Confidence intervals vs Bayesian statistics

Last updated: October 20, 2017

Recap: hypothesis testing

Hypothesis testing = statistical proof by contradiction

- There is an unknown population parameter θ. (eg: $\mu_{MA} - \mu_{BA}$
- H_0: $\theta =$(some specific value, eg, 0)
- H_A: not H_0.
- Observe evidence s

$$\text{p-value} = \mathbb{P}(Evidence|Hypothesis).$$

Recap: hypothesis testing

What are the population parameters θ in the following hypothesis tests?

- Linear regression: is β_0 significantly different from 0?
- t-test: is the mean equal to μ_0?
- z-test: is the proportion equal to p_0?
- Two-sample t-test: are the means of the two groups equal?
- F-test: are the means of all groups equal?

Recap: hypothesis testing

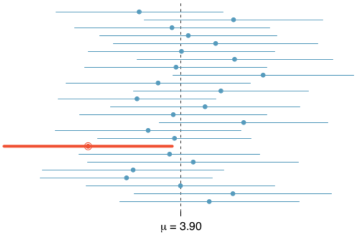
What are the population parameters θ in the following hypothesis tests?

- Linear regression: is β_0 significantly different from 0?
- t-test: is the mean equal to μ_0?
- z-test: is the proportion equal to p_0?
- Two-sample t-test: are the means of the two groups equal?
- F-test: are the means of all groups equal?

Another way to think of p-values: confidence intervals.

Confidence intervals

Definition. A 95% confidence interval for a population parameter θ is a RANDOM interval, constructed based on the data, such that there is a 95% chance that this interval contains θ.



See os3.pdf, section 4.2. **Give interpretations for the following**

- Confidence intervals for estimates of β_0 and β_1 in linear regression (eg: teacher: salary vs degree)
- Confidence interval in z-test (eg: UT Austin survey)

Recap: hypothesis testing

Example to keep in mind: teacher: salary vs degree (MA/BA).
What to do if we want...

- **A quantitative answer?** (eg: how much is $\mu_{MA} - \mu_{BA}$?).

Recap: hypothesis testing

Example to keep in mind: teacher: salary vs degree (MA/BA).
What to do if we want...

- **A quantitative answer?** (eg: how much is $\mu_{MA} - \mu_{BA}$?). Regression.

Recap: hypothesis testing

Example to keep in mind: teacher: salary vs degree (MA/BA).
What to do if we want...

- **A quantitative answer?** (eg: how much is $\mu_{MA} - \mu_{BA}$?). Regression.
- **A 'maybe' answer?** (eg: there is an 80% chance that $\mu_{MA} \neq \mu_{BA}$)

Recap: hypothesis testing

Example to keep in mind: teacher: salary vs degree (MA/BA).
What to do if we want...

- **A quantitative answer?** (eg: how much is $\mu_{MA} - \mu_{BA}$?). Regression.
- **A 'maybe' answer?** (eg: there is an 80% chance that $\mu_{MA} \neq \mu_{BA}$) Nonsense. μ_{MA} and μ_{BA} are fixed, so either they are different, or they are not.

Recap: hypothesis testing

Example to keep in mind: teacher: salary vs degree (MA/BA).
What to do if we want...

- **A quantitative answer?** (eg: how much is $\mu_{MA} - \mu_{BA}$?). Regression.
- **A 'maybe' answer?** (eg: there is an 80% chance that $\mu_{MA} \neq \mu_{BA}$) Nonsense. μ_{MA} and μ_{BA} are fixed, so either they are different, or they are not.
  BUT, you CAN construct at RANDOM interval and say: there is a 95% chance that this interval contains $\mu_{MA} - \mu_{BA}$. This is the confidence interval.

Recap: hypothesis testing

Example to keep in mind: teacher: salary vs degree (MA/BA).
What to do if we want...

- **A quantitative answer?** (eg: how much is $\mu_{MA} - \mu_{BA}$?).
  Regression.
- **A 'maybe' answer?** (eg: there is an 80% chance that $\mu_{MA} \neq \mu_{BA}$)
  Nonsense. μ_{MA} and μ_{BA} are fixed, so either they are different, or they are not.
  BUT, you CAN construct at RANDOM interval and say: there is a 95% chance that this interval contains $\mu_{MA} - \mu_{BA}$. This is the confidence interval.
- **A whole range of nulls with different certainties?** (eg: $\mu_{MA} - \mu_{BA}$ is uniformly distributed between -500 and 2000)

Recap: hypothesis testing

Example to keep in mind: teacher: salary vs degree (MA/BA).
What to do if we want...

- **A quantitative answer?** (eg: how much is $\mu_{MA} - \mu_{BA}$?). Regression.
- **A 'maybe' answer?** (eg: there is an 80% chance that $\mu_{MA} \neq \mu_{BA}$) Nonsense. μ_{MA} and μ_{BA} are fixed, so either they are different, or they are not.
  BUT, you CAN construct at RANDOM interval and say: there is a 95% chance that this interval contains $\mu_{MA} - \mu_{BA}$. This is the confidence interval.
- **A whole range of nulls with different certainties?** (eg: $\mu_{MA} - \mu_{BA}$ is uniformly distributed between -500 and 2000) Nonsense. $\mu_{MA} - \mu_{BA}$ is a fixed number so it cannot be 'uniformly distributed'

Recap: hypothesis testing

Example to keep in mind: teacher: salary vs degree (MA/BA).
What to do if we want...

- **A quantitative answer?** (eg: how much is $\mu_{MA} - \mu_{BA}$?).
  Regression.
- **A 'maybe' answer?** (eg: there is an 80% chance that $\mu_{MA} \neq \mu_{BA}$)
  Nonsense. μ_{MA} and μ_{BA} are fixed, so either they are different, or they are not.
  BUT, you CAN construct at RANDOM interval and say: there is a 95% chance that this interval contains $\mu_{MA} - \mu_{BA}$. This is the confidence interval.
- **A whole range of nulls with different certainties?** (eg: $\mu_{MA} - \mu_{BA}$ is uniformly distributed between -500 and 2000)
  Nonsense. $\mu_{MA} - \mu_{BA}$ is a fixed number so it cannot be 'uniformly distributed'

To say 'maybe', you need: Bayesian statistics

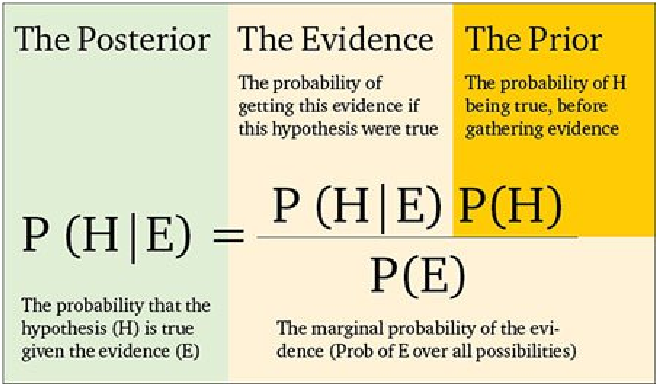
Bayesian = statistics for those who like to say "maybe"

Best explained in: `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading20.pdf`.

The Frequentist view:

$$\text{p-value} = \mathbb{P}(\textit{Evidence}|\textit{Hypothesis})$$

The Bayesian view



Bayesian vs Frequentist example

Frequentist

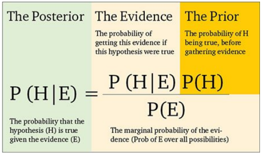
- $\mu_1 - \mu_2$ is an unknown fixed number.
- Prior 'belief' H_0: $\mu_1 - \mu_2 = 0$
- Observe data $S = s$. Compute

$$p - value = \mathbb{P}(S > s | H_0).$$

- After calculations: new 'belief': $\mu_1 - \mu_2 = 0$ or $\mu_1 - \mu_2 \neq 0$.

Bayesian:

- $\mu_1 - \mu_2$ is random
- Prior belief: $\mu_1 - \mu_2 \sim P_0$
- Observe data $S = s$. Update the belief:



- After calculations: posterior belief: $\mu_1 - \mu_2 \sim P_1$.

The Bayesian philosophy

What we think about the world after seeing data =
What we thought about the world before seeing data ×
Chance we'd see our data under different assumptions about the world

$$Pr(world|data) = Pr(world) \times \frac{Pr(data|world)}{Pr(data)}.$$

$$Posterior = Prior \times Likelihood$$

Why Bayesian?

Why NOT Bayesian?

The Bayesian philosophy

What we think about the world after seeing data =
What we thought about the world before seeing data $\times$
Chance we'd see our data under different assumptions about the world

$$Pr(world|data) = Pr(world) \times \frac{Pr(data|world)}{Pr(data)}.$$

$$Posterior = Prior \times Likelihood$$

Why Bayesian?

- Intuitive logic
- Flexible: input = a distribution, output = a distribution (instead of a single estimate or yes/no)
- Data-driven: see enough data and the prior does not matter.

Why NOT Bayesian?

The Bayesian philosophy

What we think about the world after seeing data =
What we thought about the world before seeing data ×
Chance we'd see our data under different assumptions about the world

$$Pr(world|data) = Pr(world) \times \frac{Pr(data|world)}{Pr(data)}.$$

$$Posterior = Prior \times Likelihood$$

Why Bayesian?

- Intuitive logic
- Flexible: input = a distribution, output = a distribution (instead of a single estimate or yes/no)
- Data-driven: see enough data and the prior does not matter.

Why NOT Bayesian?

- Answers depend on prior, which can be different for different people
- Likelihood hard to compute: resort to simulations or certain family of priors. So actually, not THAT flexible.

How to be Bayesian?

There is a Bayesian equivalent of regression, t-test, z-test etc.
Doable in R. See R code example: proportion estimate.