보충 자료

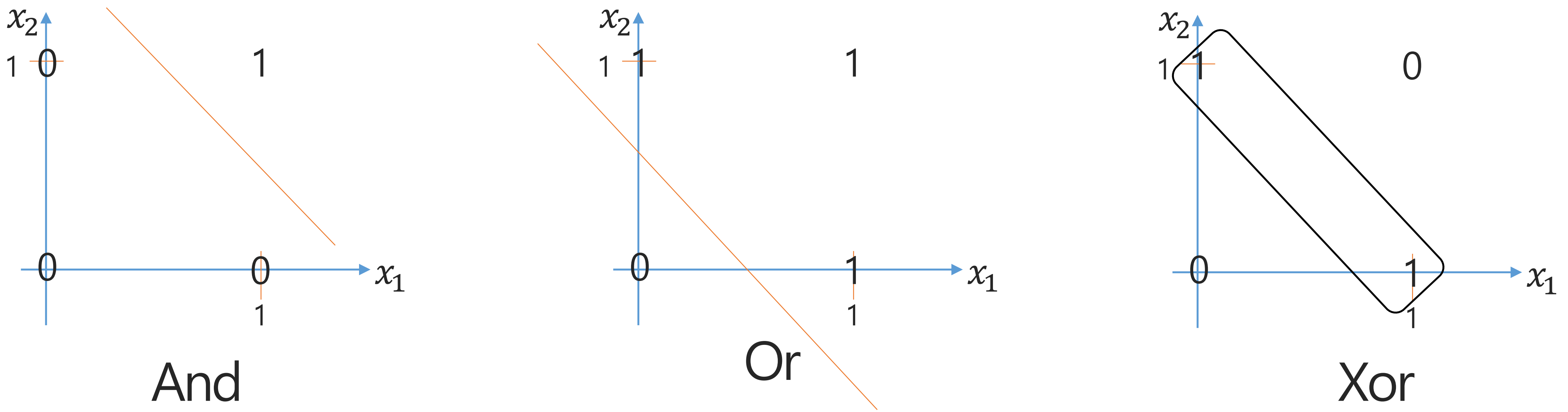
And, Or
증명

And		
X1	X2	Y
0	0	0
0	1	0
1	0	0
1	1	1

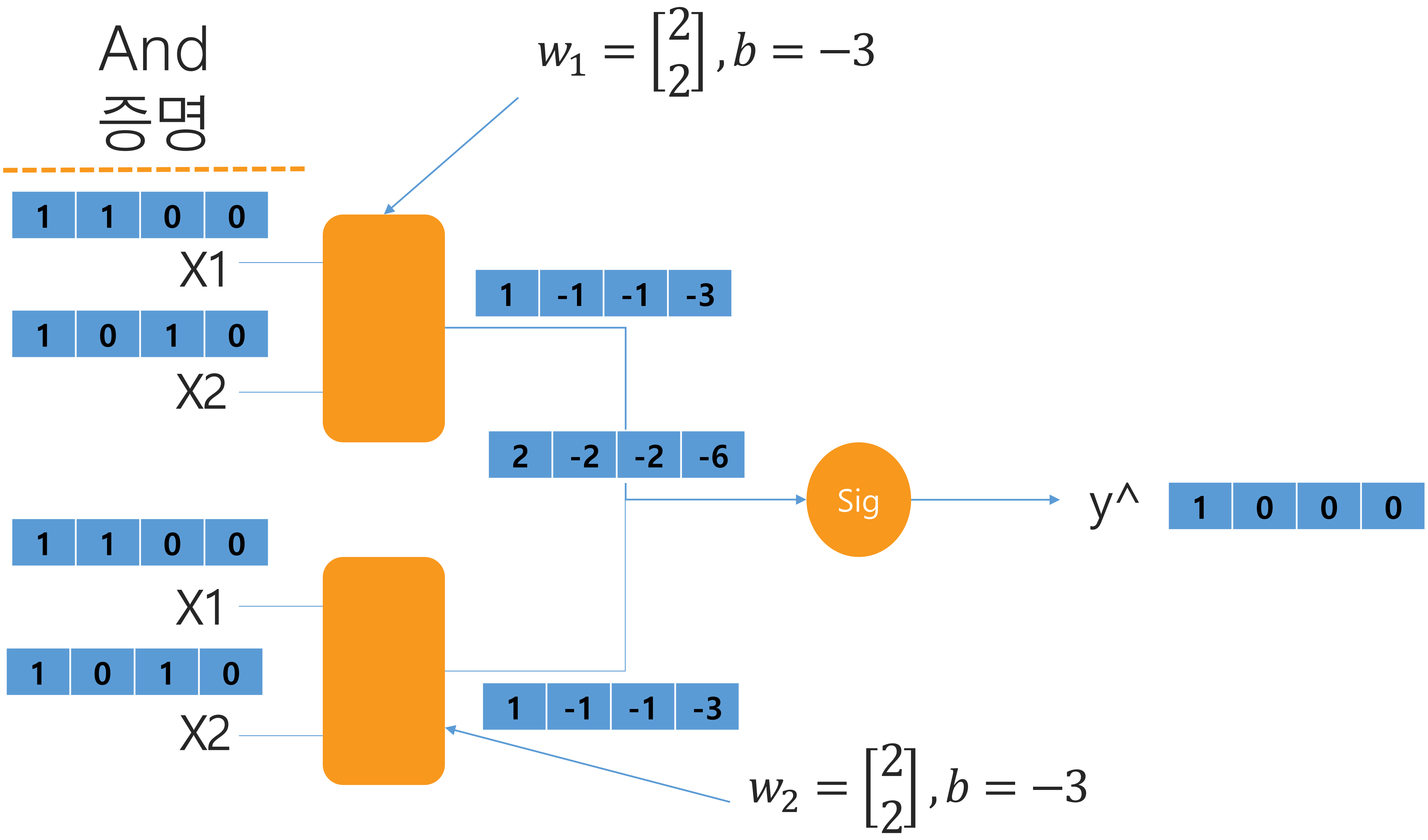
And		
X1	X2	Y
0	0	0
0	1	1
1	0	1
1	1	1

And		
X1	X2	Y
0	0	0
0	1	1
1	0	1
1	1	0

And, Or 증명

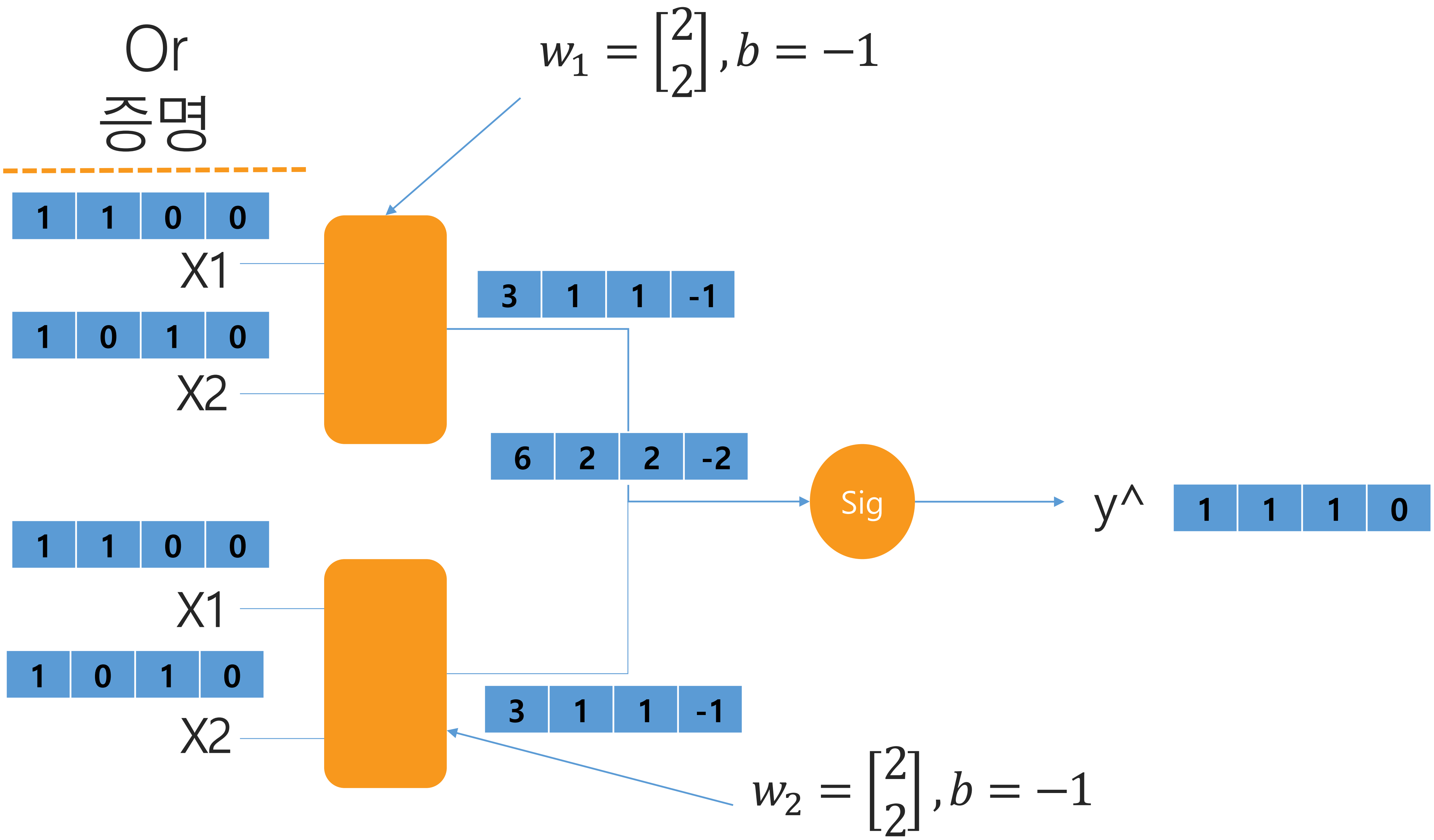


And 증명



$w_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, b = -3$
1 1 0 0
X1
1 0 1 0
X2
1 -1 -1 -3
2 -2 -2 -6
Sig
y^
1 0 0 0
1 1 0 0
X1
1 0 1 0
X2
1 -1 -1 -3
$w_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, b = -3$

Or
증명



$w_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, b = -1$
1 1 0 0
X1
1 0 1 0
X2
3 1 1 -1
6 2 2 -2
Sig
y^
1 1 1 0
1 1 0 0
X1
1 0 1 0
X2
3 1 1 -1
$w_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, b = -1$

