

# EEC269A - Error Correcting Codes I

## Project Report

Chenyue Yang, Pranav Kharche, Parisa Oftadeh

June 5, 2023

## Contents

<b>1 Workload</b>	<b>2</b>
<b>2 Source &amp; Destination</b>	<b>2</b>
2.1 Source . . . . .	2
2.1.1 Text string . . . . .	2
2.1.2 Image . . . . .	2
2.1.3 Audio . . . . .	3
2.2 Destination . . . . .	4
<b>3 Channel</b>	<b>4</b>
3.1 Binary Symmetric Channel (BSC) . . . . .	4
3.2 Additive White Gaussian Noise (AWGN) Channel . . . . .	4
<b>4 (7,4) Systematic Linear Block (Hamming) Code</b>	<b>4</b>
4.1 Syndrome Decoder . . . . .	4
4.1.1 Text string . . . . .	4
4.1.2 Image . . . . .	4
4.1.3 Audio . . . . .	4
<b>5 (<math>n, k</math>) Systematic Cyclic (Hamming) Code</b>	<b>4</b>
5.1 Syndrome Decoder . . . . .	4
5.1.1 Text string . . . . .	4
5.1.2 Image . . . . .	4
5.1.3 Audio . . . . .	4
5.2 LFSR Decoder . . . . .	4
<b>6 Appendix: Python source code</b>	<b>5</b>
6.1 Source . . . . .	5
6.2 Channel . . . . .	13
6.3 Destination . . . . .	18
6.4 Utilities . . . . .	21
6.5 Linear code . . . . .	26
6.6 Cyclic code . . . . .	29

# 1 Workload

Table 1: Workload

Function	Workload	Contributor
Source	Text string (TXT), Image (PNG), Audio (WAV)	Chenye
Encoder	(7, 4) Systematic Linear Block (Hamming) Code	Chenye
	( $n, k$ ) Systematic Cyclic (Hamming) Code	Pranav, Chenye
Channel	Binary Symmetric Channel (BSC), error probability $p$ adjustable	Chenye
Error Corrector	Syndrome Lookup Table for (7, 4) Linear Code	Chenye
	Syndrome Lookup Table for ( $n, k$ ) Cyclic Code	Chenye
	LFSR for ( $n, k$ ) Cyclic Code	TODO
Decoder	(7, 4) Systematic Linear Block (Hamming) Code	Chenye
	( $n, k$ ) Systematic Cyclic (Hamming) Code	Chenye
Destination	Text string (TXT), Image (PNG), Audio (WAV)	Chenye
Advanced	<b>Create generator matrix for (<math>n, k</math>) cyclic code</b>	Pranav
	<b>Adjustable (<math>n, k</math>)</b>	Pranav

## 2 Source & Destination

### 2.1 Source

#### 2.1.1 Text string

The very basic function of the information source is to read a hard-coded text file into a bit stream. In our text file, the following string is stored in ASCII format:

```
Hello World!
EEC269A Error Correcting Code Demo
```

In ASCII format, each character is represented by 8 bits, shown in Table 2. Then, after transformation, the bit stream is of size 376 bits:

```
0 1 0 0 1 0 0 0 0 1 1 0 0 1 0 1 ...
```

#### 2.1.2 Image

A PNG file is composed of an 8-byte signature header, followed by any number of chunks that contain control data / metadata / image data. Each chunk contains three standard fields: 4-byte length, 4-byte type code, 4-byte CRC and various internal fields that depend on the chunk type, shown in Figure 1. For example, the image file we are using has more than one image data (IDAT) chunks, each of which contains a portion of the image, shown in Table 3.

Ideally, the entire image file should be read into a bit stream and transmitted through the channel. However, if there are uncorrectable errors in the chunks other than the image data (IDAT), the received image will not be able to display. Therefore, we shall only work with the image data (IDAT) chunks for the purpose of visualization of a corrupted image.

Table 2: ASCII Table example

Character	Hexadecimal	Decimal	Binary
...	...	...	...
A	41	65	0 1 0 0 0 0 0 1
B	42	66	0 1 0 0 0 0 0 1 0
C	43	67	0 1 0 0 0 0 0 1 1
D	44	68	0 1 0 0 0 0 1 0 0
E	45	69	0 1 0 0 0 0 1 0 1
F	46	70	0 1 0 0 0 0 1 1 0
G	47	71	0 1 0 0 0 0 1 1 1
H	48	72	0 1 0 0 0 1 0 0 0
...	...	...	...

The information source is able to only extract the color information from a PNG file and convert it into a bit stream to be passed through the channel. This is done by using the Python library *numpy*.

The library *numpy* provides a method to only read out the color information of an image. The shape of the result array is typically *(height, width, channels)*, where:

1. *height* is the number of pixels in the vertical direction (i.e. the number of rows of pixels);
2. *width* is the number of pixels in the horizontal direction (i.e. the number of columns of pixels);
3. *channels* is the number of color channels per pixel. This value depends on what type of image it is:
  - In an RGB image, the three channels correspond to Red, Green, and Blue, respectively. Each channel value usually ranges from 0 to 255, where 0 indicates none of that color is present and 255 indicates that color is fully present.
  - In a grayscale image, there is typically only one channel. The value in this single channel indicates the level of gray, where 0 is black and 255 is white.
  - There are many other color spaces (HSV, LAB, etc.) that have different meanings for their channels.

The data type for each channel of each pixel is *uint8*, which is an unsigned integer that takes 8 bits. Then, the array is flattened into a bit stream by converting each channel of each pixel into an 8-bits binary number and appending them together.

For example, as for our testing image<sup>1</sup>, shown in Figure 2, the shape of the result array is *(1280, 854, 3)*, which means that the image has 1280 rows of pixels, 854 columns of pixels, and 3 color channels per pixel (RGB). Then, the array is flattened into a bit stream of 26,234,880 bits.

### 2.1.3 Audio

<sup>2</sup>

---

<sup>1</sup>The image file '*image.png*' used in this project is photographed by *Chenye Yang* at Joshua Tree National Park.

<sup>2</sup>The audio file '*file\_example\_WAV\_1MG.wav*' used in this project is free downloaded from [file-examples.com](http://file-examples.com).

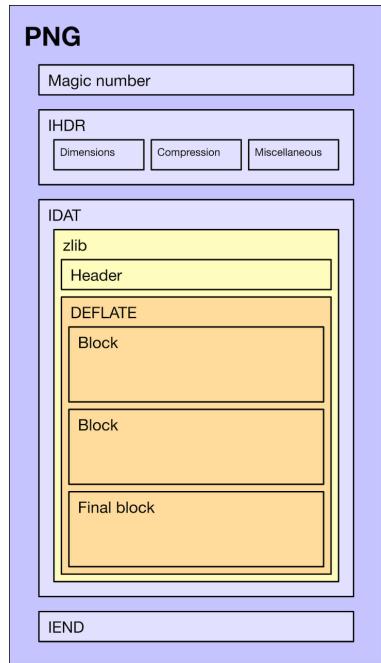


Figure 1: PNG file structure



Figure 2: Our testing PNG image

## 2.2 Destination

# 3 Channel

## 3.1 Binary Symmetric Channel (BSC)

## 3.2 Additive White Gaussian Noise (AWGN) Channel

# 4 (7,4) Systematic Linear Block (Hamming) Code

## 4.1 Syndrome Decoder

### 4.1.1 Text string

### 4.1.2 Image

### 4.1.3 Audio

# 5 ( $n, k$ ) Systematic Cyclic (Hamming) Code

## 5.1 Syndrome Decoder

### 5.1.1 Text string

### 5.1.2 Image

### 5.1.3 Audio

## 5.2 LFSR Decoder

Table 3: Our PNG file structure

Start offset	Chunk outside
0	Special: File signature; Length: 8 bytes
8	Data length: 13 bytes; Type: IHDR; Name: Image header; CRC-32: CB3954EC
33	Data length: 1 bytes; Type: sRGB; Name: Standard RGB color space; CRC-32: AECE1CE9
46	Data length: 976 bytes; Type: eXIf; Name: Exchangeable Image File (Exif) Profile; CRC-32: 47FCFA4D
1,034	Data length: 9 bytes; Type: pHYs; Name: Physical pixel dimensions; CRC-32: 5024E7F8
1,055	Data length: 4 514 bytes; Type: iTxt; Name: International textual data; CRC-32: C9C76B16
5,581	Data length: 16 384 bytes; Type: IDAT; Name: Image data; CRC-32: 8462CABD
21,977	Data length: 16 384 bytes; Type: IDAT; Name: Image data; CRC-32: 2C9D007C
...	...
1,727,161	Data length: 12 170 bytes; Type: IDAT; Name: Image data; CRC-32: 68067F52
1,739,343	Data length: 0 bytes; Type: IEND; Name: Image trailer; CRC-32: AE426082

