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Eigenvalues of Random Power law Graphs

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Abstract. Many graphs arising in various information networks exhibit the "power law" behavior — the number of vertices of degree k is proportional to $k^{-\beta}$ for some positive β. We show that if $\beta > 2.5$, the largest eigenvalue of a random power law graph is almost surely $(1+o(1))\sqrt{m}$ where m is the maximum degree. Moreover, the k largest eigenvalues of a random power law graph with exponent β have power law distribution with exponent $2\beta - 1$ if the maximum degree is sufficiently large, where k is a function depending on β, m and d, the average degree. When $2 < \beta < 2.5$, the largest eigenvalue is heavily concentrated at $cm^{3-\beta}$ for some constant c depending on β and the average degree. This result follows from a more general theorem which shows that the largest eigenvalue of a random graph with a given expected degree sequence is determined by m, the maximum degree, and $\tilde{d}$, the weighted average of the squares of the expected degrees. We show that the k-th largest eigenvalue is almost surely $(1+o(1))\sqrt{m_k}$ where m_k is the k-th largest expected degree provided m_k is large enough. These results have implications on the usage of spectral techniques in many areas related to pattern detection and information retrieval.

Keywords: random graphs, power law, eigenvalues

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