Linear

Xor

X1, X2 => 0 or 1 일 때, 0

[0, 0][w1+w2]+b`<0.5 => b`<0.5 ... ***

[1, 1][w1+w2]+b`<0.5 => $[1,1]\begin{bmatrix} w1` \\ w2` \end{bmatrix}$+b`<0.5

=> w1`+w2`< 0.5 - b ... *

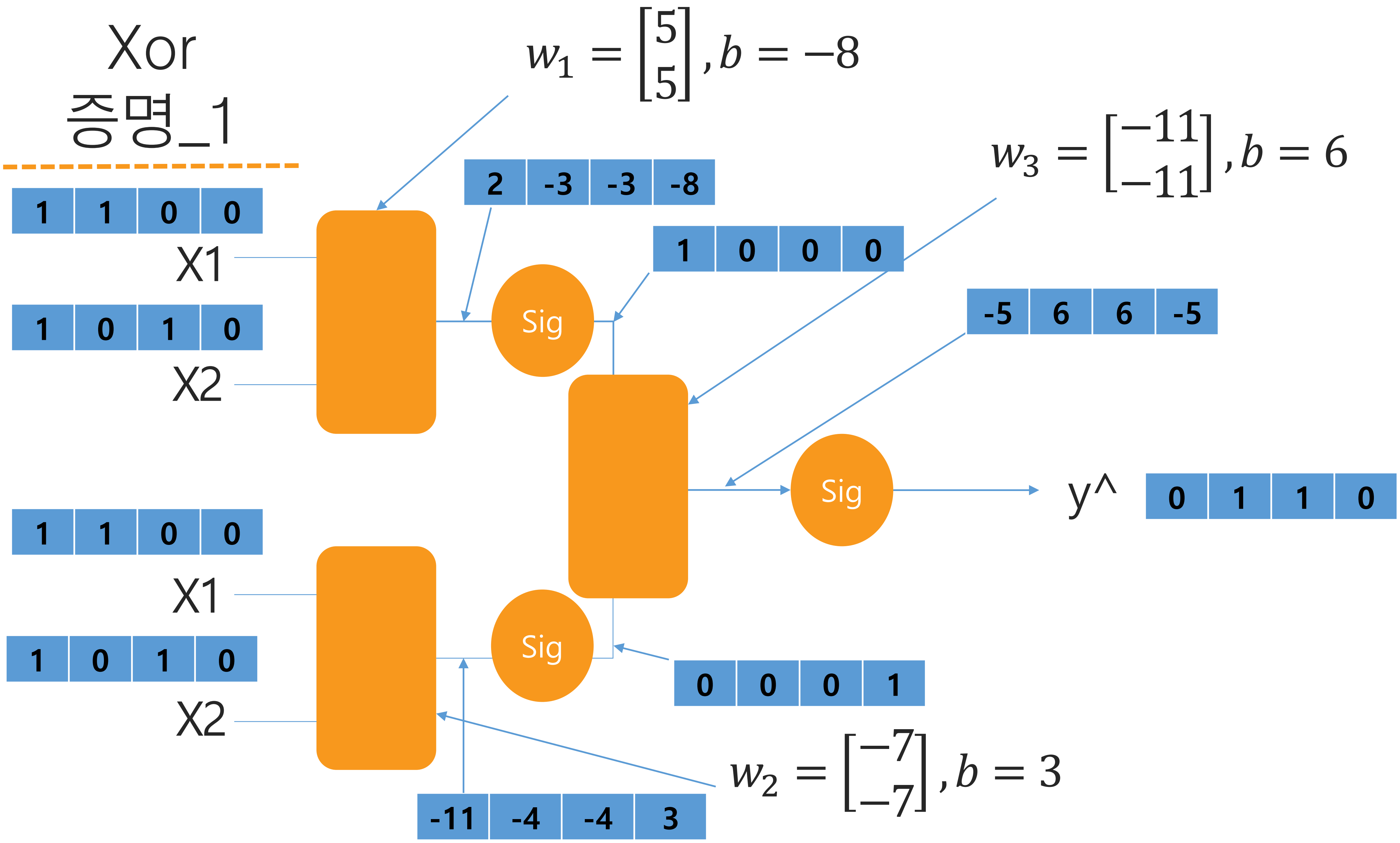
[w1+w2] => $\begin{bmatrix} w1` \\ w2` \end{bmatrix}$