Table 4: Text string encoded with Linear Hamming passed through BSC

Original	Without correction	Corrected
Hello World! EEC269A Error Correcting Code Demo	Hello Wopld! EEC269A Arror Correcting Kode Demo	Hello World! EEC269A Error Correcting Code Demo

## 6 Appendix: Python source code

### 6.1 Source

```

1 # Copyright (c) 2023 Chenye Yang
2
3 import numpy as np
4 from PIL import Image
5 import soundfile as sf
6 # from scipy.io import wavfile
7 import logging
8
9 # Create a logger in this module

```

Table 5: Text string encoded with Cyclic Hamming passed through BSC

Original	Without correction	Corrected
Hello World! EEC269A Error Correcting Code Demo	Hu,lo World! EEC269A Error Correctifg Code Demo	Hello World! EEC269A Error Correcting Code Demo



(a) Original

(b) Without correction

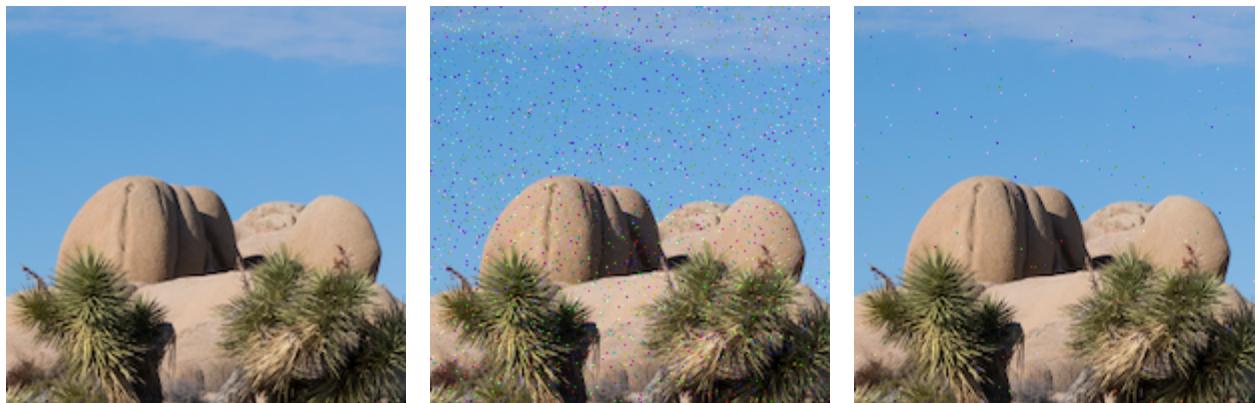
(c) Corrected

Figure 3: Image encoded with Linear Hamming passed through BSC (entire)

```

10 logger = logging.getLogger(__name__)
11
12
13 class Source:
14     """
15     The data source
16     """
17
18     def read_txt(self, src_path):
19         """
20             Read the text file as binary bits
21

```



(a) Original

(b) Without correction

(c) Corrected

Figure 4: Image encoded with Linear Hamming passed through BSC (details)

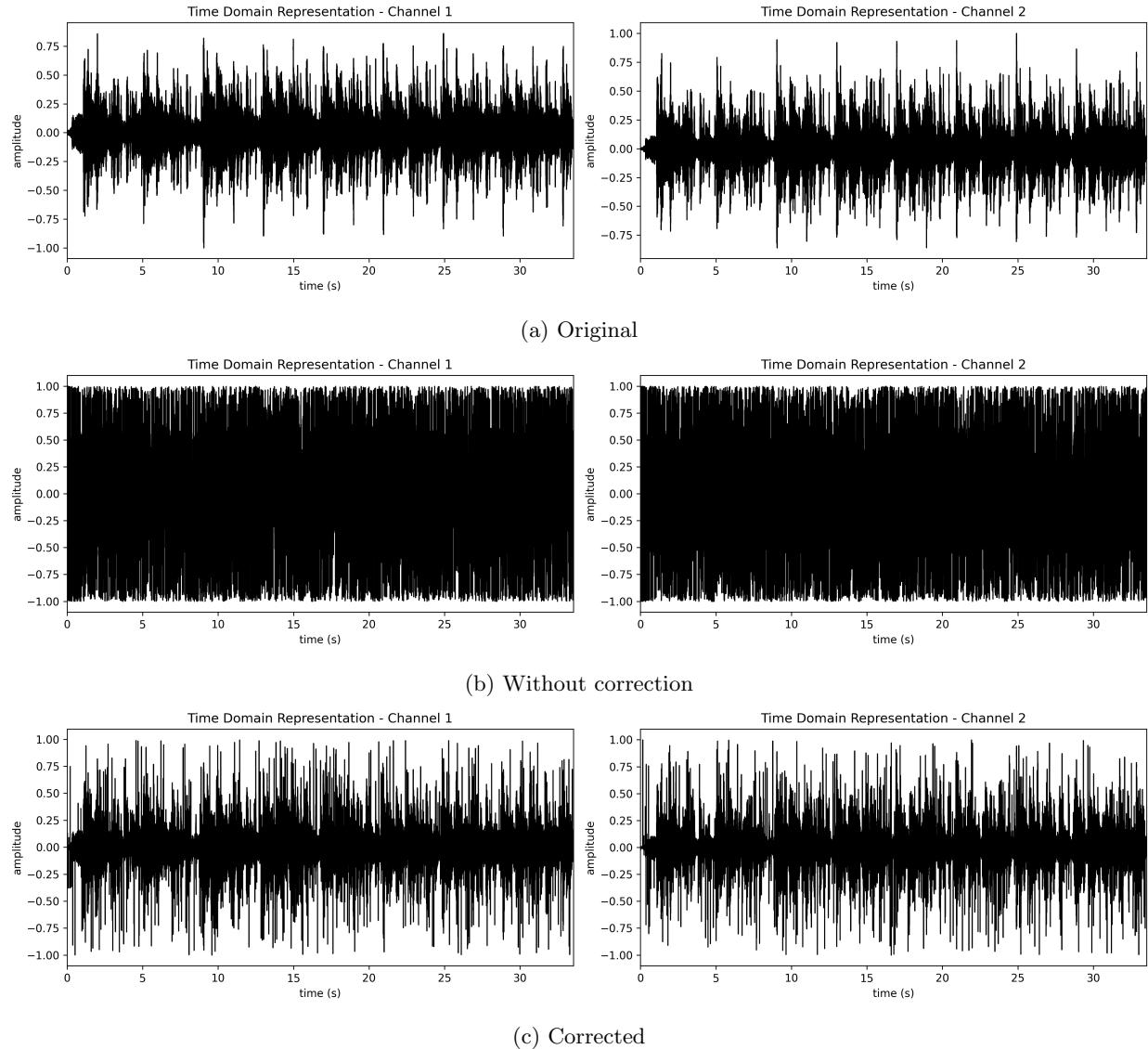


Figure 5: Audio encoded with Linear Hamming passed through BSC

```

22     @type src_path: string
23     @param src_path: source file path, with extension
24     """
25     with open(src_path, 'rb') as file:
26         byte_array = bytearray(file.read())
27         self._digital_data = np.unpackbits(np.frombuffer(byte_array, dtype
28                                         =np.uint8))
29
30     def read_png(self, src_path):
31         """
32             Read the png file,
33             store the color information as binary bits in _digital_data,
34             store the color information as ndarray in _analogue_data.
35             Return the height, width, channels of the image.

