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# Event Studies in Economics and Finance

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## 1. Introduction

ECONOMISTS are frequently asked to measure the effects of an economic event on the value of firms. On the surface this seems like a difficult task, but a measure can be constructed easily using an event study. Using financial market data, an event study measures the impact of a specific event on the value of a firm. The usefulness of such a study comes from the fact that, given rationality in the marketplace, the effects of an event will be reflected immediately in security prices. Thus a measure of the event's economic impact can be constructed using security prices observed over a relatively short time period. In contrast, direct productivity related measures may require many months or even years of observation.

The event study has many applications. In accounting and finance research, event studies have been applied to a variety of firm specific and economy wide events. Some examples include mergers and acquisitions, earnings announcements, issues of new debt or equity, and announcements of macroeconomic variables such as the trade

deficit.<sup>1</sup> However, applications in other fields are also abundant. For example, event studies are used in the field of law and economics to measure the impact on the value of a firm of a change in the regulatory environment (see G. William Schwert 1981) and in legal liability cases event studies are used to assess damages (see Mark Mitchell and Jeffry Netter 1994). In the majority of applications, the focus is the effect of an event on the price of a particular class of securities of the firm, most often common equity. In this paper the methodology is discussed in terms of applications that use common equity. However, event studies can be applied using debt securities with little modification.

Event studies have a long history. Perhaps the first published study is James Dolley (1933). In this work, he examines the price effects of stock splits, studying nominal price changes at the time of the split. Using a sample of 95 splits from 1921 to 1931, he finds that the price in-

<sup>1</sup> The first three examples will be discussed later in the paper. Grant McQueen and Vance Roley (1993) provide an illustration of the fourth using macroeconomic news announcements.

creased in 57 of the cases and the price declined in only 26 instances. Over the decades from the early 1930s until the late 1960s the level of sophistication of event studies increased. John H. Myers and Archie Bakay (1948), C. Austin Barker (1956, 1957, 1958), and John Ashley (1962) are examples of studies during this time period. The improvements included removing general stock market price movements and separating out confounding events. In the late 1960s seminal studies by Ray Ball and Philip Brown (1968) and Eugene Fama et al. (1969) introduced the methodology that is essentially the same as that which is in use today. Ball and Brown considered the information content of earnings, and Fama et al. studied the effects of stock splits after removing the effects of simultaneous dividend increases.

In the years since these pioneering studies, a number of modifications have been developed. These modifications relate to complications arising from violations of the statistical assumptions used in the early work and relate to adjustments in the design to accommodate more specific hypotheses. Useful papers which deal with the practical importance of many of the complications and adjustments are the work by Stephen Brown and Jerold Warner published in 1980 and 1985. The 1980 paper considers implementation issues for data sampled at a monthly interval and the 1985 paper deals with issues for daily data.

In this paper, event study methods are reviewed and summarized. The paper begins with discussion of one possible procedure for conducting an event study in Section 2. Section 3 sets up a sample event study which will be used to illustrate the methodology. Central to an event study is the measurement of an abnormal stock return. Section 4 details the first step—measuring the normal performance—and Section 5 follows

with the necessary tools for calculating an abnormal return, making statistical inferences about these returns, and aggregating over many event observations. The null hypothesis that the event has no impact on the distribution of returns is maintained in Sections 4 and 5. Section 6 discusses modifying this null hypothesis to focus only on the mean of the return distribution. Section 7 presents analysis of the power of an event study. Section 8 presents nonparametric approaches to event studies which eliminate the need for parametric structure. In some cases theory provides hypotheses concerning the relation between the magnitude of the event abnormal return and firm characteristics. Section 9 presents a cross-sectional regression approach that is useful to investigate such hypotheses. Section 10 considers some further issues relating event study design and the paper closes with the concluding discussion in Section 11.

## *2. Procedure for an Event Study*

At the outset it is useful to briefly discuss the structure of an event study. This will provide a basis for the discussion of details later. While there is no unique structure, there is a general flow of analysis. This flow is discussed in this section.

The initial task of conducting an event study is to define the event of interest and identify the period over which the security prices of the firms involved in this event will be examined—the event window. For example, if one is looking at the information content of an earnings with daily data, the event will be the earnings announcement and the event window will include the one day of the announcement. It is customary to define the event window to be larger than the specific period of interest. This permits examination of periods surrounding the

event. In practice, the period of interest is often expanded to multiple days, including at least the day of the announcement and the day after the announcement. This captures the price effects of announcements which occur after the stock market closes on the announcement day. The periods prior to and after the event may also be of interest. For example, in the earnings announcement case, the market may acquire information about the earnings prior to the actual announcement and one can investigate this possibility by examining pre-event returns.

After identifying the event, it is necessary to determine the selection criteria for the inclusion of a given firm in the study. The criteria may involve restrictions imposed by data availability such as listing on the New York Stock Exchange or the American Stock Exchange or may involve restrictions such as membership in a specific industry. At this stage it is useful to summarize some sample characteristics (e.g., firm market capitalization, industry representation, distribution of events through time) and note any potential biases which may have been introduced through the sample selection.

Appraisal of the event's impact requires a measure of the abnormal return. The abnormal return is the actual ex post return of the security over the event window minus the normal return of the firm over the event window. The normal return is defined as the expected return without conditioning on the event taking place. For firm  $i$  and event date  $\tau$  the abnormal return is

$$AR_{i\tau} = R_{i\tau} - E(R_{i\tau}|X_\tau) \quad (1)$$

where  $AR_{i\tau}$ ,  $R_{i\tau}$ , and  $E(R_{i\tau}|X_\tau)$  are the abnormal, actual, and normal returns respectively for time period  $\tau$ .  $X_\tau$  is the conditioning information for the normal return model. There are two common

choices for modeling the normal return—the *constant mean return model* where  $X_\tau$  is a constant, and the *market model* where  $X_\tau$  is the market return. The constant mean return model, as the name implies, assumes that the mean return of a given security is constant through time. The market model assumes a stable linear relation between the market return and the security return.

Given the selection of a normal performance model, the estimation window needs to be defined. The most common choice, when feasible, is using the period prior to the event window for the estimation window. For example, in an event study using daily data and the market model, the market model parameters could be estimated over the 120 days prior to the event. Generally the event period itself is not included in the estimation period to prevent the event from influencing the normal performance model parameter estimates.

With the parameter estimates for the normal performance model, the abnormal returns can be calculated. Next comes the design of the testing framework for the abnormal returns. Important considerations are defining the null hypothesis and determining the techniques for aggregating the individual firm abnormal returns.

The presentation of the empirical results follows the formulation of the econometric design. In addition to presenting the basic empirical results, the presentation of diagnostics can be fruitful. Occasionally, especially in studies with a limited number of event observations, the empirical results can be heavily influenced by one or two firms. Knowledge of this is important for gauging the importance of the results.

Ideally the empirical results will lead to insights relating to understanding the sources and causes of the effects (or lack

of effects) of the event under study. Additional analysis may be included to distinguish between competing explanations. Concluding comments complete the study.

### 3. *An Example of an Event Study*

The Financial Accounting Standards Board (FASB) and the Securities Exchange Commission strive to set reporting regulations so that financial statements and related information releases are informative about the value of the firm. In setting standards, the information content of the financial disclosures is of interest. Event studies provide an ideal tool for examining the information content of the disclosures.

In this section the description of an example selected to illustrate event study methodology is presented. One particular type of disclosure—quarterly earnings announcements—is considered. The objective is to investigate the information content of these announcements. In other words, the goal is to see if the release of accounting information provides information to the marketplace. If so there should be a correlation between the observed change of the market value of the company and the information.

The example will focus on the quarterly earnings announcements for the 30 firms in the Dow Jones Industrial Index over the five-year period from January 1989 to December 1993. These announcements correspond to the quarterly earnings for the last quarter of 1988 through the third quarter of 1993. The five years of data for 30 firms provide a total sample of 600 announcements. For each firm and quarter, three pieces of information are compiled: the date of the announcement, the actual earnings, and a measure of the expected earnings. The source of the date of the announcement

is Datastream, and the source of the actual earnings is Compustat.

If earnings announcements convey information to investors, one would expect the announcement impact on the market's valuation of the firm's equity to depend on the magnitude of the unexpected component of the announcement. Thus a measure of the deviation of the actual announced earnings from the market's prior expectation is required. For constructing such a measure, the mean quarterly earnings forecast reported by the Institutional Brokers Estimate System (I/B/E/S) is used to proxy for the market's expectation of earnings. I/B/E/S compiles forecasts from analysts for a large number of companies and reports summary statistics each month. The mean forecast is taken from the last month of the quarter. For example, the mean third quarter forecast from September 1990 is used as the measure of expected earnings for the third quarter of 1990.

To facilitate the examination of the impact of the earnings announcement on the value of the firm's equity, it is essential to posit the relation between the information release and the change in value of the equity. In this example the task is straightforward. If the earnings disclosures have information content, higher than expected earnings should be associated with increases in value of the equity and lower than expected earnings with decreases. To capture this association, each announcement is assigned to one of three categories: good news, no news, or bad news. Each announcement is categorized using the deviation of the actual earnings from the expected earnings. If the actual exceeds expected by more than 2.5 percent the announcement is designated as good news, and if the actual is more than 2.5 percent less than expected the announcement is designated as bad news. Those announce-

ments where the actual earnings is in the 5 percent range centered about the expected earnings are designated as no news. Of the 600 announcements, 189 are good news, 173 are no news, and the remaining 238 are bad news.

