Wigner and MP laws

Contents

Wigner

Illustration of convergence of averaged eigenvalue distribution

```
close all;
clear all;
rng('default');
```

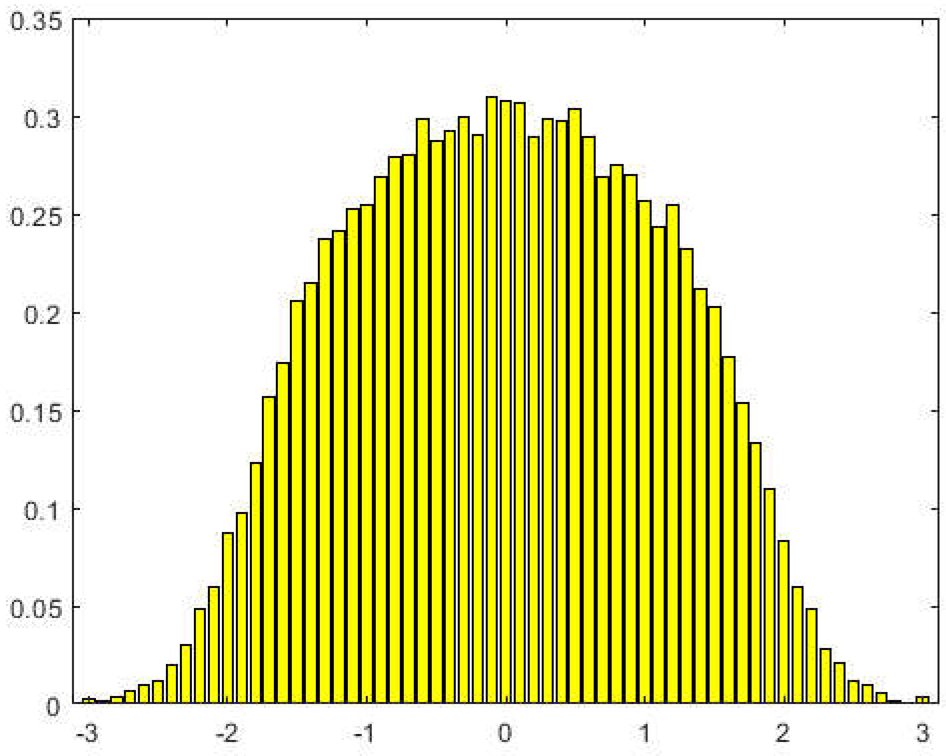
```
N = 5;
K = 1e4;
v = []; % eigenvalues
dx = 0.1; % bin size
```

Experiment

```
for i =1:K
    A =randn(N)/sqrt(N/2); % random NxN Gaussian matrix; Note: normalize by sqrt(N/2)
 and not sqrt(N) so AA will have unit variance.
    AA =(A+A')/2; % symmetrize
    v = [v; eig(AA)]; % eigenvalues
end
```

Plot

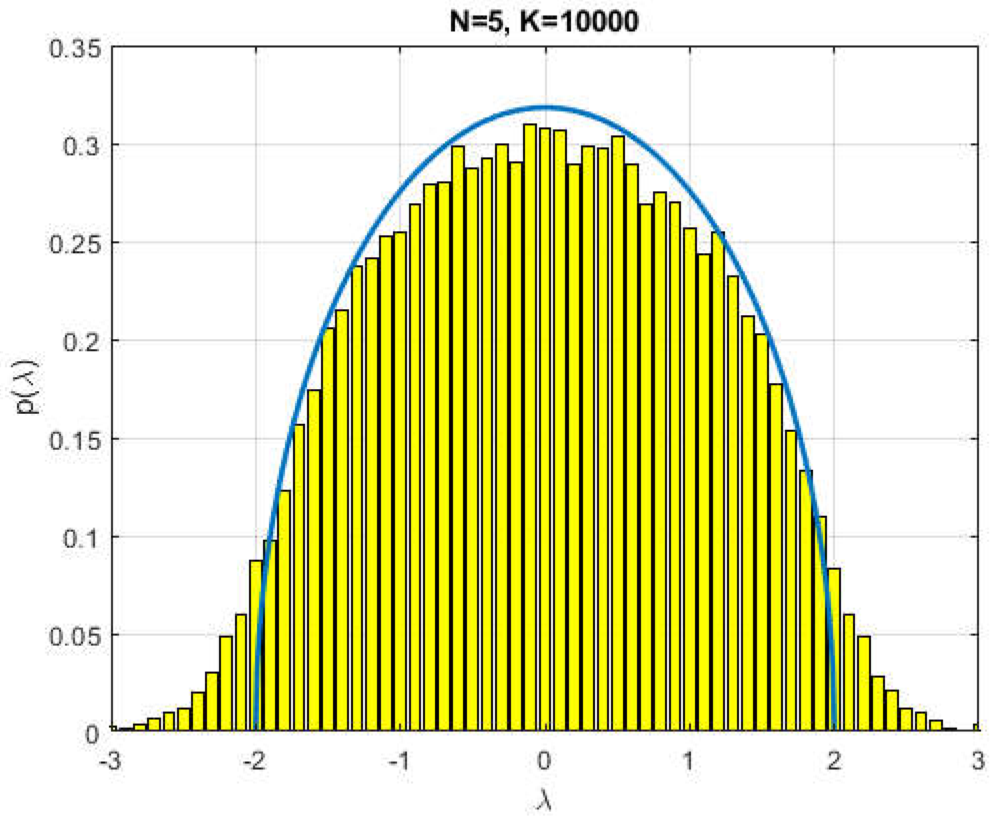
```
[count, x]= hist(v, -3:dx:3);
bar(x, count/(K*N*dx), 'y');
hold on ;
```



Theory

```
x = -2:0.01:2;
plot(x,sqrt(4-x.^2)/(2*pi), 'LineWidth', 2);
axis([-3 3 0 0.35]);
grid on;
title('N=5, K=10000');
xlabel('\lambda');
```

```
ylabel('p(\lambda)');
%%%%%%%%
%%%%%%%%
figure;
N = 20;
K = 1e4;
v = []; % eigenvalues
dx = 0.1; % bin size
```