```

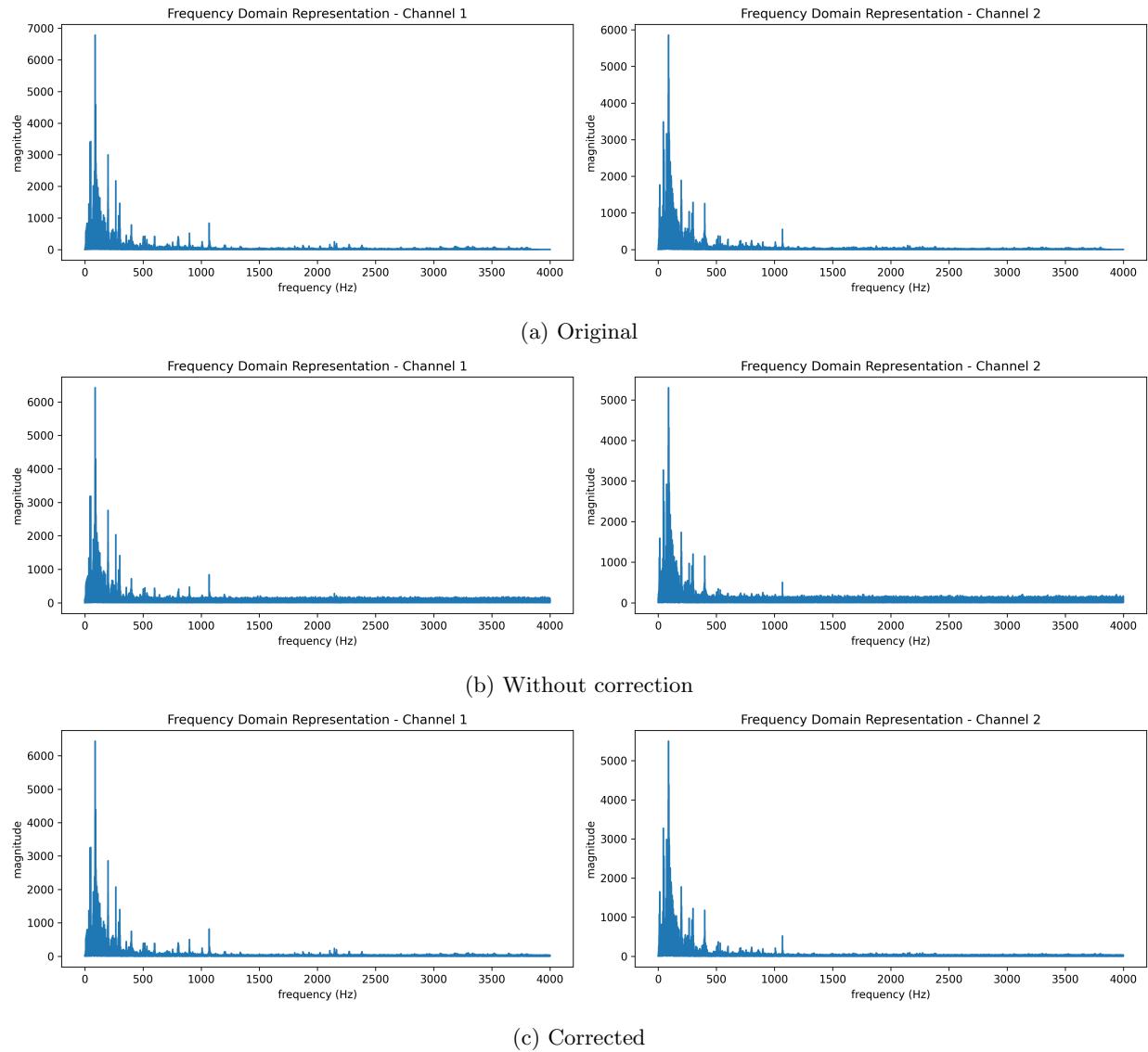


Figure 6: Audio encoded with Linear Hamming passed through BSC

```

36
37     @type  src_path: string
38     @param src_path: source file path, with extension
39
40     @rtype:    tuple
41     @return:   height, width, channels
42     """
43     # Open the image file
44     image = Image.open(src_path)
45
46     # Convert the image to a NumPy array
47     img_array = np.array(image)
48
49     # Get the shape of the array (height, width, channels)
50     height, width, channels = img_array.shape

```



(a) Original

(b) Without correction

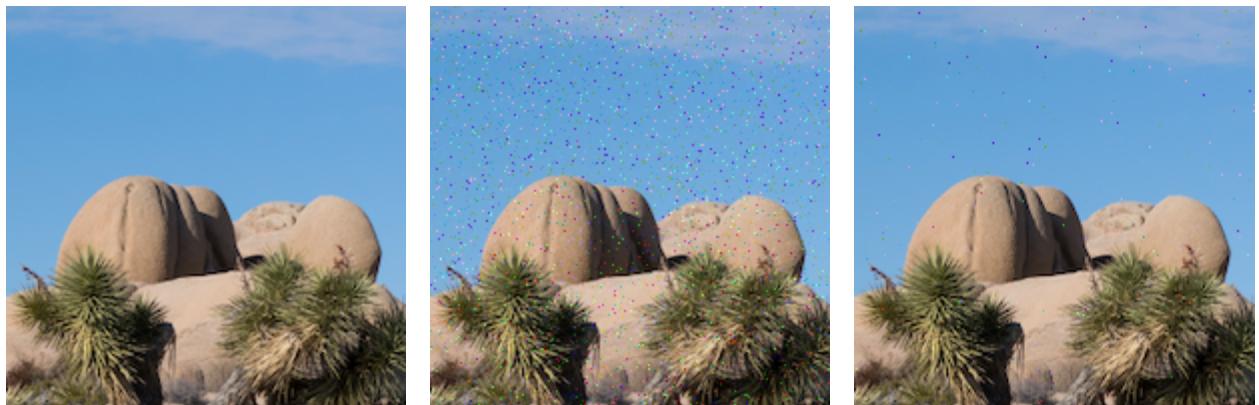
(c) Corrected

Figure 7: Image encoded with Cyclic Hamming passed through BSC (entire)

```

51
52     # Convert the img_array to binary format and flatten the array
53     bits = np.unpackbits(img_array.astype(np.uint8))
54
55     # Store the binary bits
56     self._digital_data = bits
57     # Store the analogue data
58     self._analogue_data = img_array
59
60     return height, width, channels
61
62

```



(a) Original

(b) Without correction

(c) Corrected

Figure 8: Image encoded with Cyclic Hamming passed through BSC (details)