With the announcements categorized, the next step is to specify the parameters of the empirical design to analyze the equity return, i.e., the percent change in value of the equity. It is necessary to specify a length of observation interval, an event window, and an estimation window. For this example the interval is set to one day, thus daily stock returns are used. A 41-day event window is employed, comprised of 20 pre-event days, the event day, and 20 post-event days. For each announcement the 250 trading day period prior to the event window is used as the estimation window. After presenting the methodology of an event study, this example will be drawn upon to illustrate the execution of a study.

#### 4. Models for Measuring Normal Performance

A number of approaches are available to calculate the normal return of a given security. The approaches can be loosely grouped into two categories—statistical and economic. Models in the first category follow from statistical assumptions concerning the behavior of asset returns and do not depend on any economic arguments. In contrast, models in the second category rely on assumptions concerning investors' behavior and are not based solely on statistical assumptions. It should, however, be noted that to use economic models in practice it is necessary to add statistical assumptions. Thus the potential advantage of economic models is not the absence of statistical assumptions, but the opportunity to calculate more precise measures of the normal return using economic restrictions.

For the statistical models, the assumption that asset returns are jointly multivariate normal and independently and identically distributed through time is imposed. This distributional assumption is sufficient for the constant mean return model and the market model to be correctly specified. While this assumption is strong, in practice it generally does not lead to problems because the assumption is empirically reasonable and inferences using the normal return models tend to be robust to deviations from the assumption. Also one can easily modify the statistical framework so that the analysis of the abnormal returns is autocorrelation and heteroskedasticity consistent by using a generalized method-of-moments approach.

##### A. Constant Mean Return Model

Let  $\mu_i$  be the mean return for asset  $i$ . Then the constant mean return model is

$$R_{it} = \mu_i + \zeta_{it} \quad (2)$$

$$E(\zeta_{it}) = 0 \quad \text{var}(\zeta_{it}) = \sigma_{\zeta_i}^2,$$

where  $R_{it}$  is the period- $t$  return on security  $i$  and  $\zeta_{it}$  is the time period  $t$  disturbance term for security  $i$  with an expectation of zero and variance  $\sigma_{\zeta_i}^2$ .

Although the constant mean return model is perhaps the simplest model, Brown and Warner (1980, 1985) find it often yields results similar to those of more sophisticated models. This lack of sensitivity to the model can be attributed to the fact that the variance of the abnormal return is frequently not reduced much by choosing a more sophisticated model. When using daily data the model is typically applied to nominal returns. With monthly data the model can be applied to real returns or excess returns (the return in excess of the nominal risk free return generally measured using the U.S. Treasury Bill with one month to maturity) as well as nominal returns.



### B. Market Model

The market model is a statistical model which relates the return of any given security to the return of the market portfolio. The model's linear specification follows from the assumed joint normality of asset returns. For any security  $i$  the market model is

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \quad (3)$$

$$E(\varepsilon_{it}) = 0 \quad \text{var}(\varepsilon_{it}) = \sigma_{\varepsilon_i}^2$$

where  $R_{it}$  and  $R_{mt}$  are the period- $t$  returns on security  $i$  and the market portfolio, respectively, and  $\varepsilon_{it}$  is the zero mean disturbance term.  $\alpha_i$ ,  $\beta_i$ , and  $\sigma_{\varepsilon_i}^2$  are the parameters of the market model. In applications a broad based stock index is used for the market portfolio, with the S&P 500 Index, the CRSP Value Weighted Index, and the CRSP Equal Weighted Index being popular choices.

The market model represents a potential improvement over the constant mean return model. By removing the portion of the return that is related to variation in the market's return, the variance of the abnormal return is reduced. This in turn can lead to increased ability to detect event effects. The benefit from using the market model will depend upon the  $R^2$  of the market model regression. The higher the  $R^2$  the greater is the variance reduction of the abnormal return, and the larger is the gain.

### C. Other Statistical Models

A number of other statistical models have been proposed for modeling the normal return. A general type of statistical model is the *factor model*. Factor models are motivated by the benefits of reducing the variance of the abnormal return by explaining more of the variation in the normal return. Typically the factors are portfolios of traded securities.

The market model is an example of a one factor model. Other multifactor models include industry indexes in addition to the market. William Sharpe (1970) and Sharpe, Gordon Alexander, and Jeffery Bailey (1995, p. 303) provide discussion of index models with factors based on industry classification. Another variant of a factor model is a procedure which calculates the abnormal return by taking the difference between the actual return and a portfolio of firms of similar size, where size is measured by market value of equity. In this approach typically ten size groups are considered and the loading on the size portfolios is restricted to unity. This procedure implicitly assumes that expected return is directly related to market value of equity.

Generally, the gains from employing multifactor models for event studies are limited. The reason for the limited gains is the empirical fact that the marginal explanatory power of additional factors the market factor is small, and hence, there is little reduction in the variance of the abnormal return. The variance reduction will typically be greatest in cases where the sample firms have a common characteristic, for example they are all members of one industry or they are all firms concentrated in one market capitalization group. In these cases the use of a multifactor model warrants consideration.

The use of other models is dictated by data availability. An example of a normal performance return model implemented in situations with limited data is the market-adjusted return model. For some events it is not feasible to have a pre-event estimation period for the normal model parameters, and a market-adjusted abnormal return is used. The market-adjusted return model can be viewed as a restricted market model with  $\alpha_i$  constrained to be zero and  $\beta_i$  constrained to be one. Because the model coefficients

are prespecified, an estimation period is not required to obtain parameter estimates. An example of when such a model is used is in studies of the under pricing of initial public offerings. Jay Ritter (1991) presents such an example. A general recommendation is to only use such restricted models if necessary, and if necessary, consider the possibility of biases arising from the imposition of the restrictions.

#### D. Economic Models

Economic models can be cast as restrictions on the statistical models to provide more constrained normal return models. Two common economic models which provide restrictions are the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT). The CAPM due to Sharpe (1964) and John Lintner (1965) is an equilibrium theory where the expected return of a given asset is determined by its covariance with the market portfolio. The APT due to Stephen Ross (1976) is an asset pricing theory where the expected return of a given asset is a linear combination of multiple risk factors.

The use of the Capital Asset Pricing Model is common in event studies of the 1970s. However, deviations from the CAPM have been discovered, implying that the validity of the restrictions imposed by the CAPM on the market model is questionable.<sup>2</sup> This has introduced the possibility that the results of the studies may be sensitive to the specific CAPM restrictions. Because this potential for sensitivity can be avoided at little cost by using the market model, the use of the CAPM has almost ceased.

Similarly, other studies have employed multifactor normal performance models

motivated by the Arbitrage Pricing Theory. A general finding is that with the APT the most important factor behaves like a market factor and additional factors add relatively little explanatory power. Thus the gains from using an APT motivated model versus the market model are small. See Stephen Brown and Mark Weinstein (1985) for further discussion. The main potential gain from using a model based on the arbitrage pricing theory is to eliminate the biases introduced by using the CAPM. However, because the statistically motivated models also eliminate these biases, for event studies such models dominate.

#### 5. Measuring and Analyzing Abnormal Returns

In this section the problem of measuring and analyzing abnormal returns is considered. The framework is developed using the market model as the normal performance return model. The analysis is virtually identical for the constant mean return model.

Some notation is first defined to facilitate the measurement and analysis of abnormal returns. Returns will be indexed in event time using  $\tau$ . Defining  $\tau = 0$  as the event date,  $\tau = T_1 + 1$  to  $\tau = T_2$  represents the event window, and  $\tau = T_0 + 1$  to  $\tau = T_1$  constitutes the estimation window. Let  $L_1 = T_1 - T_0$  and  $L_2 = T_2 - T_1$  be the length of the estimation window and the event window respectively. Even if the event being considered is an announcement on given date it is typical to set the event window length to be larger than one. This facilitates the use of abnormal returns around the event day in the analysis. When applicable, the post-event window will be from  $\tau = T_2 + 1$  to  $\tau = T_3$  and of length  $L_3 = T_3 - T_2$ . The timing sequence is illustrated with a time line in Figure 1.

<sup>2</sup> Eugene Fama and Kenneth French (1996) provide discussion of these anomalies.



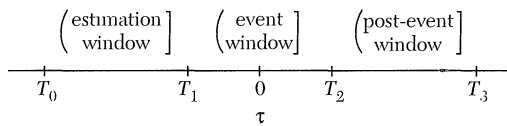


Figure 1. Time line for an event study.

It is typical for the estimation window and the event window not to overlap. This design provides estimators for the parameters of the normal return model which are not influenced by the returns around the event. Including the event window in the estimation of the normal model parameters could lead to the event returns having a large influence on the normal return measure. In this situation both the normal returns and the abnormal returns would capture the event impact. This would be problematic because the methodology is built around the assumption that the event impact is captured by the abnormal returns. On occasion, the post event window data is included with the estimation window data to estimate the normal return model. The goal of this approach is to increase the robustness of the normal market return measure to gradual changes in its parameters. In Section 6 expanding the null hypothesis to accommodate changes in the risk of a firm around the event is considered. In this case an estimation framework which uses the event window returns will be required.