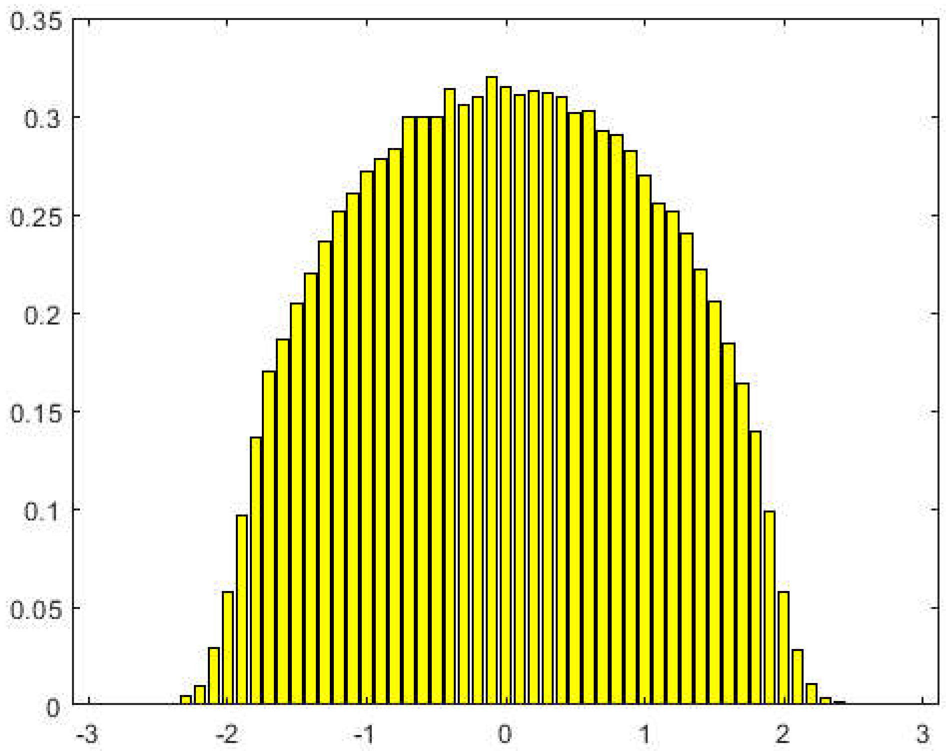
Experiment

```
for i =1:K
    A =randn(N)/sqrt(N/2); % random NxN Gaussian matrix; Note: normalize by sqrt(N/2)
 and not sqrt(N) so AA will have unit variance.
```

```
    AA =(A+A')/2; % symmetrize
    v = [v; eig(AA)]; % eigenvalues
end
```

Plot

```
[count, x]= hist(v, -3:dx:3);
bar(x, count/(K*N*dx), 'y');
hold on ;
```



Theory

```
x = -2:0.01:2;
plot(x,sqrt(4-x.^2)/(2*pi), 'LineWidth', 2);
axis([-3 3 0 0.35]);
grid on;
title('N=20, K=10000');
xlabel('\lambda');
ylabel('p(\lambda)');
```