[1, 0]
[0, 1] [w1+w2]+ b` > 0.5 => w1`+w2` > 1-2b ... **

*, ** => 2(0.5 - b) < w1`+w2`< 0.5 - b

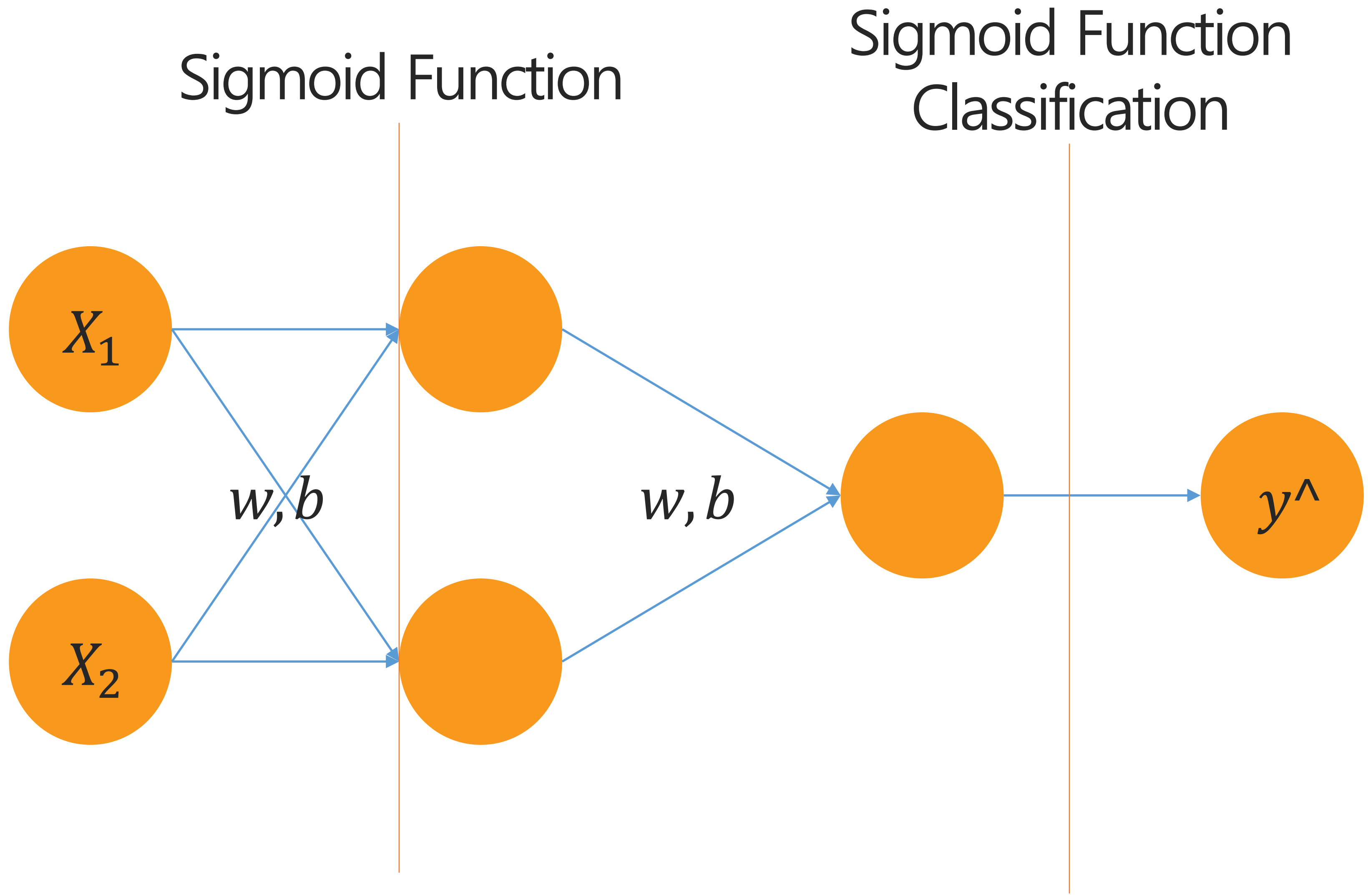
*** => 0.5 – b => 양수

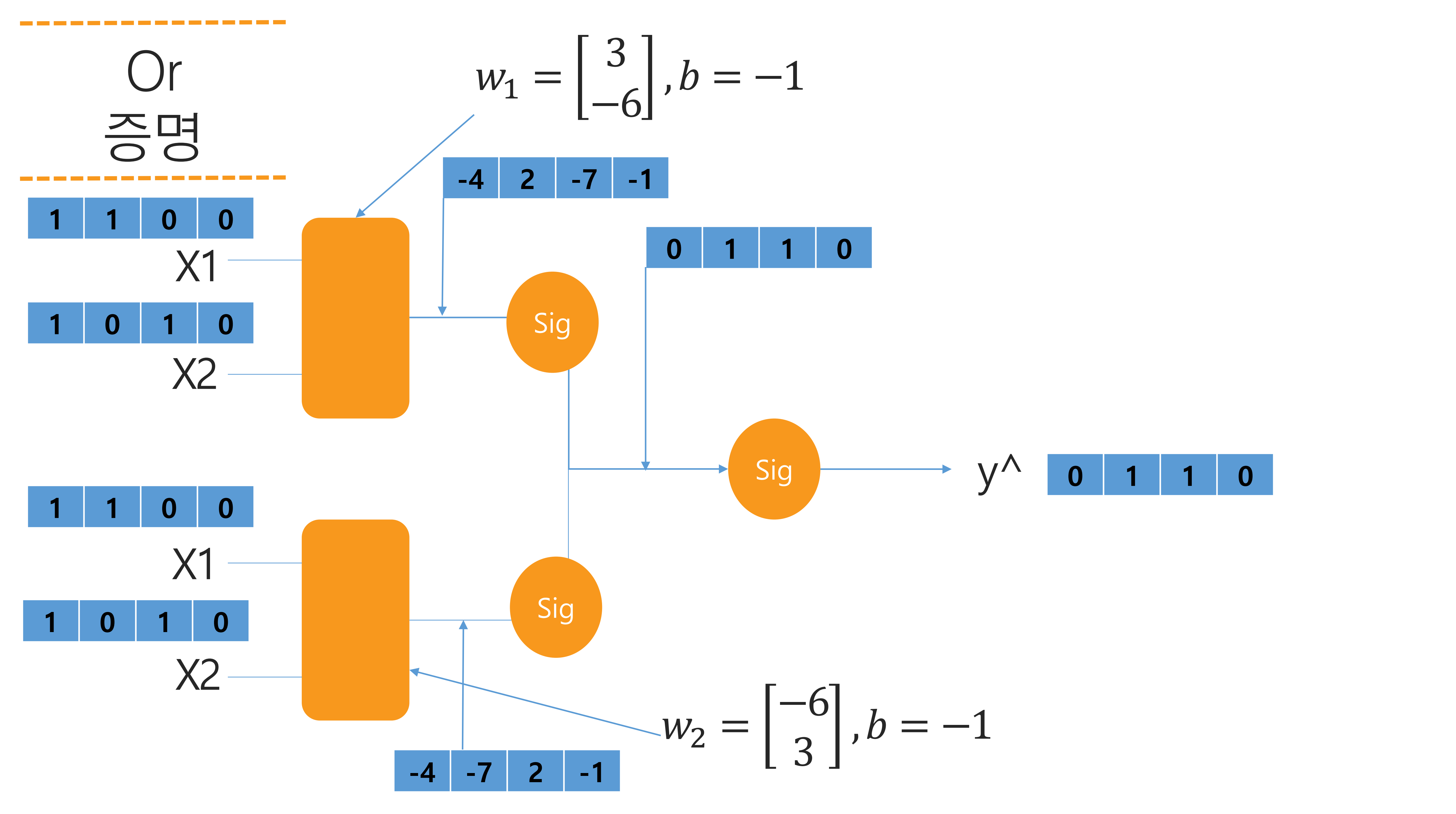
Xor 증명_1



$w_1 = \begin{bmatrix} 5 \\ 5 \end{bmatrix}, b = -8$
$w_3 = \begin{bmatrix} -11 \\ -11 \end{bmatrix}, b = 6$
$w_2 = \begin{bmatrix} -7 \\ -7 \end{bmatrix}, b = 3$
1 1 0 0
X1
1 0 1 0
X2
2 -3 -3 -8
Sig
1 0 0 0
-5 6 6 -5
Sig
y^
0 1 1 0
1 1 0 0
X1
1 0 1 0
X2
Sig
0 0 0 1
-11 -4 -4 3