#### A. Estimation of the Market Model

Under general conditions ordinary least squares (OLS) is a consistent estimation procedure for the market model parameters. Further, given the assumptions of Section 4, OLS is efficient. For the  $i^{\text{th}}$  firm in event time, the OLS estimators of the market model parameters for an estimation window of observations are

$$\hat{\beta}_i = \frac{\sum_{\tau=T_0+1}^{T_1} (R_{i\tau} - \hat{\mu}_i)(R_{m\tau} - \hat{\mu}_m)}{\sum_{\tau=T_0+1}^{T_1} (R_{m\tau} - \hat{\mu}_m)^2} \quad (4)$$

$$\hat{\alpha}_i = \hat{\mu}_i - \hat{\beta}_i \hat{\mu}_m \quad (5)$$

$$\hat{\sigma}_{\varepsilon_i}^2 = \frac{1}{L_1 - 2} \sum_{\tau=T_0+1}^{T_1} (R_{i\tau} - \hat{\alpha}_i - \hat{\beta}_i R_{m\tau})^2 \quad (6)$$

where

$$\hat{\mu}_i = \frac{1}{L_1} \sum_{\tau=T_0+1}^{T_1} R_{i\tau}$$

and 
$$\hat{\mu}_m = \frac{1}{L_1} \sum_{\tau=T_0+1}^{T_1} R_{m\tau}.$$

$R_{i\tau}$  and  $R_{m\tau}$  are the return in event period  $\tau$  for security  $i$  and the market respectively. The use of the OLS estimators to measure abnormal returns and to develop their statistical properties is addressed next. First, the properties of a given security are presented followed by consideration of the properties of abnormal returns aggregated across securities.

#### B. Statistical Properties of Abnormal Returns

Given the market model parameter estimates, one can measure and analyze the abnormal returns. Let  $AR_{i\tau}$ ,  $\tau = T_1 + 1, \dots, T_2$ , be the sample of  $L_2$  abnormal returns for firm  $i$  in the event window. Using the market model to measure the normal return, the sample abnormal return is

$$AR_{i\tau} = R_{i\tau} - \hat{\alpha}_i - \hat{\beta}_i R_{m\tau}. \quad (7)$$

The abnormal return is the disturbance term of the market model calculated on an out of sample basis. Under the null hypothesis, conditional on the event win-

dow market returns, the abnormal returns will be jointly normally distributed with a zero conditional mean and conditional variance  $\sigma^2(AR_{i\tau})$  where

$$\sigma^2(AR_{i\tau}) = \sigma_{\varepsilon_i}^2 + \frac{1}{L_1} \left[ 1 + \frac{(R_{m\tau} - \hat{\mu}_m)^2}{\hat{\sigma}_m^2} \right]. \quad (8)$$

From (8), the conditional variance has two components. One component is the disturbance variance  $\sigma_{\varepsilon_i}^2$  from (3) and a second component is additional variance due to the sampling error in  $\alpha_i$  and  $\beta_i$ . This sampling error, which is common for all the event window observations, also leads to serial correlation of the abnormal returns despite the fact that the true disturbances are independent through time. As the length of the estimation window  $L_1$  becomes large, the second term approaches zero as the sampling error of the parameters vanishes. The variance of the abnormal return will be  $\sigma_{\varepsilon_i}^2$  and the abnormal return observations will become independent through time. In practice, the estimation window can usually be chosen to be large enough to make it reasonable to assume that the contribution of the second component to the variance of the abnormal return is zero.

Under the null hypothesis,  $H_0$ , that the event has no impact on the behavior of returns (mean or variance) the distributional properties of the abnormal returns can be used to draw inferences over any period within the event window. Under  $H_0$  the distribution of the sample abnormal return of a given observation in the event window is

$$AR_{i\tau} \sim N(0, \sigma^2(AR_{i\tau})). \quad (9)$$

Next (9) is built upon to consider the aggregation of the abnormal returns.

### C. Aggregation of Abnormal Returns

The abnormal return observations must be aggregated in order to draw

overall inferences for the event of interest. The aggregation is along two dimensions—through time and across securities. We will first consider aggregation through time for an individual security and then will consider aggregation both across securities and through time. The concept of a cumulative abnormal return is necessary to accommodate a multiple period event window. Define  $CAR_i(\tau_1, \tau_2)$  as the sample cumulative abnormal return (CAR) from  $\tau_1$  to  $\tau_2$  where  $T_1 < \tau_1 \leq \tau_2 \leq T_2$ . The CAR from  $\tau_1$  to  $\tau_2$  is the sum of the included abnormal returns,

$$CAR_i(\tau_1, \tau_2) = \sum_{\tau=\tau_1}^{\tau_2} AR_{i\tau}. \quad (10)$$

Asymptotically (as  $L_1$  increases) the variance of  $CAR_i$  is

$$\sigma_i^2(\tau_1, \tau_2) = (\tau_2 - \tau_1 + 1) \sigma_{\varepsilon_i}^2. \quad (11)$$

This large sample estimator of the variance can be used for reasonable values of  $L_1$ . However, for small values of  $L_1$  the variance of the cumulative abnormal return should be adjusted for the effects of the estimation error in the normal model parameters. This adjustment involves the second term of (8) and a further related adjustment for the serial covariance of the abnormal return.

The distribution of the cumulative abnormal return under  $H_0$  is

$$CAR_i(\tau_1, \tau_2) \sim N(0, \sigma_i^2(\tau_1, \tau_2)). \quad (12)$$

Given the null distributions of the abnormal return and the cumulative abnormal return, tests of the null hypothesis can be conducted.

However, tests with one event observation are not likely to be useful so it is necessary to aggregate. The abnormal return observations must be aggregated for the event window and across observations of the event. For this aggregation,

TABLE 1

Event Day	Market Model					
	Good News		No News		Bad News	
	AR	CAR	AR	CAR	AR	CAR
-20	.093	.093	.080	.080	-.107	-.107
-19	-.177	-.084	.018	.098	-.180	-.286
-18	.088	.004	.012	.110	.029	-.258
-17	.024	.029	-.151	-.041	-.079	-.337
-16	-.018	.011	-.019	-.060	-.010	-.346
-15	-.040	-.029	.013	-.047	-.054	-.401
-14	.038	.008	.040	-.007	-.021	-.421
-13	.056	.064	-.057	-.065	.007	-.414
-12	.065	.129	.146	.081	-.090	-.504
-11	.069	.199	-.020	.061	-.088	-.592
-10	.028	.227	.025	.087	-.092	-.683
-9	.155	.382	.115	.202	-.040	-.724
-8	.057	.438	.070	.272	.072	-.652
-7	-.010	.428	-.106	.166	-.026	-.677
-6	.104	.532	.026	.192	-.013	-.690
-5	.085	.616	-.085	.107	.164	-.527
-4	.099	.715	.040	.147	-.139	-.666
-3	.117	.832	.036	.183	.098	-.568
-2	.006	.838	.226	.409	-.112	-.680
-1	.164	1.001	-.168	.241	-.180	-.860
0	.965	1.966	-.091	.150	-.679	-1.539
1	.251	2.217	-.008	.142	-.204	-1.743
2	-.014	2.203	.007	.148	.072	-1.672
3	-.164	2.039	.042	.190	.083	-1.589
4	-.014	2.024	.000	.190	.106	-1.483
5	.135	2.160	-.038	.152	.194	-1.289
6	-.052	2.107	-.302	-.150	.076	-1.213
7	.060	2.167	-.199	-.349	.120	-1.093
8	.155	2.323	-.108	-.457	-.041	-1.134
9	-.008	2.315	-.146	-.603	-.069	-1.203
10	.164	2.479	.082	-.521	.130	-1.073
11	-.081	2.398	.040	-.481	-.009	-1.082
12	-.058	2.341	.246	-.235	-.038	-1.119
13	-.165	2.176	.014	-.222	.071	-1.048
14	-.081	2.095	-.091	-.312	.019	-1.029
15	-.007	2.088	-.001	-.314	-.043	-1.072
16	.065	2.153	-.020	-.334	-.086	-1.159
17	.081	2.234	.017	-.317	-.050	-1.208
18	.172	2.406	.054	-.263	.066	-1.142
19	-.043	2.363	.119	-.144	-.088	-1.230
20	.013	2.377	.094	-.050	-.028	-1.258

TABLE 1 (Cont.)

Constant Mean Return Model					
Good News		No News		Bad News	
AR	CAR	AR	CAR	AR	CAR
.105	.105	.019	.019	-.077	-.077
-.235	-.129	-.048	-.029	-.142	-.219
.069	-.060	-.086	-.115	-.043	-.262
-.026	-.086	-.140	-.255	-.057	-.319
-.086	-.172	.039	-.216	-.075	-.394
-.183	-.355	.099	-.117	-.037	-.431
-.020	-.375	-.150	-.266	-.101	-.532
-.025	-.399	-.191	-.458	-.069	-.601
.101	-.298	.133	-.325	-.106	-.707
.126	-.172	.006	-.319	-.169	-.876
.134	-.038	.103	-.216	-.009	-.885
.210	.172	.022	-.194	.011	-.874
.106	.278	.163	-.031	.135	-.738
-.002	.277	.009	-.022	-.027	-.765
.011	.288	-.029	-.051	.030	-.735
.061	.349	-.068	-.120	.320	-.415
.031	.379	.089	-.031	-.205	-.620
.067	.447	.013	-.018	.085	-.536
.010	.456	.311	.294	-.256	-.791
.198	.654	-.170	.124	-.227	-1.018
1.034	1.688	-.164	-.040	-.643	-1.661
.357	2.045	-.170	-.210	-.212	-1.873
-.013	2.033	.054	-.156	.078	-1.795
.088	1.944	-.121	-.277	.146	-1.648
.041	1.985	.023	-.253	.149	-1.499
.248	2.233	-.003	-.256	.286	-1.214
-.035	2.198	-.319	-.575	.070	-1.143
.017	2.215	-.112	-.687	.102	-1.041
.112	2.326	-.187	-.874	.056	-.986
-.052	2.274	-.057	-.931	-.071	-1.056
.147	2.421	.203	-.728	.267	-.789
-.013	2.407	.045	-.683	.006	-.783
-.054	2.354	.299	-.384	.017	-.766
-.246	2.107	-.067	-.451	.114	-.652
-.011	2.096	-.024	-.475	.089	-.564
-.027	2.068	-.059	-.534	-.022	-.585
.103	2.171	-.046	-.580	-.084	-.670
.066	2.237	-.098	-.677	-.054	-.724
.110	2.347	.021	-.656	-.071	-.795
-.055	2.292	.088	-.568	.026	-.769
.019	2.311	.013	-.554	-.115	-.884

Abnormal returns for an event study of the information content of earnings announcements. The sample consists of a total of 600 quarterly announcements for the 30 companies in the Dow Jones Industrial Index for the five year period January 1989 to December 1993. Two models are considered for the normal returns, the market model using the CRSP value-weighted index and the constant return model. The announcements are categorized into three groups, good news, no news, and bad news. AR is the sample average abnormal return for the specified day in event time and CAR is the sample average cumulative abnormal return for day -20 to the specified day. Event time is days relative to the announcement date.

it is assumed that there is not any clustering. That is, there is not any overlap in the event windows of the included securities. The absence of any overlap and the maintained distributional assumptions imply that the abnormal returns and the cumulative abnormal returns will be independent across securities. Later inferences with clustering will be discussed.

The individual securities' abnormal returns can be aggregated using  $AR_{i\tau}$  from (7) for each event period,  $\tau = T_1 + 1, \dots, T_2$ . Given  $N$  events, the sample aggregated abnormal returns for period  $\tau$  is

$$\overline{AR}_\tau = \frac{1}{N} \sum_{i=1}^N AR_{i\tau} \quad (13)$$

and for large  $L_1$ , its variance is

$$\text{var}(\overline{AR}_\tau) = \frac{1}{N^2} \sum_{i=1}^N \sigma_{\varepsilon_i}^2 \quad (14)$$

Using these estimates, the abnormal returns for any event period can be analyzed.

The average abnormal returns can then be aggregated over the event window using the same approach as that used to calculate the cumulative abnormal return for each security  $i$ . For any interval in the event window

$$\overline{CAR}(\tau_1, \tau_2) = \sum_{\tau=\tau_1}^{\tau_2} \overline{AR}_\tau \quad (15)$$

$$\text{var}(\overline{CAR}(\tau_1, \tau_2)) = \sum_{\tau=\tau_1}^{\tau_2} \text{var}(\overline{AR}_\tau). \quad (16)$$

Observe that equivalently one can form the CAR's security by security and then aggregate through time,

$$\overline{CAR}(\tau_1, \tau_2) = \frac{1}{N} \sum_{i=1}^N CAR_i(\tau_1, \tau_2) \quad (17)$$

$$\text{var}(\overline{CAR}(\tau_1, \tau_2)) = \frac{1}{N^2} \sum_{i=1}^N \sigma_i^2(\tau_1, \tau_2). \quad (18)$$

For the variance estimators the assumption that the event windows of the  $N$  securities do not overlap is used to set the covariance terms to zero. Inferences about the cumulative abnormal returns can be drawn using

$$\overline{CAR}(\tau_1, \tau_2) \sim N[0, \text{var}(\overline{CAR}(\tau_1, \tau_2))] \quad (19)$$

to test the null hypothesis that the abnormal returns are zero. In practice, because  $\sigma_{\varepsilon_i}^2$  is unknown, an estimator must be used to calculate the variance of the abnormal returns as in (14). The usual sample variance measure of  $\sigma_{\varepsilon_i}^2$  from the market model regression in the estimation window is an appropriate choice. Using this to calculate  $\text{var}(\overline{AR}_\tau)$  in (14),  $H_0$  can be tested using

$$\theta_1 = \frac{\overline{CAR}(\tau_1, \tau_2)}{\text{var}(\overline{CAR}(\tau_1, \tau_2))^{1/2}} \sim N(0, 1). \quad (20)$$

This distributional result is asymptotic with respect to the number of securities  $N$  and the length of estimation window  $L_1$ .

Modifications to the basic approach presented above are possible. One common modification is to standardize each abnormal return using an estimator of its standard deviation. For certain alternatives, such standardization can lead to more powerful tests. James Patell (1976) presents tests based on standardization and Brown and Warner (1980, 1985) provide comparisons with the basic approach.

#### D. CAR's for the Earnings Announcement Example

The information content of earnings example previously described illustrates the use of sample abnormal residuals and sample cumulative abnormal returns. Table 1 presents the abnormal returns av-

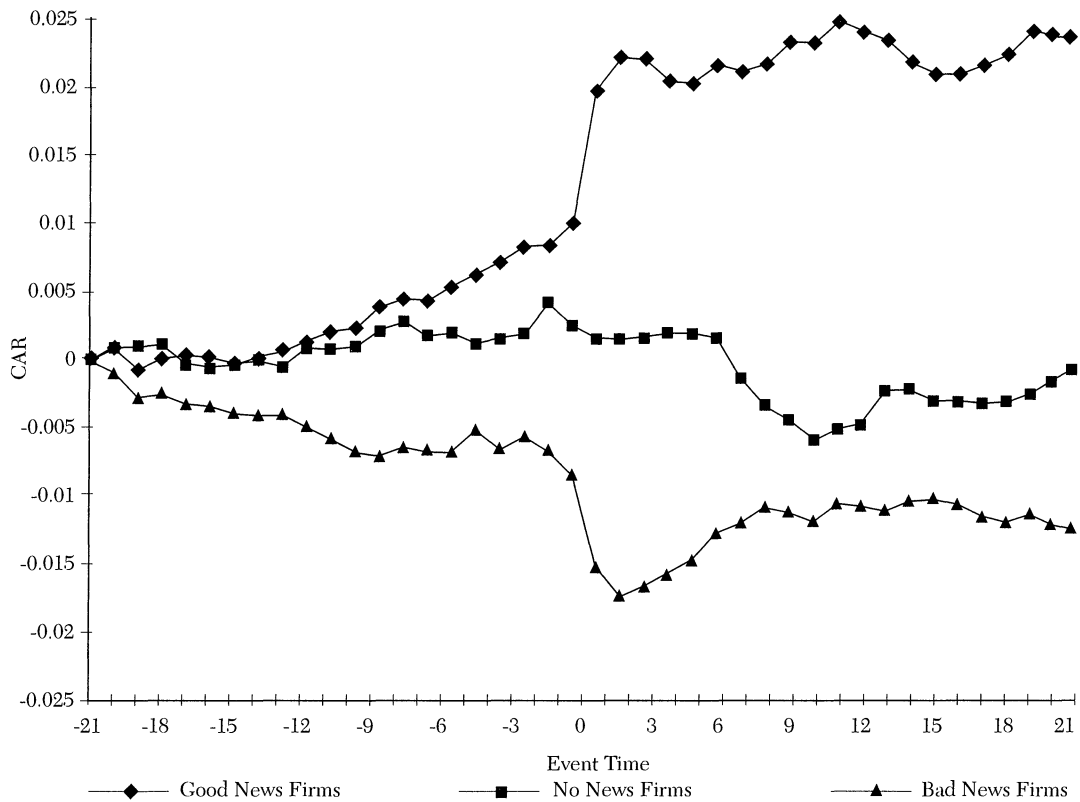


Figure 2a. Plot of cumulative abnormal return for earning announcements from event day -20 to event day 20. The abnormal return is calculated using the market model as the normal return measure.

eraged across the 600 event observations (30 firms, 20 announcements per firm) as well as the aggregated cumulative abnormal return for each of the three earnings news categories. Two normal return models are considered; the market model and for comparison, the constant mean return model. Plots of the cumulative abnormal returns are also included, with the CAR's from the market model in Figure 2a and the CAR's from the constant mean return model in Figure 2b.

The results of this example are largely consistent with the existing literature on the information content of earnings. The evidence strongly supports the hypothesis that earnings announcements do in-

deed convey information useful for the valuation of firms. Focusing on the announcement day (day 0) the sample average abnormal return for the good news firm using the market model is 0.965 percent. Given the standard error of the one day good news average abnormal return is 0.104 percent, the value of  $\theta_1$  is 9.28 and the null hypothesis that the event has no impact is strongly rejected. The story is the same for the bad news firms. The event day sample abnormal return is -0.679 percent, with a standard error of 0.098 percent, leading to  $\theta_1$  equal to -6.93 and again strong evidence against the null hypothesis. As would be expected, the abnormal return of the no news firms is small at -0.091 percent and



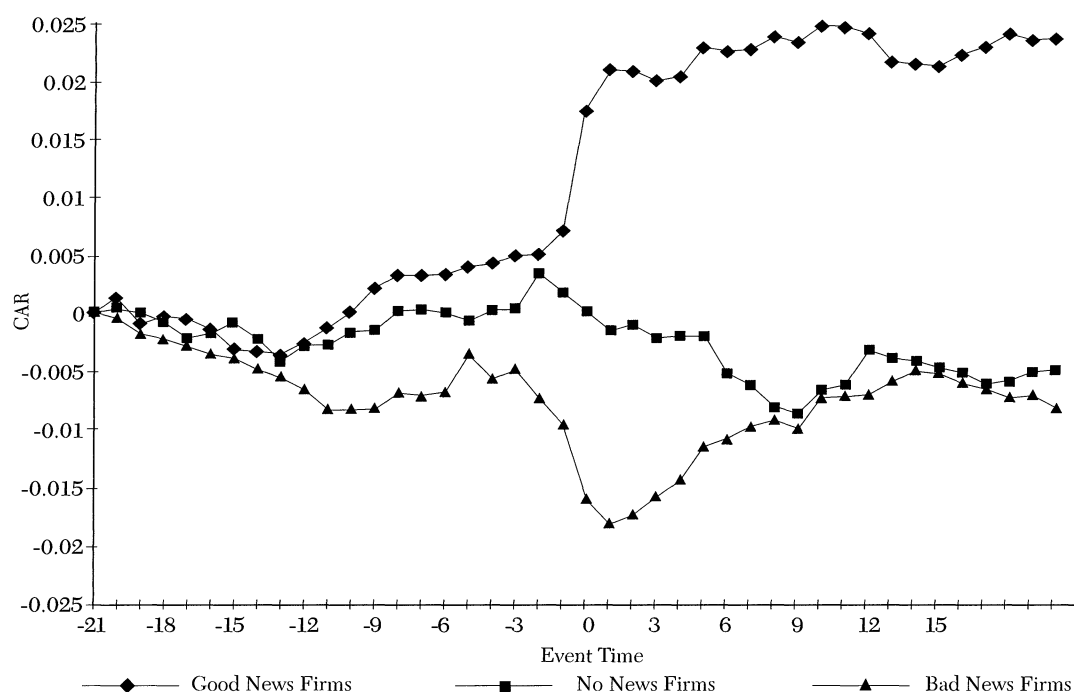


Figure 2b. Plot of cumulative abnormal return for earning announcements from event day -20 to event day 20. The abnormal return is calculated using the constant mean return model as the normal return

with a standard error of 0.098 percent is less than one standard error from zero. There is some evidence of the announcement effect on day one. The average abnormal return is 0.251 percent and  $-0.204$  percent for the good news and the bad news firms respectively. Both these values are more than two standard errors from zero. The source of these day one effects is likely to be that some of the earnings announcements are made on event day zero after the close of the stock market. In these cases, the effects will be captured in the return on day one.

