_i\cdot\frac{1}{p_0}-(3-x_i)\cdot\frac{1}{1-p_0}\right]=0$$
$$\Rightarrow (1-p_0)\sum_{i=1}^{n} w_i x_i = p_0\cdot\sum_{i=1}^{n} 3w_i-w_i x_i$$
$$\Rightarrow \sum_{i=1}^{n} w_i x_i = p_0\cdot\sum_{i=1}^{n} 3w_i$$
$$\Rightarrow p_0=\frac{\sum w_i x_i}{3\sum w_i}$$

代入 E step 的結果為 $p_0' : \dfrac{0.995 \times 3 + 0.94 \times 2}{3 \times 2.015} = \underline{0.8}$

以此類推，求 p_1 的 MLE

$$p_1 = \frac{\sum (1-w_i) x_i}{3 \sum (1-w_i)}$$

代入 E step 的結果為 $p_1' = \dfrac{0.005 \times 3 + 0.06 \times 2}{3 \times 0.005 + 3 \times 0.06 + 3 \times 0.92} = \dfrac{0.135}{2.955} = \underline{0.04}$