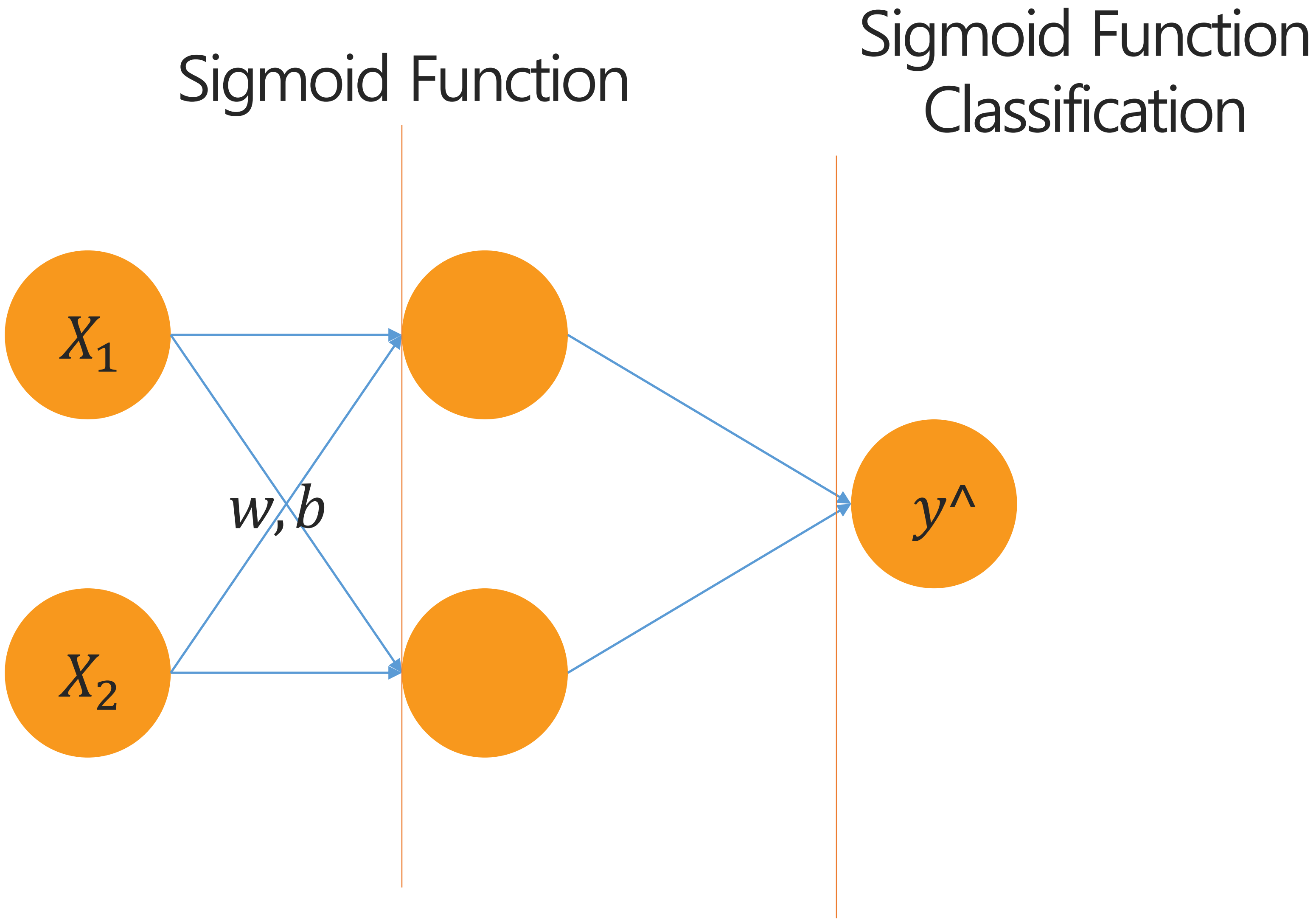
Xor
증명_1


Sigmoid Function
Sigmoid Function
Classification
X_1
X_2
w, b
w, b
y^



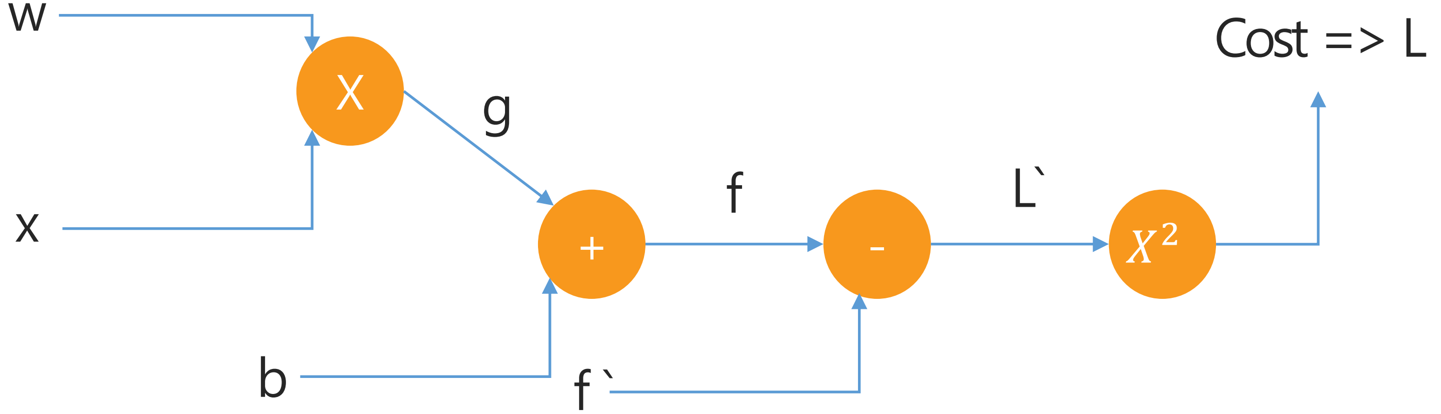
Or
증명
$w_1 = \begin{bmatrix} 3 \\ -6 \end{bmatrix}, b = -1$
-4 2 -7 -1
1 1 0 0
X1
1 0 1 0
X2
Sig
0 1 1 0
Sig
y^
0 1 1 0
1 1 0 0
X1
1 0 1 0
X2
Sig
$w_2 = \begin{bmatrix} -6 \\ 3 \end{bmatrix}, b = -1$
-4 -7 2 -1

Xor
증명_1


Sigmoid Function
Sigmoid Function
Classification
X_1
X_2
w, b
y^

Chain Rule

1. g = x * w
2. f = g + b
3. f – f ` = L`
4. $L`^2 = L$



1. $\frac{\partial g}{\partial w} = x$
2. $\frac{\partial f}{\partial g} = 1, \ \frac{\partial f}{\partial b} = 1$
3. $\frac{\partial L`}{\partial f} = 1, \ \frac{\partial L`}{\partial f`} = 1$
4. $\frac{\partial L}{\partial L`} = 2L`$

$\frac{\partial g}{\partial w}$ x $\frac{\partial f}{\partial g}$ x $\frac{\partial L`}{\partial f}$ x $\frac{\partial L}{\partial L`}$ = x*2L` = 2x(f – f`) = 2x(H(x)-y)

