Problem 1: Linear Regression from Scratch (30 points)

In [1]:
```
# import the necessary packages
import numpy as np
from matplotlib import pyplot as plt
np.random.seed(100)
```

Let's generate some data points first, by the equation $y = x - 3$.

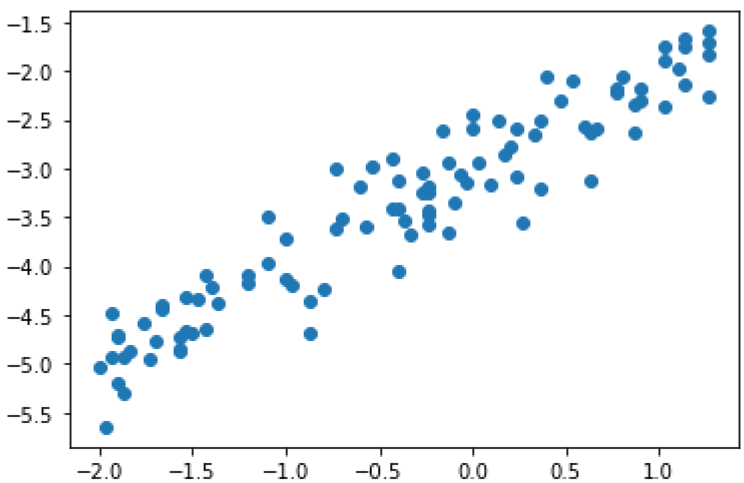
In [2]:
```
x = np.random.randint(100, size=100)/30 - 2
X = x.reshape(-1, 1)

y = x + -3 + 0.3*np.random.randn(100)
```

Let's then visualize the data points we just created.

In [3]:
```
plt.scatter(X, y)
```

Out[3]:
```
<matplotlib.collections.PathCollection at 0x18a37e2a940>
```



1.1 Gradient of vanilla linear regression model (5 points)

In the lecture, we learn that the cost function of a linear regression model can be expressed as **Equation 1**:

$$J(\theta) = \frac{1}{2m}\sum_{i}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

The gredient of it can be written as **Equation 2**:

$$\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{m}\sum_{i}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x^{(i)}$$

1.2 Gradient of vanilla regularized regression model (5 points)

After adding the L2 regularization term, the linear regression model can be expressed as **Equation 3**:

$$J(\theta) = \frac{1}{2m}\sum_{i}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j}^{n}(\theta_j)^2$$

The gredient of it can be written as **Equation 4**:

$$\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{m}\sum_{i}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x^{(i)} + \frac{\lambda}{m}(\theta)$$

1.3 Implement the cost function of a regularized regression model (5 points)

Please implement the cost function of a regularized regression model according to the above equations.

1.4 Implement the gradient of the cost function of a regularized regression model (5 points)

Please implement the gradient of the cost function of a regularized regression model according to the above equations.