The conclusions using the abnormal returns from the constant return model are consistent with those from the market model. However, there is some loss of precision using the constant return model, as the variance of the average abnormal return increases for all three

categories. When measuring abnormal returns with the constant mean return model the standard errors increase from 0.104 percent to 0.130 percent for good news firms, from 0.098 percent to 0.124 percent for no news firms, and from 0.098 percent to 0.131 percent for bad news firms. These increases are to be expected when considering a sample of large firms such as those in the Dow Index because these stocks tend to have an important market component whose variability is eliminated using the market model.

The CAR plots show that to some extent the market gradually learns about the forthcoming announcement. The average CAR of the good news firms gradually drifts up in days  $-20$  to  $-1$  and the average CAR of the bad news firms gradually drifts down over this period. In the days after the an-

nouncement the CAR is relatively stable as would be expected, although there does tend to be a slight (but statistically insignificant) increase with the bad news firms in days two through eight.

### E. Inferences with Clustering

The analysis aggregating abnormal returns has assumed that the event windows of the included securities do not overlap in calendar time. This assumption allows us to calculate the variance of the aggregated sample cumulative abnormal returns without concern about the covariances across securities because they are zero. However, when the event windows do overlap and the covariances between the abnormal returns will not be zero, the distributional results presented for the aggregated abnormal returns are no longer applicable. Victor Bernard (1987) discusses some of the problems related to clustering.

Clustering can be accommodated in two ways. The abnormal returns can be aggregated into a portfolio dated using event time and the security level analysis of Section 5 can be applied to the portfolio. This approach will allow for cross correlation of the abnormal returns.

A second method to handle clustering is to analyze the abnormal returns without aggregation. One can consider testing the null hypothesis of the event having no impact using unaggregated security by security data. This approach is applied most commonly when there is total clustering, that is, there is an event on the same day for a number of firms. The basic approach is an application of a multivariate regression model with dummy variables for the event date. This approach is developed in the papers of Katherine Schipper and Rex Thompson (1983, 1985) and Daniel Collins and Warren Dent (1984). The advantage of the approach is that, unlike the portfolio

approach, an alternative hypothesis where some of the firms have positive abnormal returns and some of the firms have negative abnormal returns can be accommodated. However, in general the approach has two drawbacks—frequently the test statistic will have poor finite sample properties except in special cases and often the test will have little power against economically reasonable alternatives. The multivariate framework and its analysis is similar to the analysis of multivariate tests of asset pricing models. MacKinlay (1987) provides analysis in that context.

## 6. Modifying the Null Hypothesis

Thus far the focus has been on a single null hypothesis—that the given event has no impact on the behavior of the returns. With this null hypothesis either a mean effect or a variance effect will represent a violation. However, in some applications one may be interested in testing for a mean effect. In these cases, it is necessary to expand the null hypothesis to allow for changing (usually increasing) variances. To allow for changing variance as part of the null hypothesis, it is necessary to eliminate the reliance on the past returns to estimate the variance of the aggregated cumulative abnormal returns. This is accomplished by using the cross section of cumulative abnormal returns to form an estimator of the variance for testing the null hypothesis. Ekkehart Boehmer, Jim Musumeci, and Annette Poulsen (1991) discuss methodology to accommodate changing variance.

The cross sectional approach to estimating the variance can be applied to the average cumulative abnormal return ( $\bar{CAR}(\tau_1, \tau_2)$ ). Using the cross-section to form an estimator of the variance gives

$$\begin{aligned} \text{var}(\overline{CAR}(\tau_1, \tau_2)) \\ = \frac{1}{N^2} \sum_{i=1}^N (CAR_i(\tau_1, \tau_2) \\ - \overline{CAR}(\tau_1, \tau_2))^2. \quad (21) \end{aligned}$$

For this estimator of the variance to be consistent, the abnormal returns need to be uncorrelated in the cross-section. An absence of clustering is sufficient for this requirement. Note that cross-sectional homoskedasticity is not required. Given this variance estimator, the null hypothesis that the cumulative abnormal returns are zero can then be tested using the usual theory.

One may also be interested in the question of the impact of an event on the risk of a firm. The relevant measure of risk must be defined before this question can be addressed. One choice as a risk measure is the market model beta which is consistent with the Capital Asset Pricing Model being appropriate. Given this choice, the market model can be formulated to allow the beta to change over the event window and the stability of the risk can be examined. Edward Kane and Haluk Unal (1988) present an application of this idea.

### 7. Analysis of Power

An important consideration when setting up an event study is the ability to detect the presence of a non-zero abnormal return. The inability to distinguish between the null hypothesis and economically interesting alternatives would suggest the need for modification of the design. In this section the question of the likelihood of rejecting the null hypothesis for a specified level of abnormal return associated with an event is addressed. Formally, the power of the test is evaluated.

Consider a two-sided test of the null hypothesis using the cumulative abnormal return based statistic  $\theta_1$  from (20). It is assumed that the abnormal returns are uncorrelated across securities; thus

the variance of  $\overline{CAR}$  is  $1/N^2 \sum_{i=1}^N \sigma_i^2(\tau_1, \tau_2)$

and  $N$  is the sample size. Because the null distribution of  $\theta_1$  is standard normal, for a two sided test of size  $\alpha$ , the null hypothesis will be rejected if  $\theta_1$  is in the critical region, that is,

$$\theta_1 < c\left(\frac{\alpha}{2}\right) \text{ or } \theta_1 > c\left(1 - \frac{\alpha}{2}\right)$$

where  $c(x) = \Phi^{-1}(x)$ .  $\Phi(\cdot)$  is the standard normal cumulative distribution function (CDF).

Given the specification of the alternative hypothesis  $H_A$  and the distribution of  $\theta_1$  for this alternative, the power of a test of size  $\alpha$  can be tabulated using the power function,

$$\begin{aligned} P(\alpha, H_A) = & pr\left(\theta_1 < c\left(\frac{\alpha}{2}\right) \mid H_A\right) \\ & + pr\left(\theta_1 > c\left(1 - \frac{\alpha}{2}\right) \mid H_A\right). \quad (22) \end{aligned}$$

The distribution of  $\theta_1$  under the alternative hypothesis considered below will be normal. The mean will be equal to the true cumulative abnormal return divided by the standard deviation of  $\overline{CAR}$  and the variance will be equal to one.

To tabulate the power one must posit economically plausible scenarios. The alternative hypotheses considered are four levels of abnormal returns, 0.5 percent, 1.0 percent, 1.5 percent, and 2.0 percent and two levels of the average variance for the cumulative abnormal return of a given security over the event period, 0.0004 and 0.0016. The

TABLE 2

Sample Size	Abnormal Return				Abnormal Return			
	.005	.010	.015	.020	.005	.010	.015	.020
	$\sigma = 0.02$				$\sigma = 0.04$			
1	0.06	0.08	0.12	0.17	0.05	0.06	0.07	0.08
2	0.06	0.11	0.19	0.29	0.05	0.06	0.08	0.11
3	0.07	0.14	0.25	0.41	0.06	0.07	0.10	0.14
4	0.08	0.17	0.32	0.52	0.06	0.08	0.12	0.17
5	0.09	0.20	0.39	0.61	0.06	0.09	0.13	0.20
6	0.09	0.23	0.45	0.69	0.06	0.09	0.15	0.23
7	0.10	0.26	0.51	0.75	0.06	0.10	0.17	0.26
8	0.11	0.29	0.56	0.81	0.06	0.11	0.19	0.29
9	0.12	0.32	0.61	0.85	0.07	0.12	0.20	0.32
10	0.12	0.35	0.66	0.89	0.07	0.12	0.22	0.35
11	0.13	0.38	0.70	0.91	0.07	0.13	0.24	0.38
12	0.14	0.41	0.74	0.93	0.07	0.14	0.25	0.41
13	0.15	0.44	0.77	0.95	0.07	0.15	0.27	0.44
14	0.15	0.46	0.80	0.96	0.08	0.15	0.29	0.46
15	0.16	0.49	0.83	0.97	0.08	0.16	0.31	0.49
16	0.17	0.52	0.85	0.98	0.08	0.17	0.32	0.52
17	0.18	0.54	0.87	0.98	0.08	0.18	0.34	0.54
18	0.19	0.56	0.89	0.99	0.08	0.19	0.36	0.56
19	0.19	0.59	0.90	0.99	0.08	0.19	0.37	0.59
20	0.20	0.61	0.92	0.99	0.09	0.20	0.39	0.61
25	0.24	0.71	0.96	1.00	0.10	0.24	0.47	0.71
30	0.28	0.78	0.98	1.00	0.11	0.28	0.54	0.78
35	0.32	0.84	0.99	1.00	0.11	0.32	0.60	0.84
40	0.35	0.89	1.00	1.00	0.12	0.35	0.66	0.89
45	0.39	0.92	1.00	1.00	0.13	0.39	0.71	0.92
50	0.42	0.94	1.00	1.00	0.14	0.42	0.76	0.94
60	0.49	0.97	1.00	1.00	0.16	0.49	0.83	0.97
70	0.55	0.99	1.00	1.00	0.18	0.55	0.88	0.99
80	0.61	0.99	1.00	1.00	0.20	0.61	0.92	0.99
90	0.66	1.00	1.00	1.00	0.22	0.66	0.94	1.00
100	0.71	1.00	1.00	1.00	0.24	0.71	0.96	1.00
120	0.78	1.00	1.00	1.00	0.28	0.78	0.98	1.00
140	0.84	1.00	1.00	1.00	0.32	0.84	0.99	1.00
160	0.89	1.00	1.00	1.00	0.35	0.89	1.00	1.00
180	0.92	1.00	1.00	1.00	0.39	0.92	1.00	1.00
200	0.94	1.00	1.00	1.00	0.42	0.94	1.00	1.00

Power of event study methodology for test of the null hypothesis that the abnormal return is zero. The power is reported for a two-sided test using  $\theta_1$  with a size of 5 percent. The sample size is the number of event observations included the study and  $\sigma$  is the square root of the average variance of the abnormal return across firms.

sample size, that is the number of securities for which the event occurs, is varied from one to 200. The power for a test with a size of 5 percent is documented. With  $\alpha = 0.05$ , the critical val-

ues calculated using  $c(\alpha/2)$  and  $c(1 - \alpha/2)$  are  $-1.96$  and  $1.96$  respectively. Of course, in applications, the power of the test should be considered when selecting the size.

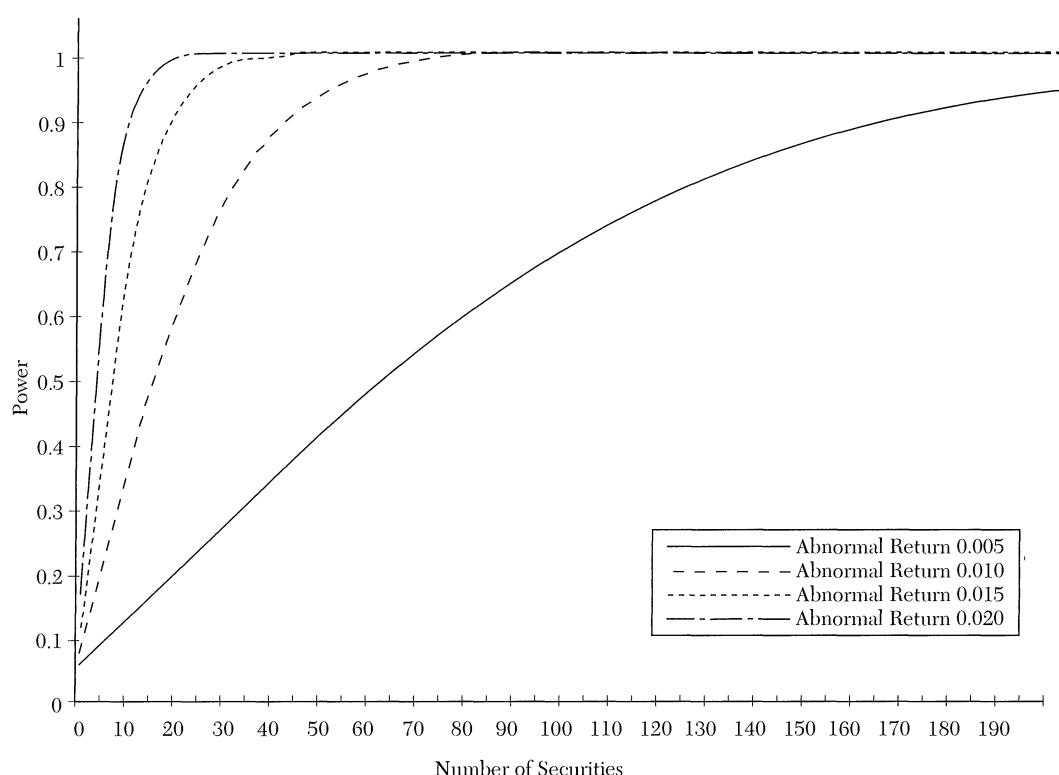


Figure 3a. Power of event study test statistic  $\theta_1$  to reject the null hypothesis that the abnormal return is zero, when the square root of the average variance of the abnormal return across firms is 2 percent.

The power results are presented in Table 2, and are plotted in Figures 3a and 3b. The results in the left panel of Table 2 and Figure 3a are for the case where the average variance is 0.0004. This corresponds to a cumulative abnormal return standard deviation of 2 percent and is an appropriate value for an event which does not lead to increased variance and can be examined using a one-day event window. In terms of having high power this is the best case scenario. The results illustrate that when the abnormal return is only 0.5 percent the power can be low. For example with a sample size of 20 the power of a 5 percent test is only 0.20. One needs a sample of over 60 firms before the power reaches 0.50. However, for a given sample size, increases in power

are substantial when the abnormal return is larger. For example, when the abnormal return is 2.0 percent the power of a 5 percent test with 20 firms is almost 1.00 with a value of 0.99. The general results for a variance of 0.0004 is that when the abnormal return is larger than 1 percent the power is quite high even for small sample sizes. When the abnormal return is small a larger sample size is necessary to achieve high power.

In the right panel of Table 2 and in Figure 3b the power results are presented for the case where the average variance of the cumulative abnormal return is 0.0016. This case corresponds roughly to either a multi-day event window or to a one-day event window with the event leading to increased variance

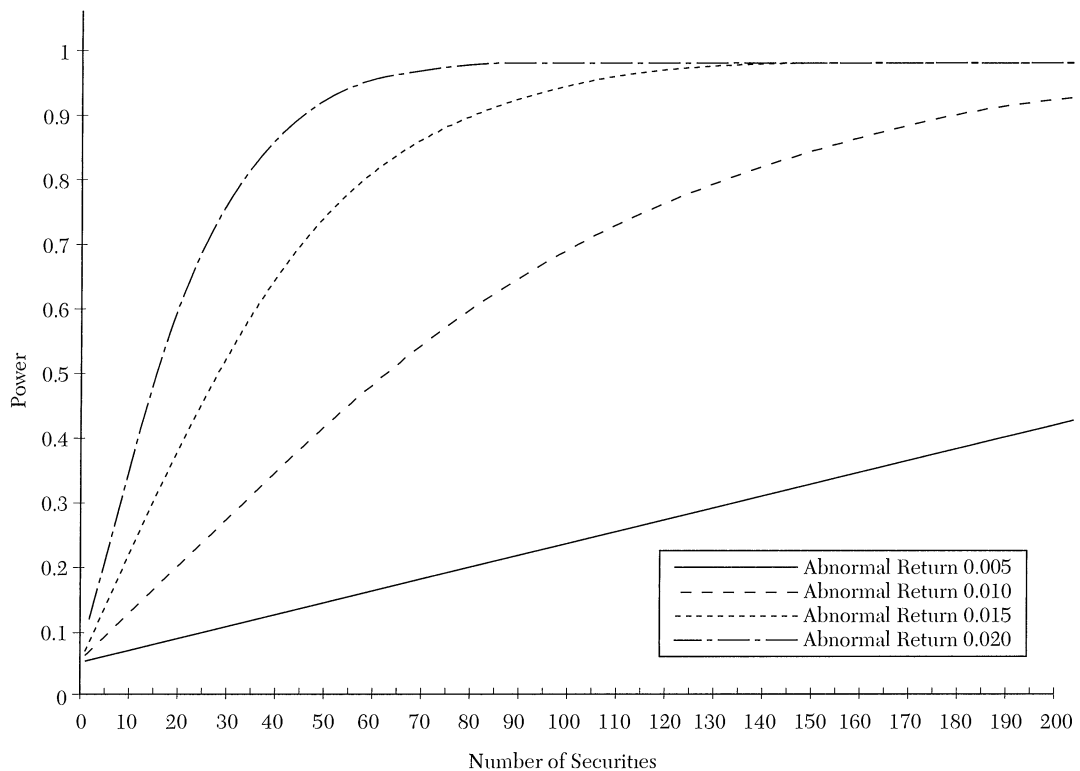


Figure 3b. Power of event study test statistic  $\theta_1$  to reject the null hypothesis that the abnormal return is zero, when the square root of the average variance of the abnormal return across firms is 4 percent.

which is accommodated as part of the null hypothesis. When the average variance of the CAR is increased from 0.0004 to 0.0016 there is a dramatic power decline for a 5 percent test. When the CAR is 0.5 percent the power is only 0.09 with 20 firms and is only 0.42 with a sample of 200 firms. This magnitude of abnormal return is difficult to detect with the larger variance. In contrast, when the CAR is as large as 1.5 percent or 2.0 percent the 5 percent test is still has reasonable power. For example, when the abnormal return is 1.5 percent and there is a sample size of 30 the power is 0.54. Generally if the abnormal return is large one will have little difficulty rejecting the null hypothesis of no abnormal return.

In the preceding analysis the power is

considered analytically for the given distributional assumptions. If the distributional assumptions are inappropriate then the results may differ. However, Brown and Warner (1985) consider this possible difference and find that the analytical computations and the empirical power are very close.

It is difficult to make general conclusions concerning the adequacy of the ability of event study methodology to detect non-zero abnormal returns. When conducting an event study it is best to evaluate the power given the parameters and objectives of the study. If the power seems sufficient then one can proceed, otherwise one should search for ways of increasing the power. This can be done by increasing the sample size, shortening the event window, or by



developing more specific predictions to test.

### 8. Nonparametric Tests

The methods discussed to this point are parametric in nature, in that specific assumptions have been made about the distribution of abnormal returns. Alternative approaches are available which are nonparametric in nature. These approaches are free of specific assumptions concerning the distribution of returns. Common nonparametric tests for event studies are the sign test and the rank test. These tests are discussed next.

The sign test, which is based on the sign of the abnormal return, requires that the abnormal returns (or more generally cumulative abnormal returns) are independent across securities and that the expected proportion of positive abnormal returns under the null hypothesis is 0.5. The basis of the test is that, under the null hypothesis, it is equally probable that the CAR will be positive or negative. If, for example, the null hypothesis is that there is a positive abnormal return associated with a given event, the null hypothesis is  $H_0: p \leq 0.5$  and the alternative is  $H_A: p > 0.5$  where  $p = \text{pr}[CAR_i \geq 0.0]$ . To calculate the test statistic we need the number of cases where the abnormal return is positive,  $N^+$ , and the total number of cases,  $N$ . Letting  $\theta_2$  be the test statistic,

$$\theta_2 = \left[ \frac{N^+}{N} - 0.5 \right] \frac{\sqrt{N}}{0.5} \sim N(0,1). \quad (23)$$

This distributional result is asymptotic. For a test of size  $(1 - \alpha)$ ,  $H_0$  is rejected if  $\theta_2 > \Phi^{-1}(\alpha)$ .

A weakness of the sign test is that it may not be well specified if the distribution of abnormal returns is skewed as can be the case with daily data. In response to this possible shortcoming,

Charles Corrado (1989) proposes a non-parametric rank test for abnormal performance in event studies. A brief description of his test of no abnormal return for event day zero follows. The framework can be easily altered for more general tests.

Drawing on notation previously introduced, consider a sample of  $L_2$  abnormal returns for each of  $N$  securities. To implement the rank test, for each security it is necessary to rank the abnormal returns from one to  $L_2$ . Define  $K_{i\tau}$  as the rank of the abnormal return of security  $i$  for event time period  $\tau$ . Recall,  $\tau$  ranges from  $T_1 + 1$  to  $T_2$  and  $\tau = 0$  is the event day. The rank test uses the fact that the expected rank of the event day is  $(L_2 + 1)/2$  under the null hypothesis. The test statistic for the null hypothesis of no abnormal return on event day zero is

$$\theta_3 = \frac{1}{N} \sum_{i=1}^N \left( K_{i0} - \frac{L_2 + 1}{2} \right) / s(K) \quad (24)$$

where

$$s(K) = \sqrt{\frac{1}{L_2} \sum_{\tau=T_1+1}^{T_2} \left( \frac{1}{N} \sum_{i=1}^N \left( K_{i\tau} - \frac{L_2 + 1}{2} \right) \right)^2}. \quad (25)$$

Tests of the null hypothesis can be implemented using the result that the asymptotic null distribution of  $\theta_3$  is standard normal. Corrado (1989) includes further discussion of details of this test.

Typically, these nonparametric tests are not used in isolation but in conjunction with the parametric counterparts. Inclusion of the nonparametric tests provides a check of the robustness of conclusions based on parametric tests. Such a check can be worthwhile as illustrated by the work of Cynthia Campbell and Charles Wasley (1993). They find that for NASDAQ stocks daily returns the nonparametric rank test provides more reliable inferences than do the standard parametric tests.

### 9. Cross-Sectional Models

Theoretical insights can result from examining the association between the magnitude of the abnormal return and characteristics specific to the event observation. Often such an exercise can be helpful when multiple hypotheses exist for the source of the abnormal return. A cross-sectional regression model is an appropriate tool to investigate this association. The basic approach is to run a cross-sectional regression of the abnormal returns on the characteristics of interest.

Given a sample of  $N$  abnormal return observations and  $M$  characteristics, the regression model is:

$$AR_j = \delta_0 + \delta_1 x_{1j} + \cdots + \delta_M x_{Mj} + \eta_j \quad (26)$$

$$E(\eta_j) = 0 \quad (27)$$

where  $AR_j$  is the  $j^{\text{th}}$  abnormal return observation,  $x_{mj}, m = 1, \dots, M$ , are  $M$  characteristics for the  $j^{\text{th}}$  observation and  $\eta_j$  is the zero mean disturbance term that is uncorrelated with the  $x$ 's.  $\delta_m, m = 0, \dots, M$  are the regression coefficients. The regression model can be estimated using OLS. Assuming the  $\eta_j$ 's are cross-sectionally uncorrelated and homoskedastic, inferences can be conducted using the usual OLS standard errors. Alternatively, without assuming homoskedasticity, heteroskedasticity-consistent  $t$ -statistics using standard errors can be derived using the approach of Halbert White (1980). The use of heteroskedasticity-consistent standard errors is advisable because there is no reason to expect the residuals of (26) to be homoskedastic.

Paul Asquith and David Mullins (1986) provide an example of this cross-sectional approach. The two day cumulative abnormal return for the announcement of an equity offering is regressed on the size of the offering as a percentage of the value of the total equity of the firm and on the cumulative abnormal re-

turn in the eleven months prior to the announcement month. They find that the magnitude of the (negative) abnormal return associated with the announcement of equity offerings is related to both these variables. Larger pre-event cumulative abnormal returns are associated with less negative abnormal returns and larger offerings are associated with more negative abnormal returns. These findings are consistent with theoretical predictions which they discuss.

Issues concerning the interpretation of the results can arise with the cross-sectional regression approach. In many situations, the event window abnormal return will be related to firm characteristics not only through the valuation effects of the event but also through a relation between the firm characteristics and the extent to which the event is anticipated. This can happen when investors rationally use the firm characteristics to forecast the likelihood of the event occurring. In these cases, a linear relation between the valuation effect of the event and the firm characteristic can be hidden. Paul Malatesta and Thompson (1985) and William Lanen and Thompson (1988) provide examples of this situation.

Technically, with the relation between the firm characteristics and the degree of anticipation of the event introduces a selection bias. The assumption that the regression residual is uncorrelated with the regressors breaks down and the OLS estimators are inconsistent. Consistent estimators can be derived by explicitly incorporating the selection bias. Sankarshan Acharya (1988) and B. Espen Eckbo, Vojislav Maksimovic, and Joseph Williams (1990) provide examples of this approach. N. R. Prabhala (1995) provides a good discussion of this problem and the possible solutions. He argues that, despite an incorrect specification, under weak conditions, the OLS ap-

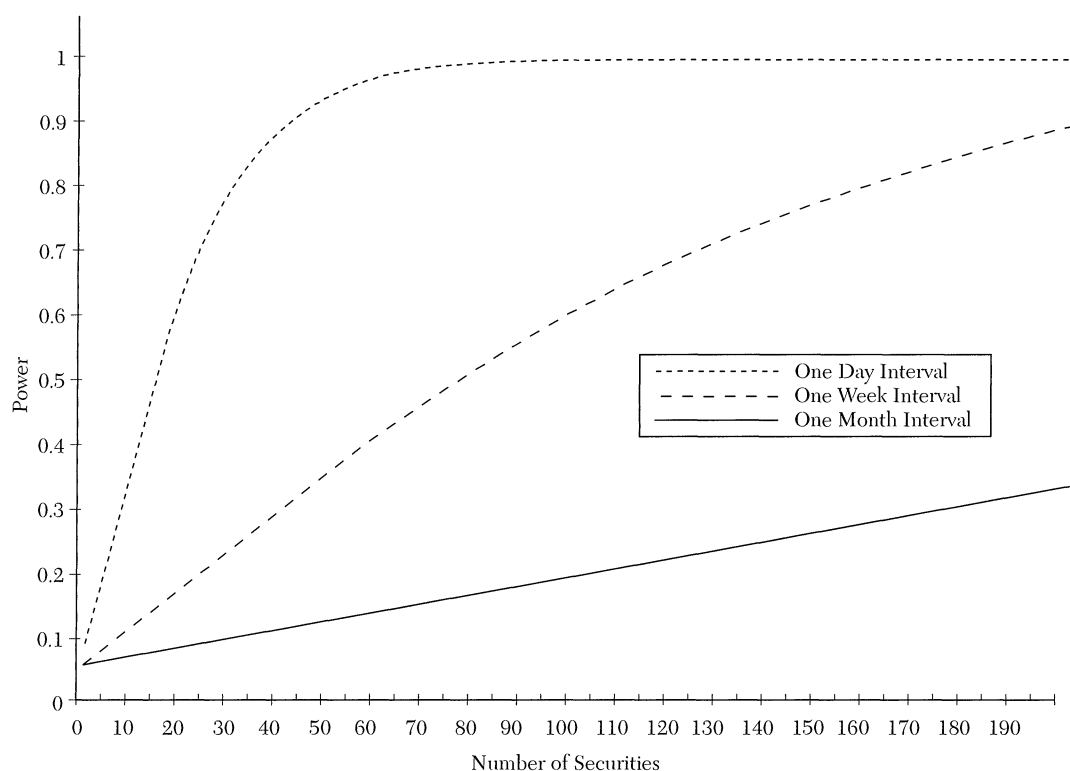


Figure 4. Power of event study test statistic  $\theta_1$  to reject the null hypothesis that the abnormal return is zero, for different sampling intervals, when the square root of the average variance of the abnormal return across firms is 4 percent for the daily interval. Size of test is 5 percent.

proach can be used for inferences and that the  $t$ -statistics can be interpreted as lower bounds on the true significance level of the estimates.