$$Cost \Rightarrow 2x(wx - y)$$

Sigmoid

$$\text{Sigmoid(x)} => \frac{d}{dx}(1+e^{-x})^{-1}$$

$$= (-1)\frac{1}{(1+e^{-x})^2}(1+e^{-x})$$

$$= (-1)\frac{1}{(1+e^{-x})^2}(0+e^{-x})\frac{d}{dx}(-x)$$

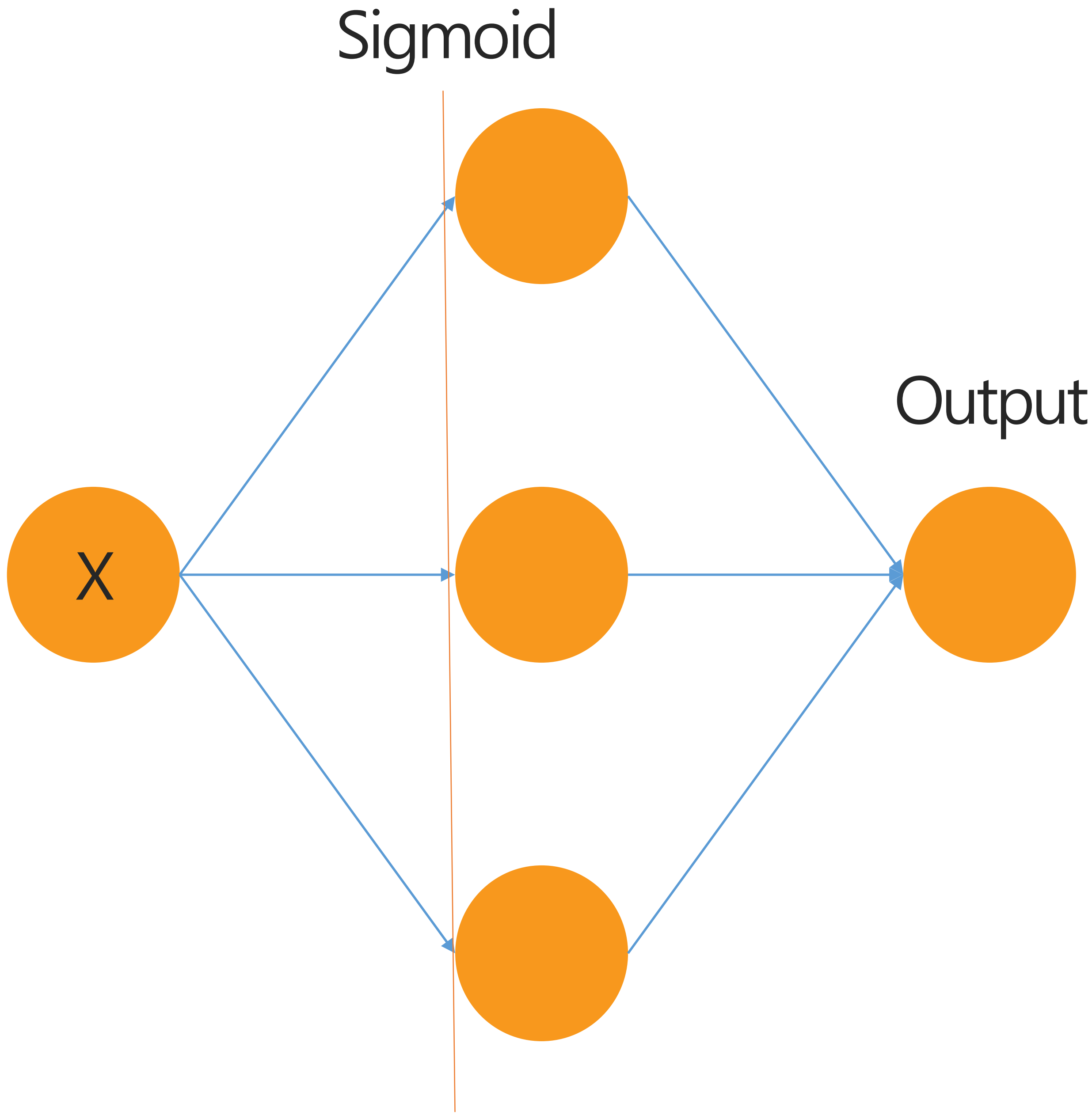
$$= (-1)\frac{1}{(1+e^{-x})^2}(e^{-x})(-1)$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1+e^{-x}-1}{(1+e^{-x})^2} = \frac{1+e^{-x}}{(1+e^{-x})^2} - \frac{1}{(1+e^{-x})^2}$$

$$= \frac{1+e^{-x}}{(1+e^{-x})^2} - \frac{1}{(1+e^{-x})^2} = \frac{1}{1+e^{-x}}(1-\frac{1}{1+e^{-x}})$$