In [16]:

```
def regularized_linear_regression(X, y, alpha=0.01, lambda_value=1, epochs=30):
    """
    :param x: feature matrix
    :param y: target vector
    :param alpha: learning rate (default:0.01)
    :param lambda_value: lambda (default:1)
    :param epochs: maximum number of iterations of the
           linear regression algorithm for a single run (default=30)
    :return: weights, list of the cost function changing overtime
    """

    m = np.shape(X)[0]  # total number of samples
    n = np.shape(X)[1]  # total number of features

    X = np.concatenate((np.ones((m, 1)), X), axis=1)
    W = np.random.randn(n + 1, )

    # stores the updates on the cost function (loss function)
```

```
    cost_history_list = []

    # iterate until the maximum number of epochs
    for current_iteration in np.arange(epochs):  # begin the process

        # compute the dot product between our feature 'X' and weight 'W'
        y_estimated = X.dot(W)

        # calculate the difference between the actual and predicted value
        error = y_estimated - y


##########################################################################################
######################################## Begin of Question 1.3 ###########################
##########################################################################################

        ##### Please write down your code here:####

        # calculate the cost (MSE) (Equation 1)

        #note: elementwise
        cost_without_regularization = ((error **2).sum()) / (2*m)


        ##### Please write down your code here:####

        # regularization term

        reg_term = lambda_value / (2*m) * (W**2).sum()


        # calculate the cost (MSE) + regularization term (Equation 3)
        cost_with_regularization = cost_without_regularization + reg_term

##########################################################################################
######################################## End of Question 1.3 #############################
##########################################################################################


##########################################################################################
######################################## Begin of Question 1.4 ###########################
##########################################################################################

        ##### Please write down your code here:####

        # calculate the gradient of the cost function with regularization term (Equati
        error2 = np.array([error, error]).T
        g_without_reg = (np.multiply(error2, X)).sum(axis = 0) / m
        g_reg = lambda_value / m * W
        gradient = g_without_reg + g_reg


        # Now we have to update our weights
        W = W - alpha * gradient

##########################################################################################
######################################## End of Question 1.4 #############################
##########################################################################################
```

```
        # keep track the cost as it changes in each iteration
        cost_history_list.append(cost_with_regularization)

    # Let's print out the cost
    print(f"Cost with regularization: {cost_with_regularization}")
    print(f"Mean square error: {cost_without_regularization}")

    return W, cost_history_list
```

Run the following code to train your model.

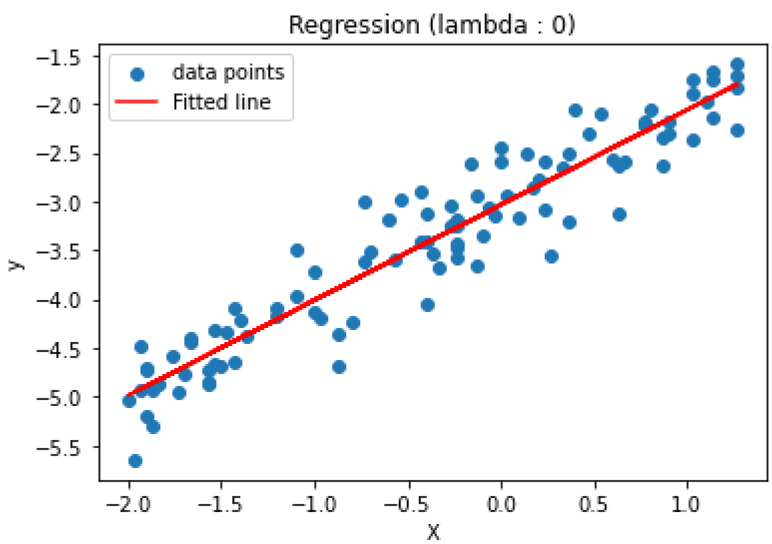
Hint: If you have the correct code written above, the cost should be 0.5181222986588751 when $\lambda = 10$.

In [17]:

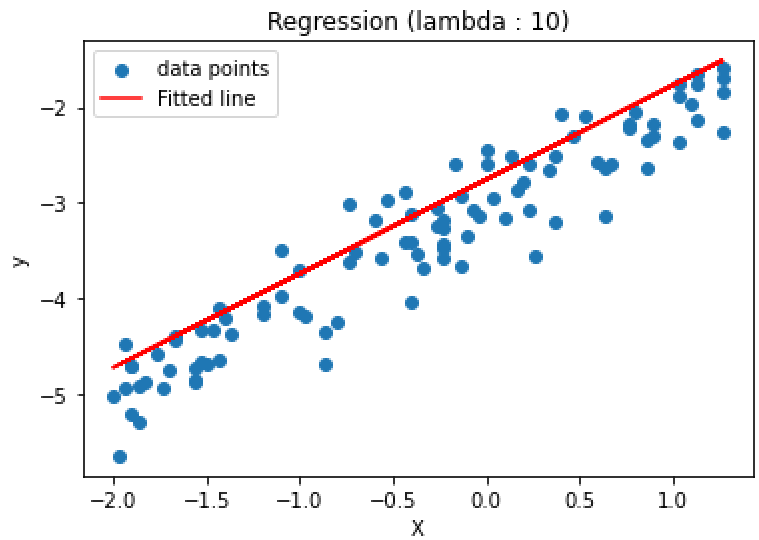
```
lambda_list = [0, 10, 100, 1000, 10000]
for lambda_ in lambda_list:
    # calls regression function with different values of lambda
    weight, _ = regularized_linear_regression(X, y, alpha=0.01,
                                             lambda_value=lambda_, epochs=1000)

    fitted_line = np.dot(X, weight[1]) + weight[0]
    plt.scatter(X, y, label='data points')
    plt.plot(X, fitted_line, color='r', label='Fitted line')
    plt.xlabel("X")
    plt.ylabel("y")
    plt.title(f"Regression (lambda : {lambda_})")
    plt.legend()
    plt.show()
```

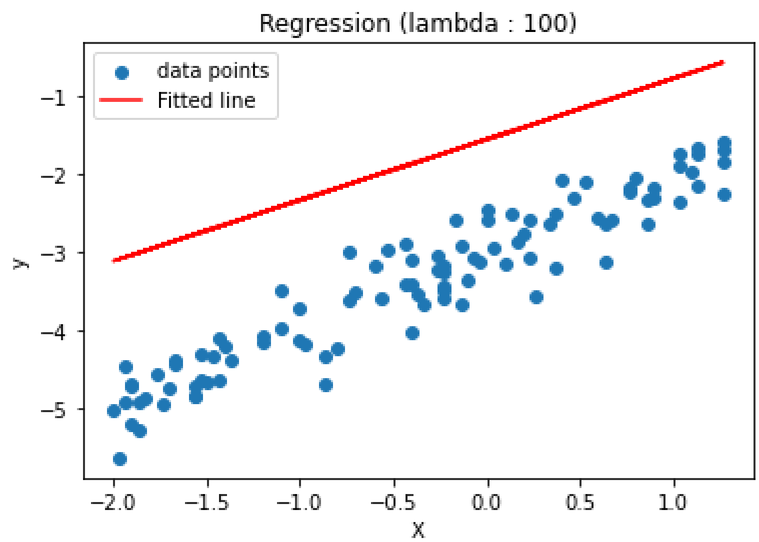
```
Cost with regularization: 0.05165921361463465
Mean square error: 0.05165921361463465
```



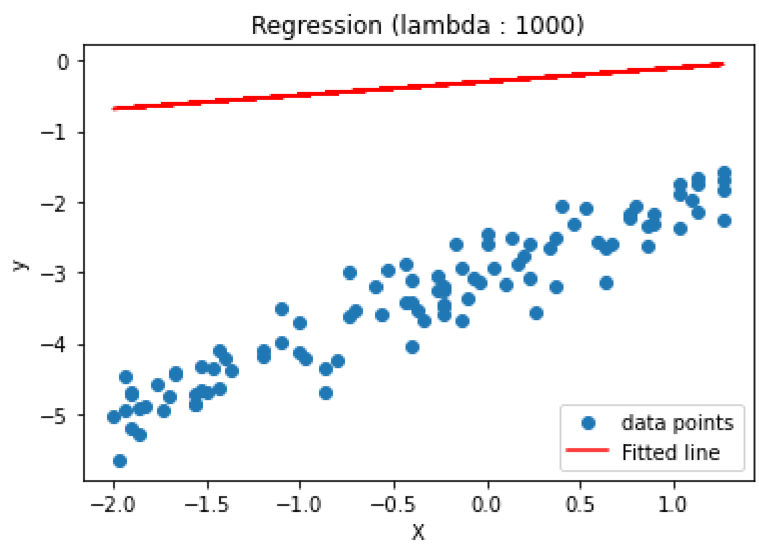

```
Cost with regularization: 0.5181225987615888
Mean square error: 0.08984032593751458
```