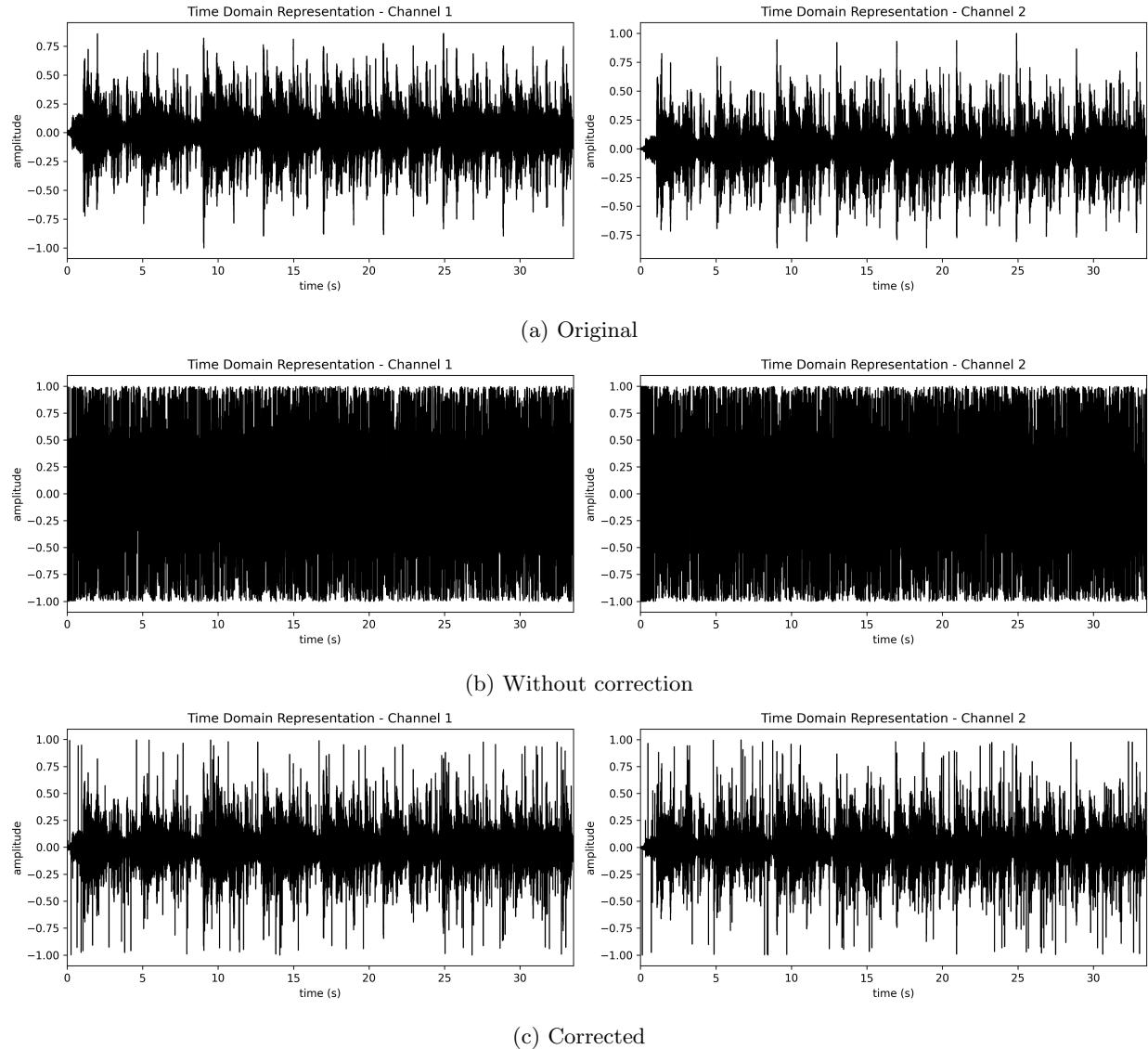


Figure 9: Audio encoded with Cyclic Hamming passed through BSC

```

63     def read_wav(self, src_path):
64         """
65             Read the wav file,
66             store the audio information as binary bits in _digital_data,
67             store the audio information as ndarray in _analogue_data.
68             Return the frame_rate, sample_width, channels of the audio.
69
70             @type src_path: string
71             @param src_path: source file path, with extension
72
73             @rtype: tuple
74             @return: frame rate, sample width, channels
75         """
76         # Open the audio file
77         # sample_rate, audio_array = wavfile.read(src_path)

```

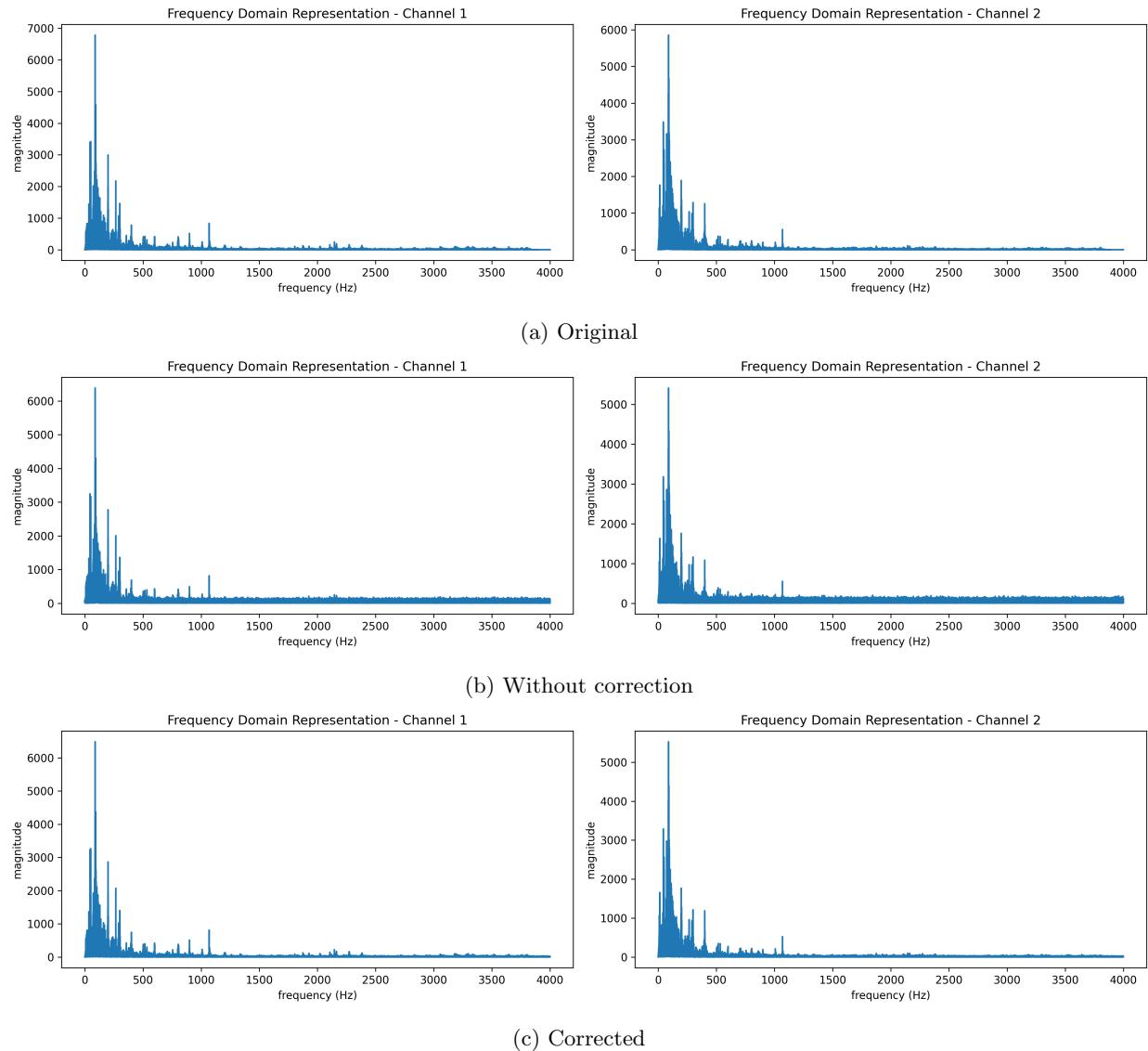


Figure 10: Audio encoded with Cyclic Hamming passed through BSC

```

78     audio_array, sample_rate = sf.read(src_path, dtype='int16')
79     shape = audio_array.shape
80
81     # Convert the audio_array to binary format and flatten the array
82     bits = np.unpackbits(audio_array.astype(np.int16).view(np.uint8))
83
84     # Store the binary bits
85     self._digital_data = bits
86     # Store the analogue data
87     self._analogue_data = audio_array
88
89     return shape, sample_rate
90
91
92     # def read_mp3(self, src_path):

```

```

93     """
94     # Read the mp3 file ,
95     # store the audio information as binary bits in _digital_data ,
96     # store the audio information as ndarray in _analogue_data .
97     # Return the frame_rate , sample_width , channels of the audio .
98
99     # @type src_path : string
100    # @param src_path: source file path , with extension
101
102    # @rtype: tuple
103    # @return: frame rate , sample width , channels
104    """
105    # Open the audio file
106    # audio = AudioSegment.from_file(src_path , format="mp3")
107    # frame_rate , sample_width , channels = audio.frame_rate , audio.
108    # sample_width , audio.channels
109
110    # Convert the audio to a NumPy array
111    # audio_array = np.array(audio.get_array_of_samples())
112    # print(audio_array.dtype)
113
114    # # Get the shape of the array: e.g. (2392270,)
115    # # shape = audio_array.shape
116
117    # # Convert the audio_array to binary format and flatten the array
118    # bits = np.unpackbits(audio_array.astype(np.uint8))
119
120    # # Store the binary bits
121    # self._digital_data = bits
122    # # Store the analogue data
123    # self._analogue_data = audio_array
124
125
126
127    def get_digital_data(self):
128        """
129            Get the bits to be transmitted
130
131            @rtype: ndarray
132            @return: data bits
133        """
134
135        return self._digital_data
136
137
138    def get_analogue_data(self):
139        """
140            Get the analogue data to be transmitted
141
142            @rtype: ndarray
143            @return: analogue data
144        """
145
146        return self._analogue_data

```

```

146
147 if __name__ == '__main__':
148     # test
149     source = Source()
150     source.read_txt("Resource/hardcoded.txt")
151     text_bits = source.get_digital_data()
152     print(text_bits)
153     print(type(text_bits))
154
155
156     source.read_png("Resource/image.png")
157     image_bits = source.get_digital_data()
158     print(image_bits[:16])
159     image_pixels = source.get_analogue_data()
160     print(image_pixels[0][0][:2])
161
162     # frame_rate, sample_width, channels = source.read_mp3("Resource/
163     # file-example-MP3_1MG.mp3")
164     # print(frame_rate, sample_width, channels)
165     # audio_bits = source.get_digital_data()
166     # print(audio_bits[:16])
167     # audio_array = source.get_analogue_data()
168     # print(audio_array[:2])
169
170     source.read_wav("Resource/file-example-WAV_1MG.wav")
171     audio_bits = source.get_digital_data()
172     print(audio_bits[:16])
173     audio_array = source.get_analogue_data()
174     print(audio_array[:2])