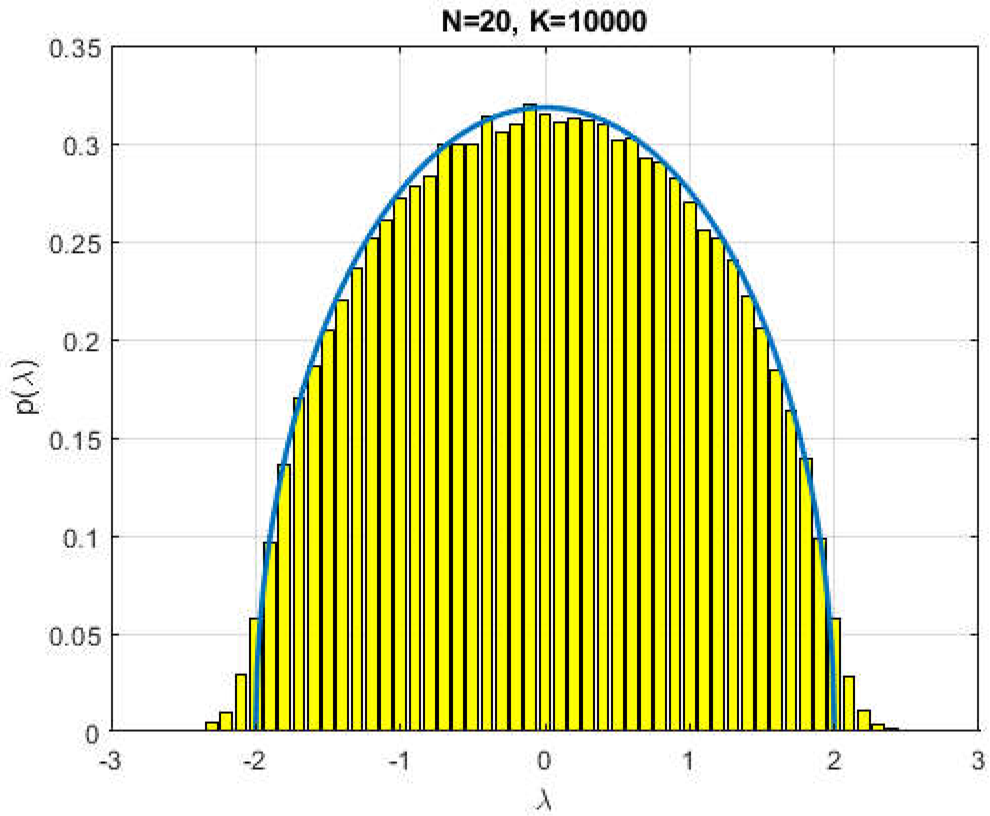


Illustration of almost sure convergence of eigenvalue distribution

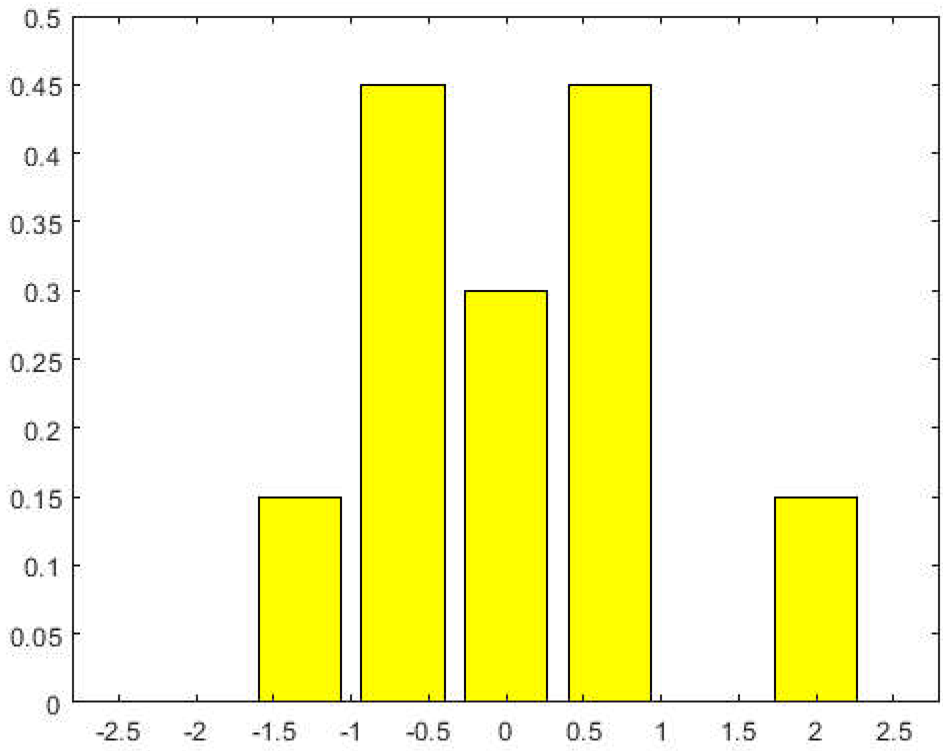
```
figure;
N = 10;
K = 1;
v = []; % eigenvalues
dx = 2/3; % bin size
```

Experiment

```
for i =1:K
    A =randn(N)/sqrt(N/2); % random NxN Gaussian matrix; Note: normalize by sqrt(N/2)
 and not sqrt(N) so AA will have unit variance.
    AA =(A+A')/2; % symmetrize
    v = [v; eig(AA)]; % eigenvalues
end
```

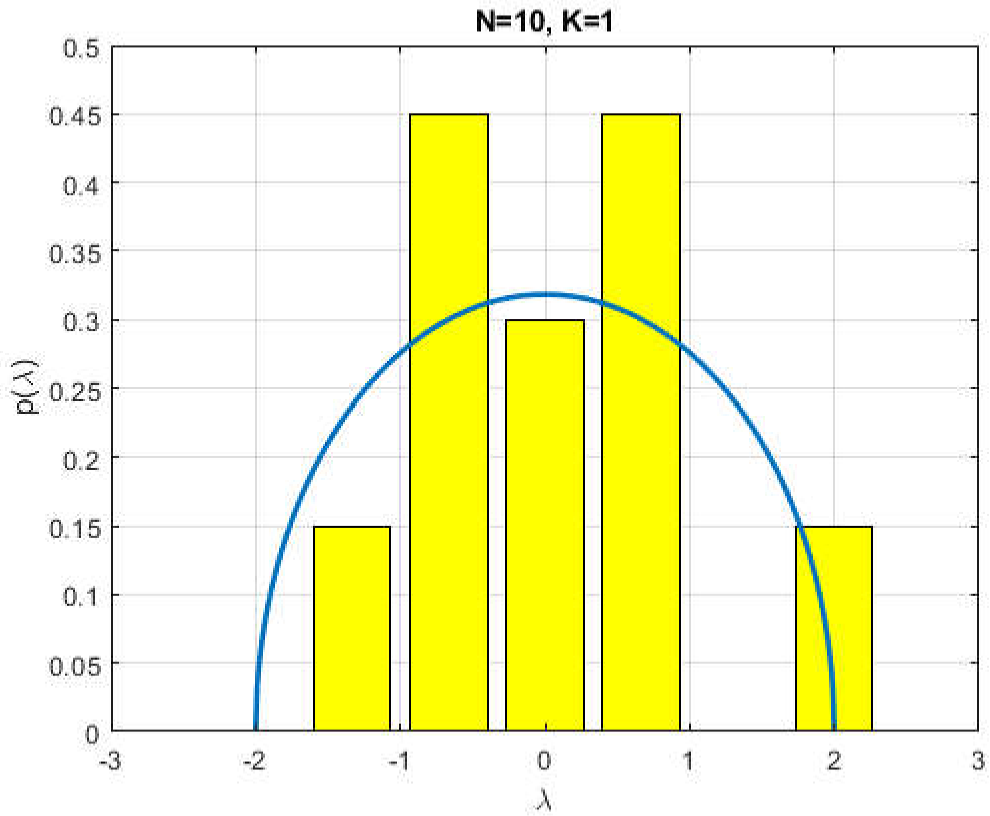
Plot

```
[count, x]= hist(v, -2:dx:2);
bar(x, count/(K*N*dx), 'y');
hold on ;
```



Theory

```
x = -2:0.01:2;
plot(x,sqrt(4-x.^2)/(2*pi), 'LineWidth', 2);
axis([-3 3 0 0.5]);
grid on;
title('N=10, K=1');
xlabel('\lambda');
ylabel('p(\lambda)');
%%%%%%%%
%%%%%%%%
figure;
N = 100;
K = 1;
v = []; % eigenvalues
dx = 0.4; % bin size
```