#### 10. Other Issues

A number of further issues often arise when conducting an event study. These issues include the role of the sampling interval, event date uncertainty, robustness, and some additional biases.

##### A. Role of Sampling Interval

Stock return data is available at different sampling intervals, with daily and monthly intervals being the most common. Given the availability of various intervals, the question of the gains of using

more frequent sampling arises. To address this question one needs to consider the power gains from shorter intervals. A comparison of daily versus monthly data is provided in Figure 4. The power of the test of no event effect is plotted against the alternative of an abnormal return of one percent for 1 to 200 securities. As one would expect given the analysis of Section 7, the decrease in power going from a daily interval to a monthly interval is severe. For example, with 50 securities the power for a 5 percent test using daily data is 0.94, whereas the power using weekly and monthly data is only 0.35 and 0.12 respectively. The clear message is that there is a substantial payoff in terms of increased power from reducing the sampling inter-

val. Dale Morse (1984) presents detailed analysis of the choice of daily versus monthly data and draws the same conclusion.

A sampling interval of one day is not the shortest interval possible. With the increased availability of transaction data, recent studies have used observation intervals of duration shorter than one day. However, the net benefit of intervals less than one day is unclear as some complications are introduced. Discussion of using transaction data for event studies is included in the work of Michael Barclay and Robert Litzenberger (1988).

#### B. Inferences with Event-Date Uncertainty

Thus far it is assumed that the event date can be identified with certainty. However, in some studies it may be difficult to identify the exact date. A common example is when collecting event dates from financial publications such as the *Wall Street Journal*. When the event announcement appears in the paper one can not be certain if the market was informed prior to the close of the market the prior trading day. If this is the case then the prior day is the event day, if not then the current day is the event day. The usual method of handling this problem is to expand the event window to two days—day 0 and day +1. While there is a cost to expanding the event window, the results in Section 6 indicated that the power properties of two day event windows are still good suggesting that the costs are worth bearing rather than to take the risk of missing the event.

Clifford Ball and Walter Torous (1988) have investigated the issue. They develop a maximum likelihood estimation procedure which accommodates event date uncertainty and examine results of their explicit procedure versus the informal procedure of expanding the event window. The results indicates that the

informal procedure works well and there is little to gain from the more elaborate estimation framework.

#### C. Robustness

The statistical analysis of Sections 4, 5, and 6 is based on assumption that returns are jointly normal and temporally independently and identically distributed. In this section, discussion of the robustness of the results to departures from this assumption is presented. The normality assumption is important for the exact finite sample results to hold. Without assuming normality, all results would be asymptotic. However, this is generally not a problem for event studies because for the test statistics, convergence to the asymptotic distributions is rather quick. Brown and Warner (1985) provide discussion of this issue.

#### D. Other Possible Biases

A number of possible biases can arise in the context of conducting an event study. Nonsynchronous trading can introduce a bias. The nontrading or nonsynchronous trading effect arises when prices, are taken to be recorded at time intervals of one length when in fact they are recorded at time intervals of other possibly irregular lengths. For example, the daily prices of securities usually employed in event studies are generally “closing” prices, prices at which the last transaction in each of those securities occurred during the trading day. These closing prices generally do not occur at the same time each day, but by calling them “daily” prices, one is implicitly and incorrectly assuming that they are equally spaced at 24-hour intervals. This nontrading effect induces biases in the moments and co-moments of returns.

The influence of the nontrading effect on the variances and covariances of individual stocks and portfolios naturally feeds into a bias for the market model

beta. Myron Scholes and Williams (1977) present a consistent estimator of beta in the presence of nontrading based on the assumption that the true return process is uncorrelated through time. They also present some empirical evidence which shows the nontrading-adjusted beta estimates of thinly traded securities to be approximately 10 to 20 percent larger than the unadjusted estimates. However, for actively traded securities, the adjustments are generally small and unimportant.

Prem Jain (1986) considers the influence of thin trading on the distribution of the abnormal returns from the market model with the beta estimated using the Scholes-Williams approach. When comparing the distribution of these abnormal returns to the distribution of the abnormal returns using the usual OLS betas finds that the differences are minimal. This suggests that in general the adjustments for thin trading are not important.

The methodology used to compute the cumulative abnormal returns can induce an upward bias. The bias arises from the observation by observation rebalancing to equal weights implicit in the calculation of the aggregate cumulative abnormal return combined with the use of transaction prices which can represent both the bid and the offer side of the market. Marshall Blume and Robert Stambaugh (1983) analyze this bias and show that it can be important for studies using low market capitalization firms which have, in percentage terms, wide bid offer spreads. In these cases the bias can be eliminated by considering cumulative abnormal returns which represent buy and hold strategies.

### 11. *Concluding Discussion*

In closing, examples of event study successes and limitations are presented. Perhaps the most successful applications

have been in the area of corporate finance. Event studies dominate the empirical research in this area. Important examples include the wealth effects of mergers and acquisitions and the price effects of financing decisions by firms. Studies of these events typically focus on the abnormal return around the date of first announcement.

In the 1960s there was a paucity of empirical evidence on the wealth effects of mergers and acquisitions. For example, Henry Manne (1965) discusses the various arguments for and against mergers. At that time the debate centered on the extent to which mergers should be regulated in order to foster competition in the product markets. Manne argued that mergers represent a natural outcome in an efficiently operating market for corporate control and consequently provide protection for shareholders. He downplayed the importance of the argument that mergers reduce competition. At the conclusion of his article Manne suggested that the two competing hypotheses for mergers could be separated by studying the price effects of the involved corporations. He hypothesized that, if mergers created market power, one would observe price increases for both the target and acquirer. In contrast, if the merger represented the acquiring corporation paying for control of the target, one would observe a price increase for the target only and not for the acquirer. However, Manne concludes, in reference to the price effects of mergers, that "no data are presently available on this subject."

Since that time an enormous body of empirical evidence on mergers and acquisitions has developed which is dominated by the use of event studies. The general result is that, given a successful takeover, the abnormal returns of the targets are large and positive and the abnormal returns of the acquirer are close



to zero. Gregg Jarrell and Poulsen (1989) document that the average abnormal return for target shareholders exceeds 20 percent for a sample of 663 successful takeovers from 1960 to 1985. In contrast the abnormal returns for acquirers is close to zero. For the same sample, Jarrell and Poulsen find an average abnormal return of 1.14 percent for acquirers. In the 1980s they find the average abnormal return is negative at -1.10 percent. Eckbo (1983) explicitly addresses the role of increased market power in explaining merger related abnormal returns. He separates mergers of competing firms from other mergers and finds no evidence that the wealth effects for competing firms are different. Further, he finds no evidence that rivals of firms merging horizontally experience negative abnormal returns. From this he concludes that reduced competition in the product market is not an important explanation for merger gains. This leaves competition for corporate control a more likely explanation. Much additional empirical work in the area of mergers and acquisitions has been conducted. Michael Jensen and Richard Ruback (1983) and Jarrell, James Brickley, and Netter (1988) provide detailed surveys of this work.

A number of robust results have been developed from event studies of financing decisions by corporations. When a corporation announces that it will raise capital in external markets there is, on average, a negative abnormal return. The magnitude of the abnormal return depends on the source of external financing. Asquith and Mullins (1986) find for a sample of 266 firms announcing an equity issue in the period 1963 to 1981 the two day average abnormal return is -2.7 percent and on a sample of 80 firms for the period 1972 to 1982 Wayne Mikkelsen and Megan Partch (1986) find the two day average abnormal return is

-3.56 percent. In contrast, when firms decide to use straight debt financing, the average abnormal return is closer to zero. Mikkelsen and Partch (1986) find the average abnormal return for debt issues to be -0.23 percent for a sample of 171 issues. Findings such as these provide the fuel for the development of new theories. For example, in this case, the findings motivate the pecking order theory of capital structure developed by Stewart Myers and Nicholas Majluf (1984).

A major success related to those in the corporate finance area is the implicit acceptance of event study methodology by the U.S. Supreme Court for determining materiality in insider trading cases and for determining appropriate disgorgement amounts in cases of fraud. This implicit acceptance in the 1988 Basic, Incorporated v. Levinson case and its importance for securities law is discussed in Mitchell and Netter (1994).

There have also been less successful applications. An important characteristic of a successful event study is the ability to identify precisely the date of the event. In cases where the event date is difficult to identify or the event date is partially anticipated, studies have been less useful. For example, the wealth effects of regulatory changes for affected entities can be difficult to detect using event study methodology. The problem is that regulatory changes are often debated in the political arena over time and any accompanying wealth effects generally will gradually be incorporated into the value of a corporation as the probability of the change being adopted increases.

Larry Dann and Christopher James (1982) discuss this issue in the context of the impact of deposit interest rate ceilings for thrift institutions. In their study of changes in rate ceilings, they decide not to consider a change in 1973 because it was due to legislative action. Schipper



and Thompson (1983, 1985) also encounter this problem in a study of merger related regulations. They attempt to circumvent the problem of regulatory changes being anticipated by identifying dates when the probability of a regulatory change being passed changes. However, they find largely insignificant results leaving open the possibility the of absence of distinct event dates as the explanation of the lack of wealth effects.

Much has been learned from the body of research based on the use of event study methodology. In a general context, event studies have shown that, as would be expected in a rational marketplace, prices do respond to new information. As one moves forward, it is expected that event studies will continue to be a valuable and widely used tool in economics and finance.

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