```

## 6.2 Channel

```

1 # Copyright (c) 2023 Chenye Yang
2
3 import numpy as np
4 import logging
5
6 # Create a logger in this module
7 logger = logging.getLogger(__name__)
8
9
10 def create_parity_check_matrix(G):
11     """
12         Create the parity-check matrix,  $H = [I_{n-k} \mid P \cdot T]$ 
13
14         @type G: ndarray
15         @param G: generator matrix in systematic form,  $G = [P \mid I_k]$ 
16
17         @rtype: ndarray
18         @return: parity-check matrix
19     """
20
21     # Get the size of the generator matrix

```

```

22     k, n = G.shape
23
24     # Create the parity-check matrix
25     #  $G = [P \mid I_{n-k}]$ , Extract P
26     P = G[:, :n-k]
27
28     #  $H = [I_{n-k} \mid P.T]$ 
29     H = np.hstack((np.eye(n-k, dtype=np.uint8), P.T))
30
31     return H
32
33
34 def create_syndrome_table(H):
35     """
36         Create a table of all possible syndromes and their corresponding error
37         vectors (syndrome look-up table)
38
39         @type H: ndarray
40         @param H: parity-check matrix
41
42     """
43
44     # Get the size of the parity-check matrix
45     _, n = H.shape
46
47     # Create a table of all possible syndromes and their corresponding error
48         vectors (syndrome look-up table)
49     coset_leader = np.vstack((np.zeros(n, dtype=np.uint8), np.eye(n, dtype=np.
50         uint8)))
51     possible_syndromes = (coset_leader @ H.T) % 2
52     syndrome_table = {str(possible_syndromes[i]) : coset_leader[i] for i in
53         range(n+1)}
54
55     return syndrome_table
56
57
58 class Linear_Code:
59     """
60         (7, 4) Systematic Linear Block (Hamming) Code
61
62     def __init__(self):
63         self.n, self.k = 7, 4
64         self.G = np.array([[1, 1, 0, 1, 0, 0, 0],
65                           [0, 1, 1, 0, 1, 0, 0],
66                           [1, 1, 1, 0, 0, 1, 0],
67                           [1, 0, 1, 0, 0, 0, 1]], dtype=np.uint8)
68         self.H = np.array([[1, 0, 0, 1, 0, 1, 1],
69                           [0, 1, 0, 1, 1, 1, 0],
70                           [0, 0, 1, 0, 1, 1, 1]], dtype=np.uint8)
71         self.syndrome_table = {'[0_0_0]' : np.array([0, 0, 0, 0, 0, 0, 0]),
72                               '[1_0_0]' : np.array([1, 0, 0, 0, 0, 0, 0]),
73                               '[0_1_0]' : np.array([0, 1, 0, 0, 0, 0, 0]),
74                               '[0_0_1]' : np.array([0, 0, 1, 0, 0, 0, 0]),
75                               '[1_1_0]' : np.array([0, 0, 0, 1, 0, 0, 0]),
76                               '[0_1_1]' : np.array([0, 0, 0, 0, 1, 0, 0])}

```

```

72             '[1_1_1]' : np.array([0, 0, 0, 0, 0, 1, 0]),
73             '[1_0_1]' : np.array([0, 0, 0, 0, 0, 0, 1])}
74
75     def encoder_systematic(self, bits):
76         """
77             Systematic – Encode the to-be-transmitted binary bits message with
78             (7,4) hamming encoder, return the to-be-transmitted codewords
79
80             @type bits: ndarray
81             @param bits: TX message
82
83             @rtype: ndarray
84             @return: TX codewords
85             """
86
87         encoded_array = np.zeros((len(bits) // self.k * self.n), dtype=np.
88             uint8)
89
90         for i in range(0, len(bits), self.k):
91             message = bits[i:i + self.k]
92             codeword = np.dot(message, self.G) % 2
93             encoded_array[i // self.k * self.n:(i // self.k + 1) * self.n] =
94                 codeword
95
96         return encoded_array
97
98
99     def decoder_systematic(self, encoded_array):
100        """
101            Systematic – Decode the received binary bits codeword with (7,4)
102            Hamming decoder, return the received message
103
104            @type encoded_array: ndarray
105            @param encoded_array: RX codewords
106
107            @rtype: ndarray
108            @return: RX message
109            """
110
111         decoded_array = np.zeros((len(encoded_array) // self.n * self.k),
112             dtype=np.uint8)
113
114         err_count = 0
115
116         for i in range(0, len(encoded_array), self.n):
117             received_codeword = encoded_array[i:i + self.n]
118             syndrome = np.dot(self.H, received_codeword) % 2
119             if np.any(syndrome): # If there are errors, count and keep it
120                 err_count += 1
121             decoded_array[i // self.n * self.k : (i // self.n + 1) * self.k] =
122                 received_codeword[self.n - self.k:self.n]
123
124         logger.info(f"Error_codeword_rate:{err_count/(len(encoded_array)//
125             self.n)}")
126
127         return decoded_array

```

```

119
120
121     def corrector_syndrome(self, received_array):
122         """
123             Systematic – Correct the received binary bits codeword with (7,4)
124                 Hamming syndrome look-up table corrector,
125             return the estimated TX codeword = (RX codeword + error pattern)
126
127             @type received_array: ndarray
128             @param received_array: RX codewords
129
130             @rtype: ndarray
131             @return: estimated TX codewords
132         """
133
134         corrected_array = received_array.copy()
135
136         for i in range(0, len(received_array), self.n):
137             received_codeword = received_array[i:i + self.n]
138             syndrome = np.dot(self.H, received_codeword) % 2
139             if np.any(syndrome):
140                 corrected_array[i:i + self.n] = (received_codeword + self.
141                     syndrome_table[str(syndrome)]) % 2
142
143
144     class Cyclic_Code(Linear_Code):
145         """
146             (n, k) Systematic Cyclic (Hamming) Code
147         """
148         def __init__(self, G):
149             self.G = G
150             self.k, self.n = self.G.shape
151             self.H = create_parity_check_matrix(self.G)
152
153             self.syndrome_table = create_syndrome_table(self.H)
154
155
156         def corrector_lfsr(self, received_array):
157             """
158                 Systematic – Correct the received binary bits codeword with (n, k)
159                 Hamming LFSR corrector,
160             return the estimated TX codeword = (RX codeword + error pattern)
161
162             @type received_array: ndarray
163             @param received_array: RX codewords
164
165             @rtype: ndarray
166             @return: estimated TX codewords
167         """
168
169         corrected_array = received_array.copy()
170         pass

```

```

170
171 class Channel:
172     """
173     Channel
174     """
175     def binary_symmetric_channel(self, input_bits, p):
176         """
177             BSC - binary symmetric channel with adjustable error probability
178
179             @type input_bits: ndarray
180             @param input_bits: TX codewords
181
182             @type p: float
183             @param p: error_probability
184
185             @rtype: ndarray
186             @return: RX codewords
187         """
188
189         output_bits = np.copy(input_bits)
190         for i in range(len(output_bits)):
191             if np.random.random() < p:
192                 output_bits[i] = 1 - output_bits[i] # flip the bit
193
194
195     if __name__ == '__main__':
196         # test
197         tx_msg = np.array([0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1], dtype=np.uint8)
198         print("Original_bits:", tx_msg)
199
200
201         # Channel
202         channel = Channel()
203
204         # Linear Code
205         linear_code = Linear_Code()
206
207         # Encoding
208         tx_codewords = linear_code.encoder_systematic(tx_msg)
209         print("Encoded_bits:", tx_codewords)
210
211         # Passing through the channel
212         rx_codewords = channel.binary_symmetric_channel(tx_codewords, 0.1)
213         print("Bits_after_channel:", rx_codewords)
214
215         # Correction with syndrome look-up table
216         estimated_tx_codewords = linear_code.corrector_syndrome(rx_codewords)
217         print("Estimated_TX_bits:", estimated_tx_codewords)
218
219         # Decoding
220         rx_msg = linear_code.decoder_systematic(estimated_tx_codewords)
221         print("Decoded_bits:", rx_msg)