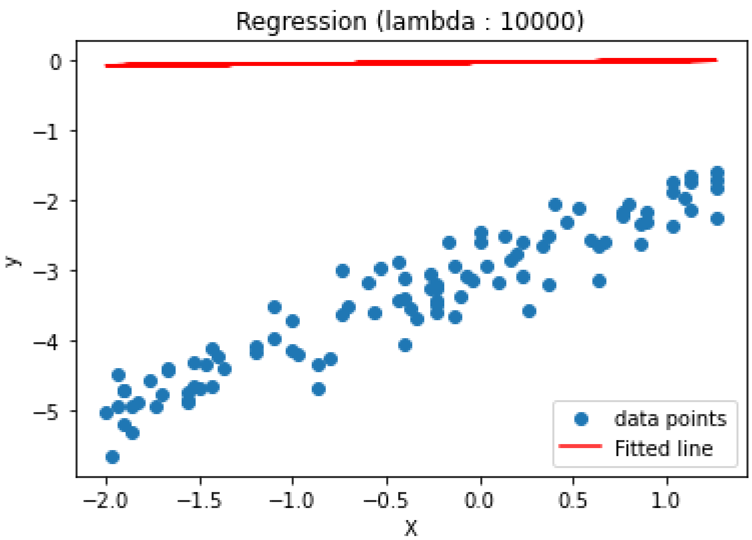

```
Cost with regularization: 2.79317248874003
Mean square error: 1.2785107553109407
```




```
Cost with regularization: 5.591464362606628
Mean square error: 4.946888025066497
```




```
Cost with regularization: 6.242695626933973
Mean square error: 6.161442583355813
```



1.5 Analyze your results (10 points)

According to the above figures, what's the best choice of λ?

Why the regressed line turns to be flat as we increase λ?

Your answer:

1. The best choice of λ is 0 according to the above figures
2. As we increases \labmda, we will get more loss if the model has higher coefficient. Hence, the model will be trained to use smaller coefficient, which makes the regressed line flat.

Problem 2: Getting familiar with PyTorch (30 points)

In [9]:
```
import torch
import numpy as np
import mltools as ml
from matplotlib import pyplot as plt
```

1. Load the " data/curve80.txt " data set, and split it into 75% / 25% training/test. We will use degree=5 for all the polynomial features

In [10]:
```
# Your code:
data = np.genfromtxt("data/curve80.txt")
X = data[:,0]
X = np.atleast_2d(X).T # code expects shape (M,N) so make sure it's 2-dimensional
Y = data[:,1] # doesn't matter for Y
Xtr,Xte,Ytr,Yte = ml.splitData(X,Y,0.75) # split data set 75/25

degree = 5
XtrP = ml.transforms.fpoly(Xtr, degree=degree, bias=False)
XtrP,params = ml.transforms.rescale(XtrP)
```

Transform numpy arrays to tensor. Make sure the XtrP_tensor has the shape of (60, 5) while Ytr_tensor has the shape of (60, 1). (5 points)

In [11]:
```
XtrP_tensor = torch.from_numpy(XtrP)
Ytr_tensor = torch.from_numpy(Ytr)

XtrP_tensor = XtrP_tensor.float()
Ytr_tensor = Ytr_tensor.float()

Ytr_tensor = torch.unsqueeze(Ytr_tensor, 1)
```

1. Initialize our linear regressor. (5 points)

In [12]:
```
linear_regressor = torch.nn.Linear(in_features=5, out_features=1)
```

1. Set up the criterion and optimizer

In [13]:
```
criterion = torch.nn.MSELoss()
optimizer = torch.optim.SGD(linear_regressor.parameters(), lr=0.1)
epochs = 100000
```