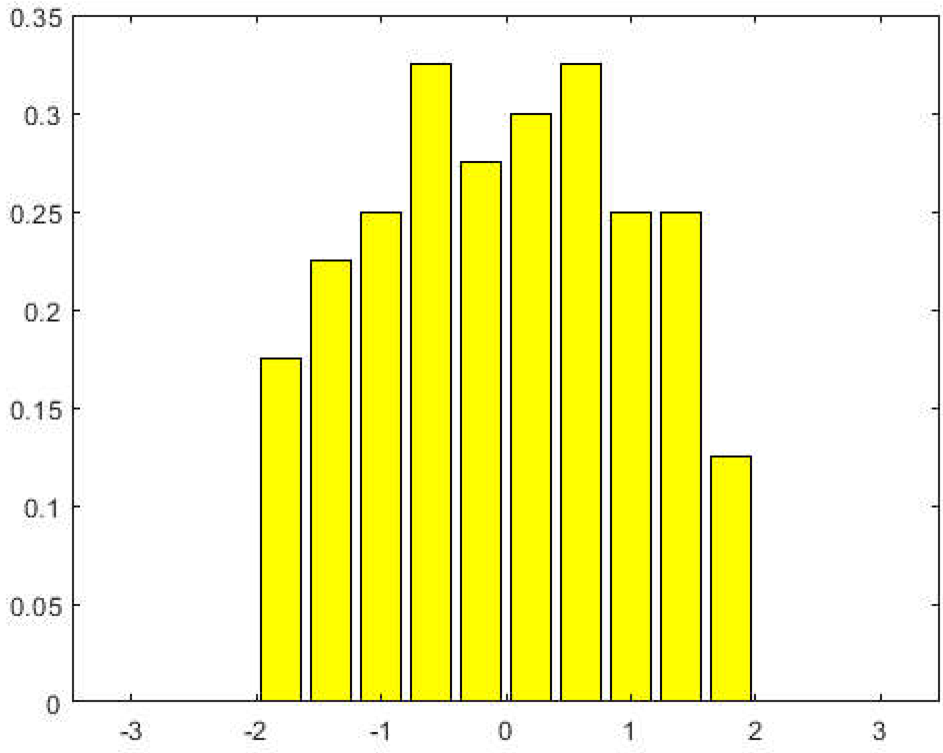
Experiment

```
for i =1:K
    A =randn(N)/sqrt(N/2); % random NxN Gaussian matrix; Note: normalize by sqrt(N/2)
 and not sqrt(N) so AA will have unit variance.
```

```
    AA =(A+A')/2; % symmetrize
    v = [v; eig(AA)]; % eigenvalues
end
```

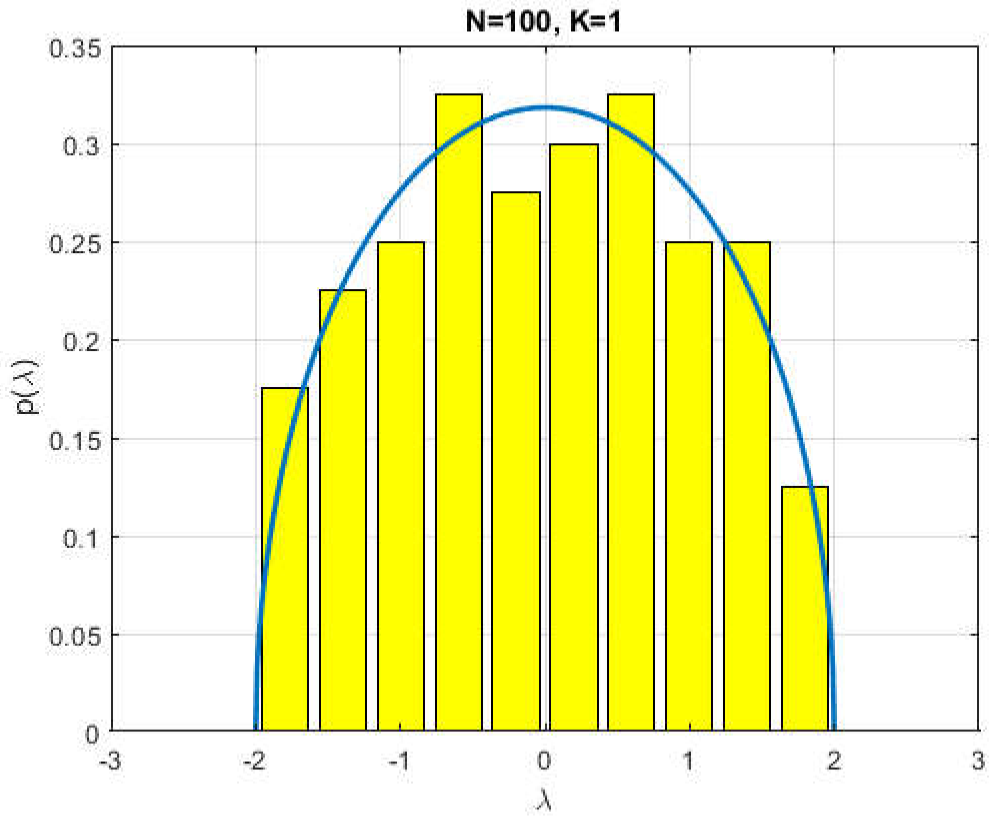
Plot

```
[count, x]= hist(v, -3:dx:3);
bar(x, count/(K*N*dx), 'y');
hold on ;
```



Theory

```
x = -2:0.01:2;
plot(x,sqrt(4-x.^2)/(2*pi), 'LineWidth', 2);
axis([-3 3 0 0.35]);
grid on;
title('N=100, K=1');
xlabel('\lambda');
ylabel('p(\lambda)');
%%%%%%%%
%%%%%%%%
figure;
N = 1000;
K = 1;
v = []; % eigenvalues
dx = 1/6; % bin size
```



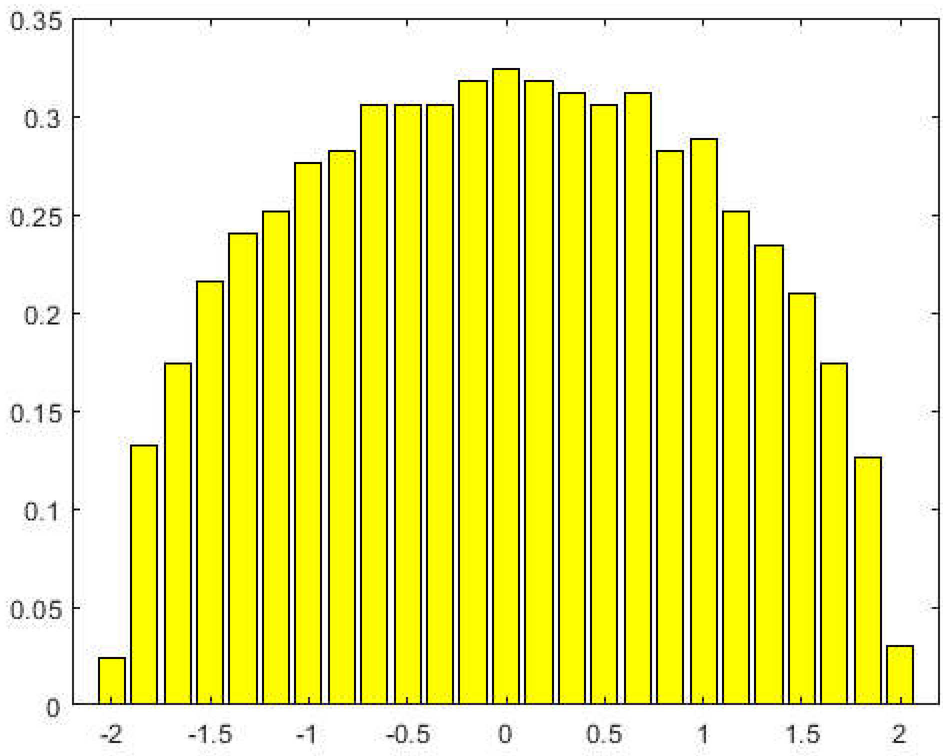
Experiment

```
for i =1:K
    A =randn(N)/sqrt(N/2); % random NxN Gaussian matrix; Note: normalize by sqrt(N/2)
 and not sqrt(N) so AA will have unit variance.
```

```
    AA =(A+A')/2; % symmetrize
    v = [v; eig(AA)]; % eigenvalues
end
```

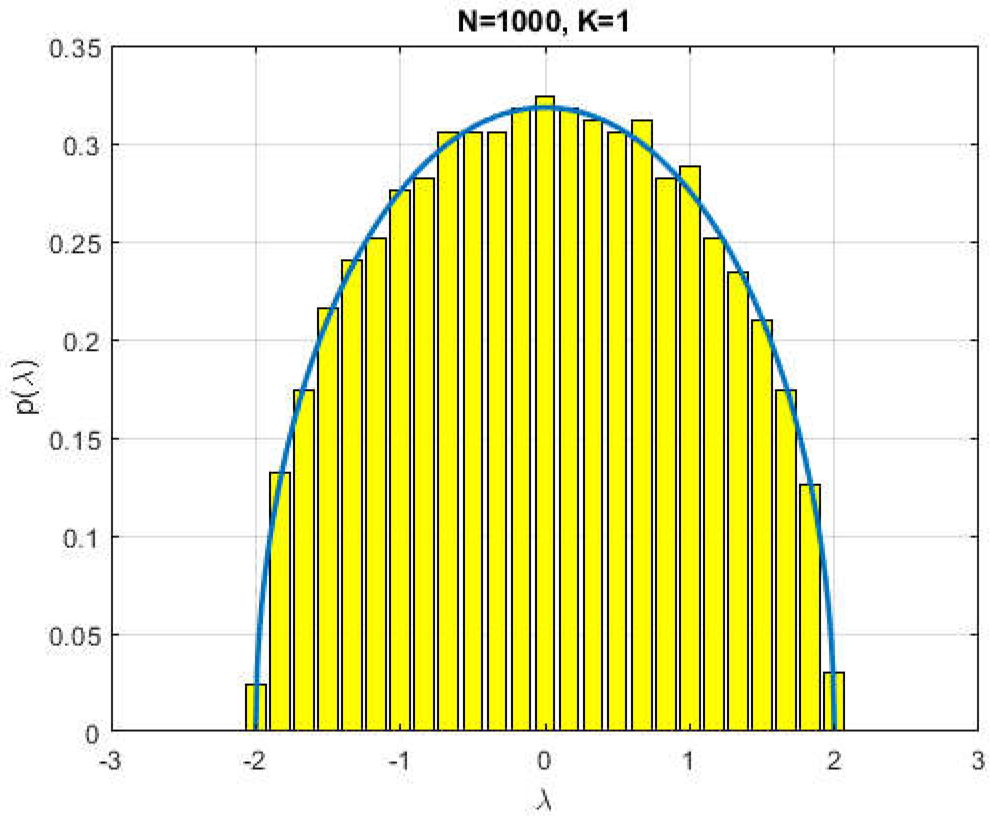
Plot

```
[count, x]= hist(v, -2:dx:2);
bar(x, count/(K*N*dx), 'y');
hold on ;
```



Theory

```
x = -2:0.01:2;
plot(x,sqrt(4-x.^2)/(2*pi), 'LineWidth', 2);
axis([-3 3 0 0.35]);
grid on;
title('N=1000, K=1');
xlabel('\lambda');
ylabel('p(\lambda)');
%%%%%%%%
%%%%%%%%
figure;
N = 4000;
K = 1;
v = []; % eigenvalues
dx = 0.1; % bin size
```



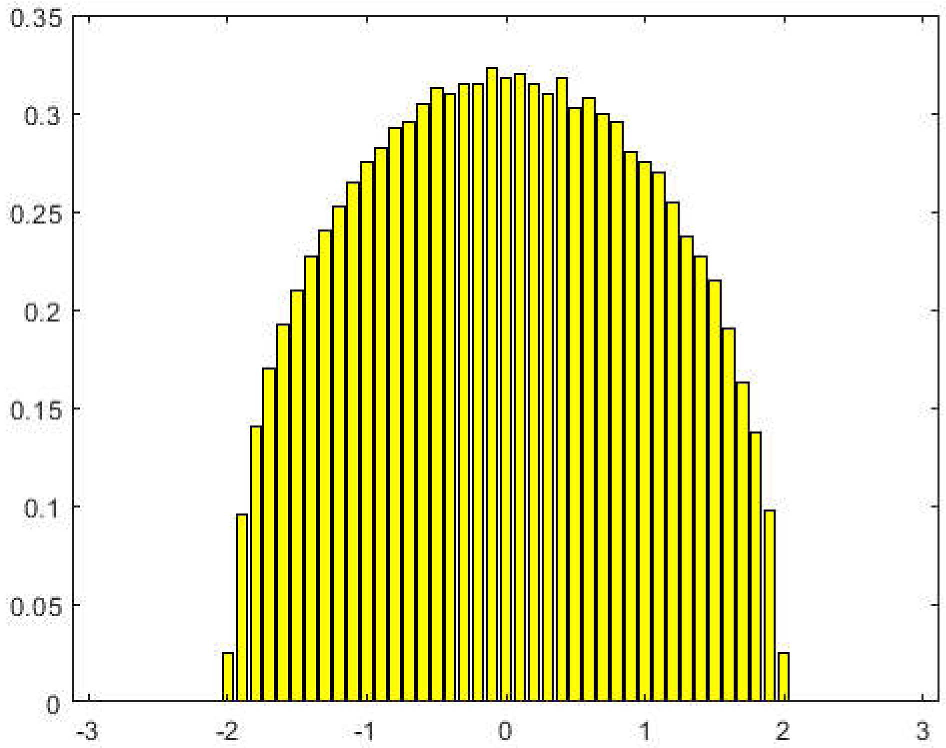
Experiment

```
for i =1:K
    A =randn(N)/sqrt(N/2); % random NxN Gaussian matrix; Note: normalize by sqrt(N/2)
 and not sqrt(N) so AA will have unit variance.
```

```
    AA =(A+A')/2; % symmetrize
    v = [v; eig(AA)]; % eigenvalues
end
```

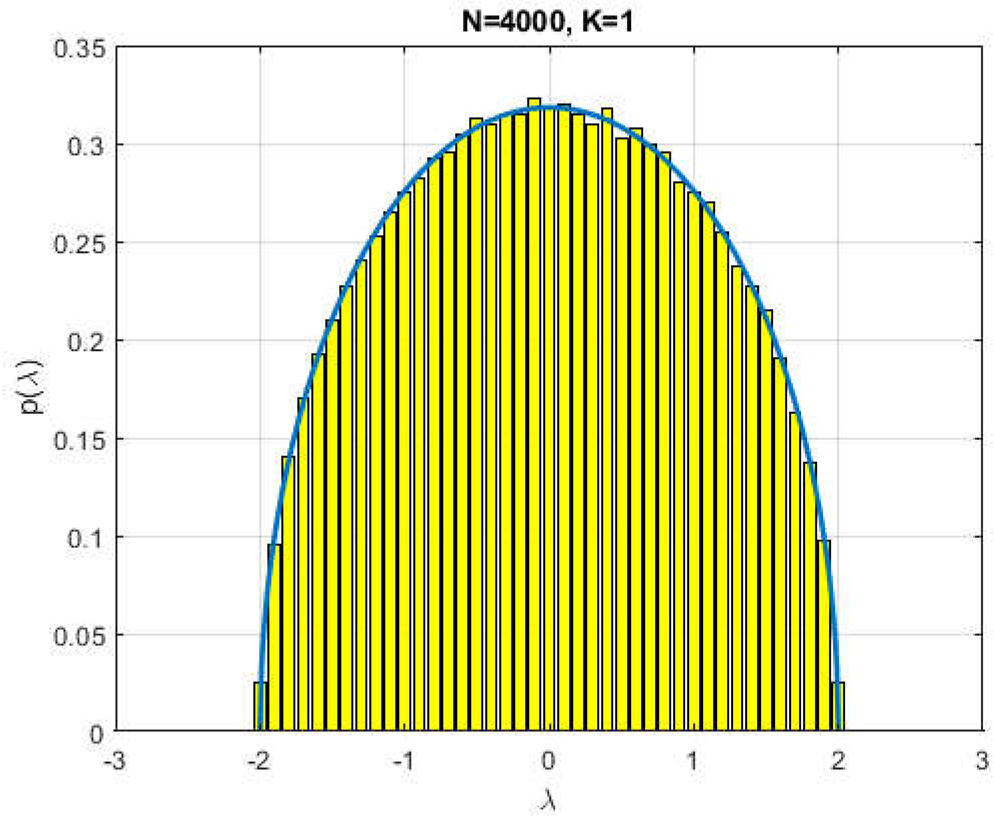
Plot

```
[count, x]= hist(v, -3:dx:3);
bar(x, count/(K*N*dx), 'y');
hold on ;
```



Theory

```
x = -2:0.01:2;
plot(x,sqrt(4-x.^2)/(2*pi), 'LineWidth', 2);
axis([-3 3 0 0.35]);
grid on;
title('N=4000, K=1');
xlabel('\lambda');
ylabel('p(\lambda)');
```



MP law

Illustration of convergence of averaged eigenvalue distribution

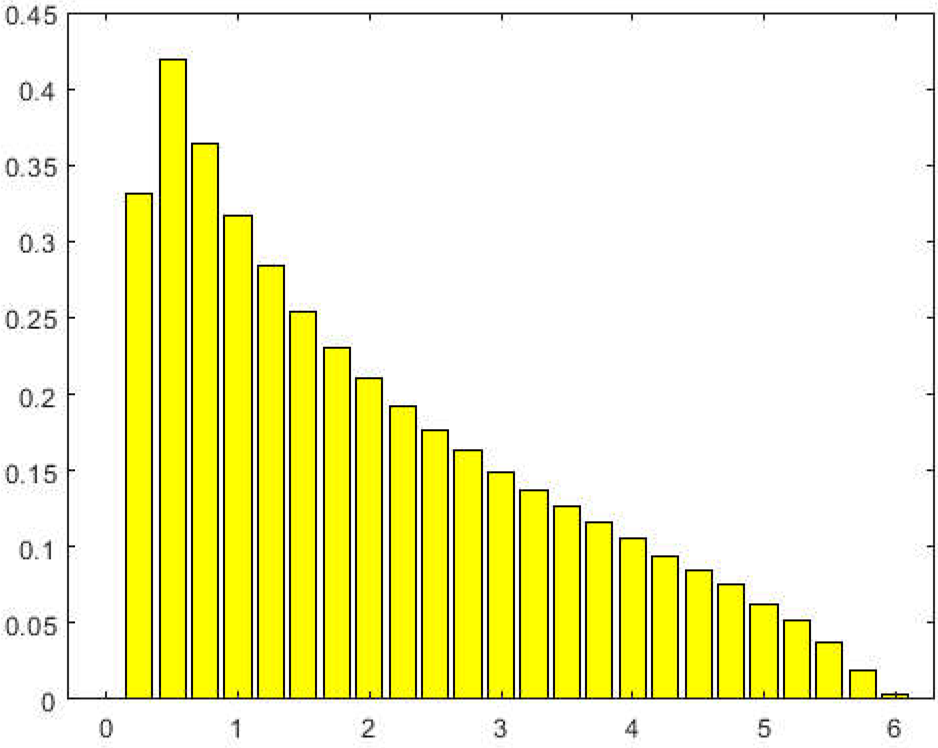
```
figure;
N = 100;
M = 200;
K = 3000;
v = []; % eigenvalues
dx = 0.25; % bin size
```

Experiment

```
for i =1:K
    X =randn(N,M); % random NxN Gaussian matrix;
    A =X*X'/N; % Wishart
    v = [v; eig(A)]; % eigenvalues
end
```

Plot

```
alpha = M/N;
%[count, x]= hist(v, (1-sqrt(alpha))^2:dx:(1+sqrt(alpha))^2);
[count, x]= hist(v, 0:dx:6);
bar(x, count/(K*N*dx), 'y');
hold on ;
```



Theory

```
x = (1-sqrt(alpha))^2:0.01:(1+sqrt(alpha))^2;
plot(x,sqrt(4*alpha-(x-1-alpha).^2)./(2*pi*x), 'LineWidth', 2);
axis([-0 6 0 0.5]);
grid on;
title('N=100, \alpha=2, K=3000');
xlabel('\lambda');
ylabel('p(\lambda)');
```

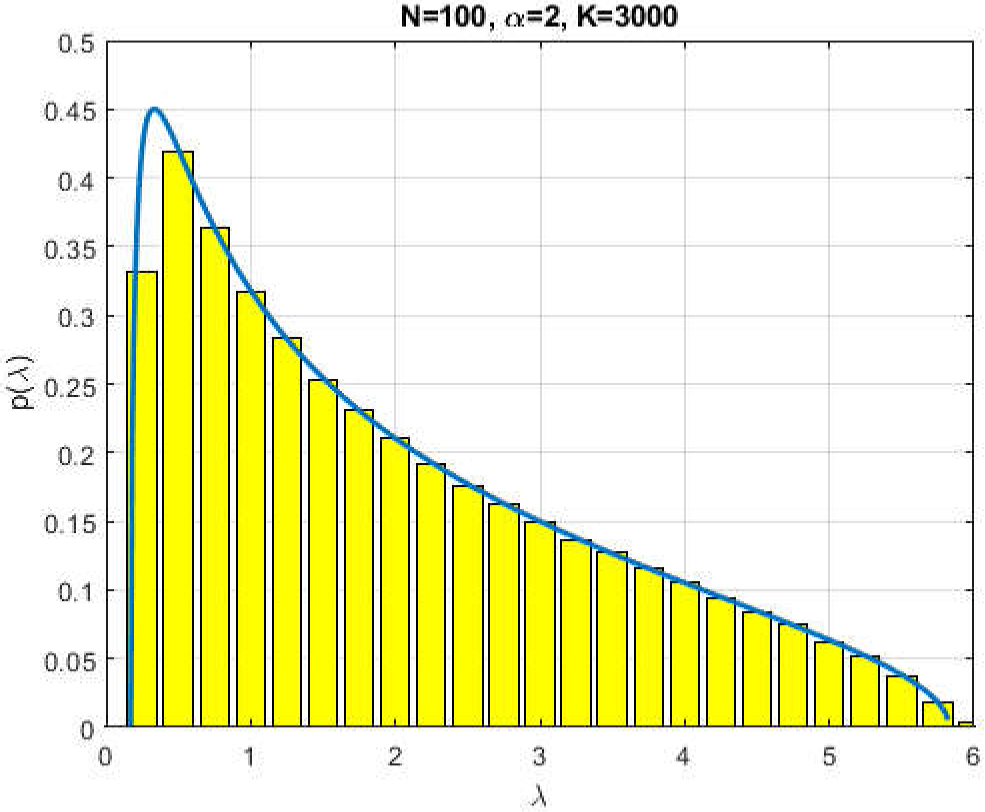


Illustration of almost sure convergence of eigenvalue distribution

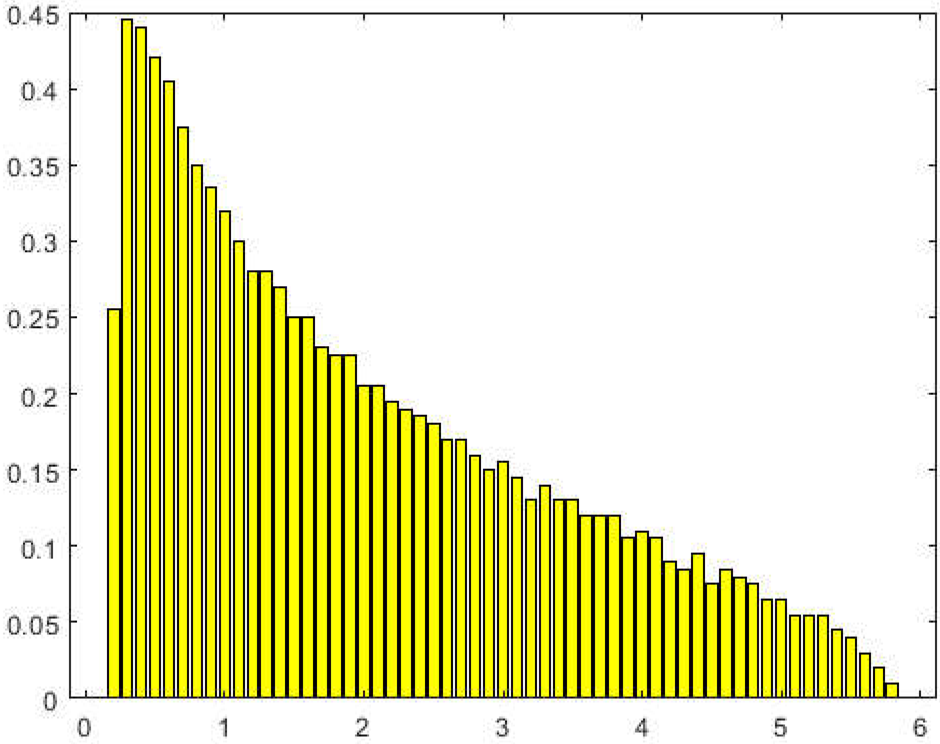
```
figure;
N = 2000;
M = 4000;
K = 1;
v = []; % eigenvalues
dx = 0.1; % bin size
```

Experiment

```
for i =1:K
    X =randn(N,M); % random NxN Gaussian matrix;
    A =X*X'/N; % Wishart
    v = [v; eig(A)]; % eigenvalues
end
```

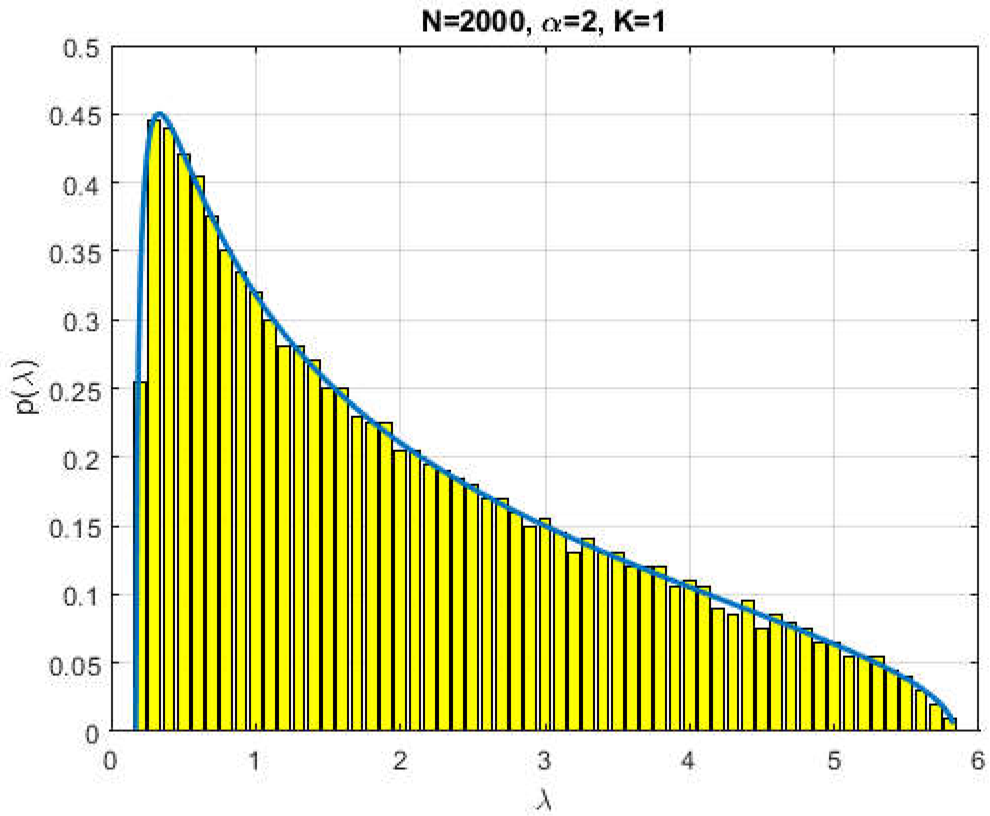
Plot

```
alpha = M/N;
%[count, x]= hist(v, (1-sqrt(alpha))^2:dx:(1+sqrt(alpha))^2);
[count, x]= hist(v, 0:dx:6);
bar(x, count/(K*N*dx), 'y');
hold on ;
```



Theory

```
x = (1-sqrt(alpha))^2:0.01:(1+sqrt(alpha))^2;
plot(x,sqrt(4*alpha-(x-1-alpha).^2)./(2*pi*x), 'LineWidth', 2);
axis([-0 6 0 0.5]);
grid on;
title('N=2000, \alpha=2, K=1');
xlabel('\lambda');
ylabel('p(\lambda)');
```



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