Homework #3: Statistical Mechanics-based Analysis

ELEN E6880: RMT with Applications

1. Proof of McKay's Law via Cavity Method. Let the $n \times n$ matrix $\mathbf{A}$ be the adjacency matrix of a regular graph of finite degree $k \geq 2$ (with no short cycles).

   (a) Draw an example of such a regular sparse graph and its corresponding matrix $\mathbf{A}$.

   (b) Write down the self-consistency equations for the cavity variances $\Delta_i^{(j)}$, when n grows large (sum of rows/columns is identical).

   (c) Solve the self-consistency equations to find explicitly $\Delta_i^{(j)}$. How did you choose the solution?

   (d) Derive the McKay Law for regular sparse matrices

   $$p(\lambda) = \frac{k\sqrt{4(k-1) - \lambda^2}}{2\pi(k^2 - \lambda^2)}, \qquad |\lambda| \leq 2\sqrt{k-1}.$$

   (e) Draw the spectral density $p(\lambda)$ for $k = 2, 3, 4, 5, 10$.