$$\Rightarrow Sig(x)(1-Sig(x))$$

Xor
증명_1



Chain Rule

$$\frac{\partial g'}{\partial w'} = x, \frac{\partial g'}{\partial x} = w'$$

$$\frac{\partial g''}{\partial w''} = x, \frac{\partial g''}{\partial x} = w''$$

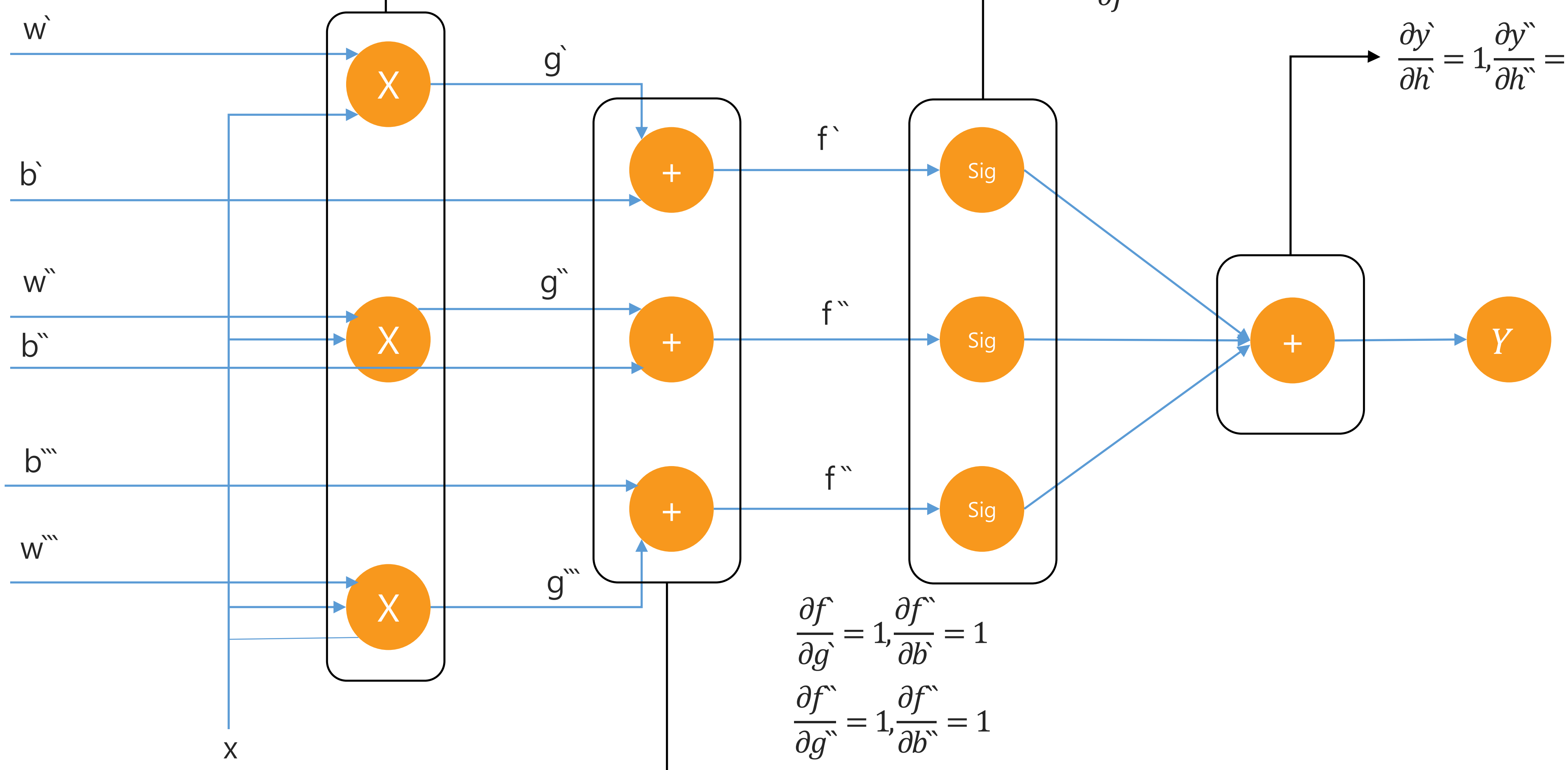
$$\frac{\partial g'''}{\partial w'''} = x, \frac{\partial g'''}{\partial x} = w'''$$

$$\frac{\partial h'}{\partial f'} = Sigmoid(f')(1 - Sigmoid(f'))$$

$$\frac{\partial h''}{\partial f''} = Sigmoid(f'')(1 - Sigmoid(f''))$$

$$\frac{\partial h'''}{\partial f'''} = Sigmoid(f''')(1 - Sigmoid(f'''))$$

$$\frac{\partial y'}{\partial h'} = 1, \frac{\partial y''}{\partial h''} = 1, \frac{\partial y'''}{\partial h'''} = 1,$$



$$\frac{\partial f'}{\partial g'} = 1, \frac{\partial f'}{\partial b'} = 1$$

$$\frac{\partial f''}{\partial g''} = 1, \frac{\partial f''}{\partial b''} = 1$$

$$\frac{\partial f'''}{\partial g'''} = 1, \frac{\partial f'''}{\partial b'''} = 1$$

Cost

$$W := W - \alpha \frac{\partial}{\partial W} Cost(w)$$

$$W := W - \alpha \frac{\partial}{\partial W} \frac{1}{m} \sum (H(x) - y)^2$$

$$W := W - \alpha \frac{1}{m} \sum 2(H(x) - y) H`(x)$$

W = w`+w``+w```

$$H`(x) = \frac{\partial y}{\partial w`} + \frac{\partial y}{\partial w``} + \frac{\partial y}{\partial w```}$$

$$\frac{\partial y}{\partial h`} \times \frac{\partial h`}{\partial f`} \times \frac{\partial f`}{\partial g`} \times \frac{\partial g`}{\partial w`} = \frac{\partial y}{\partial w`} = xSig(f`)(1 - Sig(f`))$$

$$\frac{\partial y}{\partial h`} \times \frac{\partial h`}{\partial f`} \times \frac{\partial f`}{\partial g`} \times \frac{\partial g`}{\partial w``} = \frac{\partial y}{\partial w``} = xSig(f``)(1 - Sig(f``))$$

$$\frac{\partial y}{\partial h`} \times \frac{\partial h`}{\partial f`} \times \frac{\partial f`}{\partial g`} \times \frac{\partial g`}{\partial w```} = \frac{\partial y}{\partial w```} = xSig(f```)(1 - Sig(f```))$$

Thank You