# Homework #0: Numerical Simulations*

ELEN E6880: RMT with Applications

1. ***Gaussian random matrices and Wigner's semi-circle law:*** Consider an ensemble of $K$ realizations of $N \times N$ Gaussian matrices, $\mathbf{A}$, with (symmetric) entries $a_{i \leq j} \sim \mathcal{N}(0, 1/N)$.

   (a) Plot the averaged histograms of the eigenvalues of $\mathbf{A}$ for: $K = 10000$, $N = 5, 20$.

   (b) Plot the averaged histograms of the eigenvalues of $\mathbf{A}$ for: $K = 1$, $N = 10, 100, 1000, 4000$.

   (c) Plot, on top of your simulated histograms, the analytical curve (a.k.a. Wigner's semi-circle law of the limiting distribution):

   $$p(\lambda) = \begin{cases} \frac{1}{2\pi}\sqrt{4-\lambda^2} & |\lambda| < 2 \\ 0 & \text{else} \end{cases} \tag{1}$$

   (d) How can you characterize the difference between these two types of convergence ((a) and (b))?

2. ***Wishart random matrices and Marčenko-Pastur law:*** Consider an ensemble of $K$ realizations of $N \times N$ Wishart matrices of the form $\mathbf{A} \triangleq \frac{1}{N}\mathbf{X}\mathbf{X}^H$, where the $N \times M$ matrix $\mathbf{X} \sim \mathcal{N}(0, 1)$.

   (a) Plot the averaged histograms of the eigenvalues of $\mathbf{A}$ for: $M = 2N$, $K = 3000$, $N = 100$.

   (b) Plot the averaged histograms of the eigenvalues of $\mathbf{A}$ for: $M = 2N$, $K = 1$, $N = 2000$.

*Use a numerical SW/language of choice (Matlab, Octave, Python, R,...). Source code for generating the plots should be also provided. DUE: 09/27/19

(c) Plot, on top of your simulated histograms, the analytical curve (a.k.a. Marčenko-Pastur law of the limiting distribution):

$$p(\lambda) = \begin{cases} \frac{\sqrt{4\alpha-(\lambda-1-\alpha)^2}}{2\pi\lambda} & (1-\sqrt{\alpha})^2 \leq \lambda \leq (1+\sqrt{\alpha})^2 \\ 0 & \text{else} \end{cases} \tag{2}$$

and $\alpha \triangleq \frac{M}{N}$.

(d) Again, how can you characterize the difference between these two types of convergence ((a) and (b))?