```

### 6.3 Destination

```
1 # Copyright (c) 2023 Chenye Yang
2
3 import numpy as np
4 from PIL import Image
5 import soundfile as sf
6 # from scipy.io import wavfile
7 import logging
8
9 # Create a logger in this module
10 logger = logging.getLogger(__name__)
11
12
13 class Destination:
14     """
15     The destination
16     """
17
18     def set_digital_data(self, bits):
19         """
20             Record the received digital data
21
22             @type bits: ndarray
23             @param bits: data bits
24         """
25         self._digital_data = bits
26
27
28     def set_analogue_data(self, array):
29         """
30             Record the received analogue data
31
32             @type array: ndarray
33             @param array: analogue data array
34         """
35         self._analogue_data = array
36
37
38     def get_digital_data(self):
39         """
40             Get the received digital data
41
42             @rtype: ndarray
43             @return: data bits
44         """
45         return self._digital_data
46
47
48     def get_analogue_data(self):
49         """
50             Get the received analogue data
51
```

```

52         @rtype:    ndarray
53         @return:   analogue data
54     """
55
56     return self._analogue_data
57
58 def write_txt(self, dest_path):
59     """
60     Write data to a text file
61
62     @type dest_path: string
63     @param dest_path: destination file path, with extension
64     """
65     byte_array = bytearray(np.packbits(self._digital_data).tobytes())
66     with open(dest_path, 'wb') as file:
67         file.write(byte_array)
68
69
70 def write_png_from_digital(self, dest_path, height, width, channels):
71     """
72     Write color information in bits array to a png file, at the same time
73     store the color information as ndarray in _analogue_data
74
75     @type dest_path: string
76     @param dest_path: destination file path, with extension
77
78     @type height: int
79     @param height: height of the image
80
81     @type width: int
82     @param width: width of the image
83
84     @type channels: int
85     @param channels: channels of the image
86     """
87     # Convert the bit array to uint8 array
88     uint8_array = np.packbits(self._digital_data)
89
90     # Reshape the array to a 3D array of pixels (height, width, channels)
91     pixels = uint8_array.reshape(height, width, channels)
92
93     # Store the analogue data
94     self._analogue_data = pixels
95
96     # Create a new image from the pixel values
97     new_image = Image.fromarray(pixels)
98
99     # Save the new image to a file
100    new_image.save(dest_path)
101
102 def write_png_from_analogue(self, dest_path):
103     """

```

```

104     Write color information in (height, width, channels) array to a png
105     file, at the same time store the color information as bit array in
106     _digital_data
107
108     @type dest_path: string
109     @param dest_path: destination file path, with extension
110     """
111
112     # Create a new image from the pixel values
113     new_image = Image.fromarray(self._analogue_data)
114
115     # Save the new image to a file
116     new_image.save(dest_path)
117
118     # Convert the img_array to binary format and flatten the array
119     bits = np.unpackbits(self._analogue_data.astype(np.uint8))
120
121
122     def write_wav_from_digital(self, shape, sample_rate, dest_path):
123         """
124             Write audio information in bits array to a wav file, at the same time
125             store the audio information as ndarray in _analogue_data
126
127             @type dest_path: string
128             @param dest_path: destination file path, with extension
129             """
130             # Convert the bit array to int16 array
131             int16_array = np.packbits(self._digital_data).view(np.int16)
132
133             # Reshape the array to a 2D array of samples (channels, samples)
134             audio_array = int16_array.reshape(shape)
135
136             # Store the analogue data
137             self._analogue_data = audio_array
138
139             # Write the array to a wav file
140             # wavfile.write(dest_path, sample_rate, audio_array)
141             sf.write(dest_path, audio_array, sample_rate)
142
143     def write_wav_from_analogue(self, sample_rate, dest_path):
144         """
145             Write audio information in audio array to a wav file, at the same time
146             store the audio information as bit array in _digital_data
147
148             @type dest_path: string
149             @param dest_path: destination file path, with extension
150             """
151             # Write the array to a wav file
152             # wavfile.write(dest_path, sample_rate, self._analogue_data)
153             sf.write(dest_path, self._analogue_data, sample_rate)

```

```

154     # Convert the audio_array to binary format and flatten the array
155     bits = np.unpackbits(self._analogue_data.astype(np.int16).view(np.
156         uint8))
157
158     # Store the binary bits
159     self._digital_data = bits
160
161     # def write_mp3_from_digital(self, frame_rate, sample_width, channels,
162     # dest_path):
163     # """
164     #     Write audio information in bits array to a mp3 file
165     #
166     #     @type dest_path: string
167     #     @param dest_path: destination file path, with extension
168     # """
169     #     # Convert the bit array to uint16 array
170     #     uint16_array = np.packbits(self._digital_data).astype(np.int16)
171
172     #     # Create a new AudioSegment object from the modified audio array
173     #     modified_audio = AudioSegment(uint16_array.tobytes(), frame_rate,
174     #     sample_width, channels)
175
176     #     # Export the modified audio to a new MP3 file
177     #     modified_audio.export(dest_path, format="mp3")
178
179     # def write_mp3_from_analogue(self, frame_rate, sample_width, channels,
180     # dest_path):
181     # """
182     #     Write audio information in audio array to a mp3 file
183     #
184     #     @type dest_path: string
185     #     @param dest_path: destination file path, with extension
186     # """
187     #     # Create a new AudioSegment object from the modified audio array
188     #     modified_audio = AudioSegment(self._analogue_data.tobytes(),
189     #     frame_rate, sample_width, channels)
190
191     #     # Export the modified audio to a new MP3 file
192     #     modified_audio.export(dest_path, format="mp3")
193
194     if __name__ == '__main__':
195         # test
196         bits = np.array([0, 1, 0, 0, 1, 0, 0, 0]) # H
197         destination = Destination()
198         destination.set_data(bits)
199         destination.write_file("output.txt")

```

## 6.4 Utilities

```

1 # Copyright (c) 2023 Chenye Yang
2
3 import numpy as np
4 import matplotlib.pyplot as plt
5 from scipy.fft import fft
6
7
8 def plot_wav_time_domain(audio_array, sample_rate, dest_path):
9     """
10     Plot the audio signal in the time domain.
11
12     @type audio_array: ndarray
13     @param audio_array: audio data array
14
15     @type sample_rate: int
16     @param sample_rate: sample rate of the audio
17
18     @type dest_path: string
19     @param dest_path: destination file path, with extension
20     """
21
22     # Copy the audio array
23     data = audio_array.copy()
24     # Normalize to [-1, 1]
25     data = data / np.max(np.abs(data), axis=0)
26
27     times = np.arange(len(data))/float(sample_rate)
28
29     # Prepare the subplots
30     fig, axs = plt.subplots(1, 2, figsize=(15, 4))
31
32     # Time domain representation for channel 1
33     axs[0].fill_between(times, data[:, 0], color='k')
34     axs[0].set_xlim(times[0], times[-1])
35     axs[0].set_xlabel('time_(s)')
36     axs[0].set_ylabel('amplitude')
37     axs[0].set_title('Time-Domain_Representation--Channel_1')
38
39     # Time domain representation for channel 2
40     axs[1].fill_between(times, data[:, 1], color='k')
41     axs[1].set_xlim(times[0], times[-1])
42     axs[1].set_xlabel('time_(s)')
43     axs[1].set_ylabel('amplitude')
44     axs[1].set_title('Time-Domain_Representation--Channel_2')
45
46     # Display the plot
47     plt.tight_layout()
48     plt.savefig(dest_path, dpi=300)
49     plt.close()
50
51 def plot_wav_frequency_domain(audio_array, sample_rate, dest_path):
52     """
53     Plot the audio signal in the frequency domain.

```

```

54
55     @type audio_array: ndarray
56     @param audio_array: audio data array
57
58     @type sample_rate: int
59     @param sample_rate: sample rate of the audio
60
61     @type dest_path: string
62     @param dest_path: destination file path, with extension
63 """
64
65 # Copy the audio array
66 data = audio_array.copy()
67 # Normalize to [-1, 1]
68 data = data / np.max(np.abs(data), axis=0)
69
70 # Prepare the subplots
71 fig, axs = plt.subplots(1, 2, figsize=(15, 4))
72
73 # Frequency domain representation for channel 1
74 fft_out_channel_1 = fft(data[:, 0])
75 magnitude_spectrum_channel_1 = np.abs(fft_out_channel_1)
76 frequencies_channel_1 = np.linspace(0, sample_rate, len(
77     magnitude_spectrum_channel_1))
78
79 axs[0].plot(frequencies_channel_1[:int(len(frequencies_channel_1)/2)],
80             magnitude_spectrum_channel_1[:int(len(magnitude_spectrum_channel_1)/2)])
81 # plot only first half of frequencies
82 axs[0].set_xlabel('frequency_(Hz)')
83 axs[0].set_ylabel('magnitude')
84 axs[0].set_title('Frequency-Domain-Representation--Channel-1')
85
86 # Frequency domain representation for channel 2
87 fft_out_channel_2 = fft(data[:, 1])
88 magnitude_spectrum_channel_2 = np.abs(fft_out_channel_2)
89 frequencies_channel_2 = np.linspace(0, sample_rate, len(
90     magnitude_spectrum_channel_2))
91
92 axs[1].plot(frequencies_channel_2[:int(len(frequencies_channel_2)/2)],
93             magnitude_spectrum_channel_2[:int(len(magnitude_spectrum_channel_2)/2)])
94 # plot only first half of frequencies
95 axs[1].set_xlabel('frequency_(Hz)')
96 axs[1].set_ylabel('magnitude')
97 axs[1].set_title('Frequency-Domain-Representation--Channel-2')
98
99 # Display the plot
100 plt.tight_layout()
101 plt.savefig(dest_path, dpi=300)
102 plt.close()

```

```

1 # Copyright (c) 2023 Pranav Kharche, Chenye Yang
2 # Toolbox for polynomials and CRC
3
4 import numpy as np

```

```

5 import logging
6
7 # Create a logger in this module
8 logger = logging.getLogger(__name__)
9
10
11 def order(p):
12     p = p >> 1
13     order = 0
14     while p:
15         p = p >> 1
16         order += 1
17     return order
18
19 def bitRev(p, pOrder = None):
20     if p == 0:
21         return 0
22     if pOrder == None:
23         pOrder = order(p)
24
25     retVal = 0
26     for i in range(pOrder+1):
27         retVal = (retVal << 1) + (p % 2)
28         p = p >> 1
29     return retVal
30
31 def polyDiv(dividend, divisor, dividendOrder = None, divisorOrder = None):
32     if dividendOrder == None or divisorOrder == None:
33         dividendOrder = order(dividend)
34         divisorOrder = order(divisor)
35
36     sizeDiff = dividendOrder - divisorOrder
37     rem = dividend
38     remOrder = dividendOrder
39     result = 0
40     for i in range(sizeDiff, -1, -1):
41         if(rem >> remOrder):
42             rem = rem ^ (divisor << i)
43             result = result + (1 << i)
44     remOrder -= 1
45
46     # if rem:
47     #     print(f'{dividend:b}', ':', f'{divisor:b}', '=', f'{result:b}', 'R', f'{rem:0{divisorOrder}b}', sep=' ')
48     # else:
49     #     print(f'{dividend:b}', ':', f'{divisor:b}', '=', f'{result:b}')
50     return result, rem
51
52 def findGen(n,k):
53     target = (1 << n) + 1
54     gen = (1 << (n-k)) + 3
55     parity, rem = polyDiv(target, gen, n, n-k)
56     while rem:

```

```

57             gen += 2
58             if gen >= target:
59                 return None, None
60             parity, rem = polyDiv(target, gen, n, n-k)
61             logger.debug('generator =', f'{gen:b}')
62             logger.debug('parity =', f'{parity:b}')
63             return gen, parity
64
65     def buildMatrix(n, k, gen, parity):
66         genMatrix = [gen]
67         # print(f'{gen:0{n}b}')
68         for i in range(k-1):
69             nextLine = genMatrix[i] << 1
70             if (nextLine >> (n-k))%2:
71                 nextLine = nextLine ^ gen
72             genMatrix.append(nextLine)
73             # print(f'{nextLine:0{n}b}')
74         logger.debug('Generator matrix')
75         for i in range(k):
76             genMatrix[i] = bitRev(genMatrix[i], n-1)
77             logger.debug(f'{genMatrix[i]:0{n}b}')
78         return genMatrix
79
80     def findMatrix(n,k):
81         gen, parity = findGen(n,k)
82         if gen == None:
83             return None
84         return buildMatrix(n, k, gen, parity)
85
86     def encode(data, gen):
87         if order(data)+1 > len(gen):
88             return None
89         result = 0
90         data = bitRev(data, len(gen)-1)
91         for row in gen:
92             result = result ^ ((data & 1)*row)
93             data = data >> 1
94         return result
95
96
97     def genMatrixDecimal2Ndarray(genMatrix_decimal, n):
98         """
99             Convert the generator matrix in decimal form to numpy array form
100
101             @type genMatrix_decimal: list
102             @param genMatrix_decimal: generator matrix in decimal form
103
104             @type n: int
105             @param n: number of bits in a codeword
106
107             @rtype: ndarray
108             @return: generator matrix in numpy array form
109         """
110         # Convert the generator matrix in decimal form to binary form

```

```

111     genMatrix_binary = [[int(bit) for bit in f'{num:0{n}b}'] for num in
112         genMatrix_decimal]
113
114     # Convert the binary generator matrix to a numpy array
115     G = np.array(genMatrix_binary, dtype=np.uint8)
116
117     return G

```

## 6.5 Linear code

```

1  # Copyright (c) 2023 Chenye Yang
2
3 import logging
4
5 from Source import source
6 from Channel import channel
7 from Destination import destination
8 from Utils import plot_wav
9
10
11 # Configure the logging
12 logging.basicConfig(level=logging.INFO, format='%(asctime)s - %(name)s - %(levelname)s - %(message)s')
13
14 # Create a logger in the main module
15 logger = logging.getLogger(__name__)
16
17
18 def linear_txt():
19     """
20         Source - Channel encoder - Channel - Channel decoder - Destination
21
22         txt      - (7,4) linear      - bsc      - (7,4) linear      - txt
23     """
24     src = source.Source()
25     chl = channel.Channel()
26     linear_code = channel.Linear_Code()
27     dest = destination.Destination()
28
29
30     src.read_txt("Resource/hardcoded.txt")
31     tx_msg = src.get_digital_data()
32
33     tx_codeword = linear_code.encoder_systematic(tx_msg)
34
35     rx_codeword = chl.binary_symmetric_channel(tx_codeword, 0.01)
36
37     # without error correction
38     rx_msg = linear_code.decoder_systematic(rx_codeword)
39     dest.set_digital_data(rx_msg)
40     dest.write_txt("Result/linear-bsc-output.txt")
41
42

```

```

43     # with error correction
44     estimated_tx_codeword = linear_code.corrector_syndrome(rx_codeword)
45     rx_msg = linear_code.decoder_systematic(estimated_tx_codeword)
46     dest.set_digital_data(rx_msg)
47     dest.write_txt("Result/linear-bsc-output-syndrome-corrected.txt")
48
49
50 def linear_png():
51     """
52     Source - Channel encoder - Channel - Channel decoder - Destination
53
54     png      - (7,4) linear      - bsc      - (7,4) linear      - png
55     """
56     src = source.Source()
57     chl = channel.Channel()
58     linear_code = channel.Linear_Code()
59     dest = destination.Destination()
60
61
62     height, width, channels = src.read_png("Resource/image.png")
63     tx_msg = src.get_digital_data()
64
65     tx_codeword = linear_code.encoder_systematic(tx_msg)
66
67     rx_codeword = chl.binary_symmetric_channel(tx_codeword, 0.01)
68
69     # without error correction
70     rx_msg = linear_code.decoder_systematic(rx_codeword)
71     dest.set_digital_data(rx_msg)
72     dest.write_png_from_digital("Result/linear-bsc-output.png", height, width,
73                               channels)
74
75     # with error correction
76     estimated_tx_codeword = linear_code.corrector_syndrome(rx_codeword)
77     rx_msg = linear_code.decoder_systematic(estimated_tx_codeword)
78     dest.set_digital_data(rx_msg)
79     dest.write_png_from_digital("Result/linear-bsc-output-syndrome-corrected.
80                               png", height, width, channels)
81
82
83 def linear_wav():
84     """
85     Source - Channel encoder - Channel - Channel decoder - Destination
86
87     wav      - (7,4) linear      - bsc      - (7,4) linear      - wav
88     """
89     src = source.Source()
90     chl = channel.Channel()
91     linear_code = channel.Linear_Code()
92     dest = destination.Destination()
93
94
95     shape, sample_rate = src.read_wav("Resource/file-example_WAV_1MG.wav")

```

```

94     plot_wav.plot_wav_time_domain(src.get_analogue_data(), sample_rate, "
95         Result/wav-time-domain-TX.png")
96     plot_wav.plot_wav_frequency_domain(src.get_analogue_data(), sample_rate, "
97         Result/wav-frequency-domain-TX.png")
98
99     # tx_msg = src.get_analogue_data()
100    # rx_msg = tx_msg
101    # dest.set_analogue_data(rx_msg)
102    # plot_wav.plot_wav_time_domain(dest.get_analogue_data(), sample_rate, "
103        Result/wav-time-domain-RX.png")
104    # plot_wav.plot_wav_frequency_domain(dest.get_analogue_data(), sample_rate,
105        "Result/wav-frequency-domain-RX.png")
106    # dest.write_wav_from_analogue(sample_rate, "Result/hamming-bsc-output.wav"
107        ")
108
109
110
111    tx_msg = src.get_digital_data()
112
113    tx_codeword = linear_code.encoder_systematic(tx_msg)
114
115    rx_codeword = chl.binary_symmetric_channel(tx_codeword, 0.01)
116
117    # without error correction
118    rx_msg = linear_code.decoder_systematic(rx_codeword)
119
120    dest.set_digital_data(rx_msg)
121    dest.write_wav_from_digital(shape, sample_rate, "Result/linear-bsc-output.
122        wav")
123
124
125    plot_wav.plot_wav_time_domain(dest.get_analogue_data(), sample_rate, "
126        Result/linear-bsc-wav-time-domain-RX.png")
127    plot_wav.plot_wav_frequency_domain(dest.get_analogue_data(), sample_rate,
128        "Result/linear-bsc-wav-frequency-domain-RX.png")
129
130
131    # with error correction
132    estimated_tx_codeword = linear_code.corrector_syndrome(rx_codeword)
133    rx_msg = linear_code.decoder_systematic(estimated_tx_codeword)
134
135    dest.set_digital_data(rx_msg)
136    dest.write_wav_from_digital(shape, sample_rate, "Result/linear-bsc-output-
137        syndrome-corrected.wav")
138
139
140    plot_wav.plot_wav_time_domain(dest.get_analogue_data(), sample_rate, "
141        Result/linear-bsc-wav-time-domain-RX-syndrome-corrected.png")
142    plot_wav.plot_wav_frequency_domain(dest.get_analogue_data(), sample_rate,
143        "Result/linear-bsc-wav-frequency-domain-RX-syndrome-corrected.png")
144
145
146    if __name__ == '__main__':
147        linear_txt()
148        linear_png()
149        linear_wav()

```

## 6.6 Cyclic code

```
1 # Copyright (c) 2023 Chenye Yang
2
3 import logging
4
5 from Source import source
6 from Channel import channel
7 from Destination import destination
8 from Utils import plot_wav, polyTools
9
10
11 # Configure the logging
12 logging.basicConfig(level=logging.INFO, format='%(asctime)s - %(name)s - %(levelname)s - %(message)s')
13
14 # Create a logger in the main module
15 logger = logging.getLogger(__name__)
16
17
18
19 # Work with (3, 1) cyclic code
20 n = 3
21 k = 1
22
23 # Create the generator matrix
24 genMatrix_decimal = polyTools.findMatrix(n, k)
25 G = polyTools.genMatrixDecmial2Ndarray(genMatrix_decimal, n)
26
27
28 def cyclic_txt():
29     """
30         Source - Channel encoder - Channel - Channel decoder - Destination
31
32         txt      - (n, k) cyclic      - bsc      - (n, k) cyclic      - txt
33     """
34     src = source.Source()
35     chl = channel.Channel()
36     cyclic_code = channel.Cyclic_Code(G)
37     dest = destination.Destination()
38
39
40     src.read_txt("Resource/hardcoded.txt")
41     tx_msg = src.get_digital_data()
42
43     tx_codeword = cyclic_code.encoder_systematic(tx_msg)
44
45     rx_codeword = chl.binary_symmetric_channel(tx_codeword, 0.01)
46
47     # without error correction
48     rx_msg = cyclic_code.decoder_systematic(rx_codeword)
49     dest.set_digital_data(rx_msg)
50     dest.write_txt("Result/cyclic-bsc-output.txt")
```

```

51
52 # with error correction
53 estimated_tx_codeword = cyclic_code.corrector_syndrome(rx_codeword)
54 rx_msg = cyclic_code.decoder_systematic(estimated_tx_codeword)
55 dest.set_digital_data(rx_msg)
56 dest.write_txt("Result/cyclic-bsc-output-syndrome-corrected.txt")
57
58
59 def cyclic_png():
60     """
61     Source - Channel encoder - Channel - Channel decoder - Destination
62
63     png      - (n, k) cyclic      - bsc      - (n, k) cyclic      - png
64     """
65     src = source.Source()
66     chl = channel.Channel()
67     cyclic_code = channel.Cyclic_Code(G)
68     dest = destination.Destination()
69
70
71     height, width, channels = src.read_png("Resource/image.png")
72     tx_msg = src.get_digital_data()
73
74     tx_codeword = cyclic_code.encoder_systematic(tx_msg)
75
76     rx_codeword = chl.binary_symmetric_channel(tx_codeword, 0.01)
77
78 # without error correction
79 rx_msg = cyclic_code.decoder_systematic(rx_codeword)
80 dest.set_digital_data(rx_msg)
81 dest.write_png_from_digital("Result/cyclic-bsc-output.png", height, width,
82                             channels)
83
84 # with error correction
85 estimated_tx_codeword = cyclic_code.corrector_syndrome(rx_codeword)
86 rx_msg = cyclic_code.decoder_systematic(estimated_tx_codeword)
87 dest.set_digital_data(rx_msg)
88 dest.write_png_from_digital("Result/cyclic-bsc-output-syndrome-corrected.
89                             png", height, width, channels)
90
91
92 def cyclic_wav():
93     """
94     Source - Channel encoder - Channel - Channel decoder - Destination
95
96     wav      - (n, k) cyclic      - bsc      - (n, k) cyclic      - wav
97     """
98     src = source.Source()
99     chl = channel.Channel()
100    cyclic_code = channel.Cyclic_Code(G)
101    dest = destination.Destination()
102
103
104    shape, sample_rate = src.read_wav("Resource/file-example_WAV_1MG.wav")

```

```

103 plot_wav.plot_wav_time_domain(src.get_analogue_data(), sample_rate, "
104     Result/wav-time-domain-TX.png")
105 plot_wav.plot_wav_frequency_domain(src.get_analogue_data(), sample_rate, "
106     Result/wav-frequency-domain-TX.png")
107
108 # tx_msg = src.get_analogue_data()
109 # rx_msg = tx_msg
110 # dest.set_analogue_data(rx_msg)
111 # plot_wav.plot_wav_time_domain(dest.get_analogue_data(), sample_rate, "
112     Result/wav-time-domain-RX.png")
113 # plot_wav.plot_wav_frequency_domain(dest.get_analogue_data(), sample_rate,
114     , "Result/wav-frequency-domain-RX.png")
115 # dest.write_wav_from_analogue(sample_rate, "Result/hamming-bsc-output.wav"
116     ")
117
118 tx_msg = src.get_digital_data()
119
120 tx_codeword = cyclic_code.encoder_systematic(tx_msg)
121
122 rx_codeword = chl.binary_symmetric_channel(tx_codeword, 0.01)
123
124 # without error correction
125 rx_msg = cyclic_code.decoder_systematic(rx_codeword)
126
127 dest.set_digital_data(rx_msg)
128 dest.write_wav_from_digital(shape, sample_rate, "Result/cyclic-bsc-output.
129     wav")
130
131 plot_wav.plot_wav_time_domain(dest.get_analogue_data(), sample_rate, "
132     Result/cyclic-bsc-wav-time-domain-RX.png")
133 plot_wav.plot_wav_frequency_domain(dest.get_analogue_data(), sample_rate,
134     "Result/cyclic-bsc-wav-frequency-domain-RX.png")
135
136 # with error correction
137 estimated_tx_codeword = cyclic_code.corrector_syndrome(rx_codeword)
138 rx_msg = cyclic_code.decoder_systematic(estimated_tx_codeword)
139
140 dest.set_digital_data(rx_msg)
141 dest.write_wav_from_digital(shape, sample_rate, "Result/cyclic-bsc-output-
142     syndrome-corrected.wav")
143
144 plot_wav.plot_wav_time_domain(dest.get_analogue_data(), sample_rate, "
145     Result/cyclic-bsc-wav-time-domain-RX-syndrome-corrected.png")
146 plot_wav.plot_wav_frequency_domain(dest.get_analogue_data(), sample_rate,
147     "Result/cyclic-bsc-wav-frequency-domain-RX-syndrome-corrected.png")
148
149 if __name__ == '__main__':
150     cyclic_txt()
151     cyclic_png()
152     cyclic_wav()

```