1. Training the regressor using gradient descent. 10 points

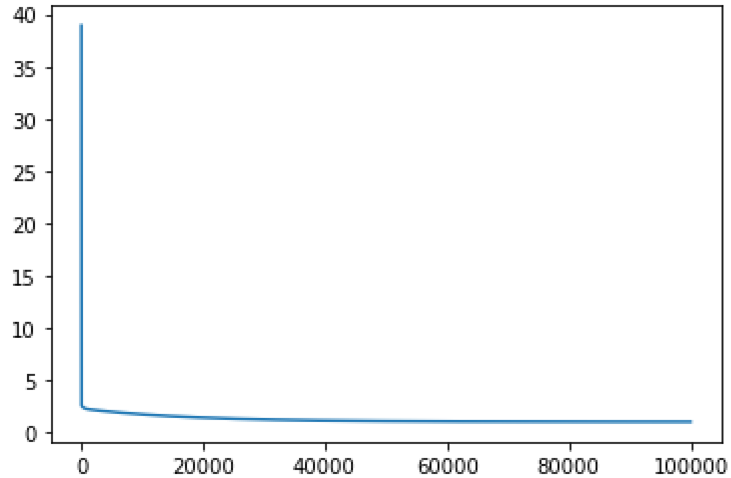
In [14]:
```
loss_record = []
for _ in range(epochs):
    optimizer.zero_grad() # set gradient to zero
    pred_y = linear_regressor(XtrP_tensor)
    loss = criterion(pred_y, Ytr_tensor) # calculate loss function

    loss.backward() # backpropagate gradient
    loss_record.append(loss.item())
    optimizer.step()  # update the parameters in the linear regressor
```

1. Plot the loss v.s. epochs. Show the plot here. 5 points.

In [15]:
```
plt.plot(range(epochs), (loss_record))
```

Out[15]:
```
[<matplotlib.lines.Line2D at 0x1e8cde8d820>]
```

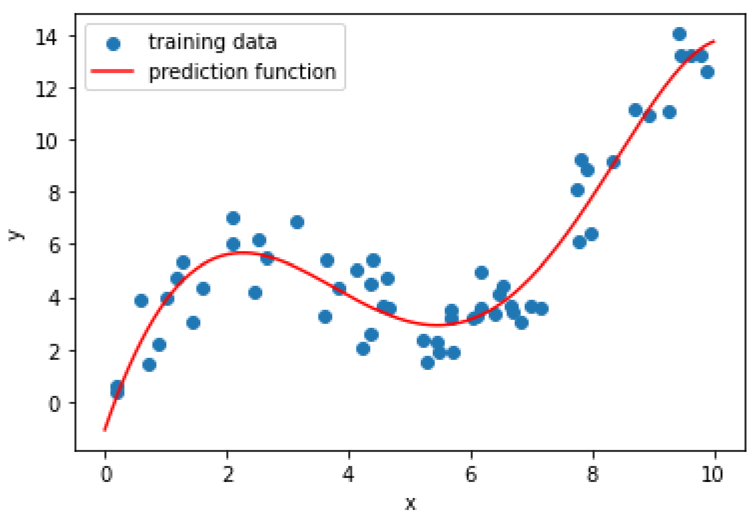


1. Visualize the trained linear regressor. 5 points.

In [16]:
```
xs = np.linspace(0,10,200)
xs = xs[:,np.newaxis]
xsP, _ = ml.transforms.rescale(ml.transforms.fpoly(xs,degree=degree,bias=False), param
xsP_tensor = torch.from_numpy(xsP).float()
ys = linear_regressor(xsP_tensor)

plt.scatter(Xtr,Ytr,label="training data")
plt.plot(xs,ys.detach().numpy(),label="prediction function",color ='red')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
```

Out[16]: <matplotlib.legend.Legend at 0x1e8ce830be0>



Statement of Collaboration

Sabina Yang - discussed about the usage of pytorch

In [ ]: