Homework 4-2

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Simplex Method

$\Rightarrow$ standard form

$$\text{Max } Z = -2x_1 + 3x_2 + 4x_3$$

$$\begin{aligned} \text{s.t. } -x_1 + x_2 - x_3 &\le -10 \\ -x_1 + 2x_2 - 3x_3 &\le -6 \\ -3x_1 + 4x_2 - 5x_3 &\le -15 \end{aligned}$$

At first I turned the question into the standard form. However it seemed to not comfirm to use the Simplex Method here. As you can see, the b in this question are negative but it must be positive in the standard form. So we could not solve the question by using the Simplex Method.

TWO-Phase

$\text{Max } \{-x_0\}$

$\text{Max } \bar{z} = -2x_1 - 3x_2 - 4x_3$

$\text{s.t. } s_1 = -10 + x_1 - x_2 + x_3 + x_0$

$s_2 = -6 + x_1 - 2x_2 + x_3 + x_0$

$s_3 = -15 + 3x_1 - 4x_2 + 5x_3 + x_0$

$\Rightarrow$

$\text{Max } -15 + 3x_1 - 4x_2 + 5x_3 - s_3$

$\text{Max } \bar{z} = -2x_1 - 3x_2 - 4x_3$

$\text{s.t. } s_1 = 5 - 2x_1 + 3x_2 - 4x_3 + s_3$

$s_2 = 9 - 2x_1 + 2x_2 - 2x_3 + s_3$

$-x_0 = -15 + 3x_1 - 4x_2 + 5x_3 - s_3$

$\Rightarrow \text{Max } -\frac{35}{4} + \frac{1}{2}x_1 - \frac{1}{4}x_2 - \frac{5}{4}s_1 + \frac{1}{4}s_3$

$\text{Max } \bar{z} = -5 - 6x_2 + s_1 - s_3$

$\text{s.t. } x_3 = \frac{5}{4} - \frac{1}{2}x_1 + \frac{3}{4}x_2 - \frac{1}{4}s_1 + \frac{1}{4}s_3$

$s_2 = \frac{13}{2} - x_1 + \frac{1}{2}x_2 + \frac{1}{2}s_1 + \frac{1}{2}s_3$

$x_0 = \frac{35}{4} - \frac{1}{2}x_1 + \frac{1}{4}x_2 + \frac{5}{4}s_1 - \frac{1}{4}s_3$

$\Rightarrow \text{Max } -\frac{15}{2} + \frac{1}{2}x_2 - x_3 - \frac{3}{2}s_1 + \frac{1}{2}s_3$

$\text{Max } \bar{z} = -5 - 6x_2 + s_1 - s_3$

$\text{s.t. } x_1 = \frac{5}{2} + \frac{3}{2}x_2 - 2x_3 - \frac{1}{2}s_1 + \frac{1}{2}s_3$

$s_2 = \frac{8}{2} - x_2 + 2x_3 + s_1$

$x_0 = \frac{15}{2} - \frac{1}{2}x_2 + \frac{3}{2}s_1 - \frac{1}{2}s_3$

$\rightarrow \text{Max } -\frac{11}{2} - s_1 - \frac{1}{2}s_2 + \frac{1}{2}s_3$

$\text{Max } \bar{z} = -29 - 12x_3 - 5s_1 + 6s_2 - s_3$

$\text{s.t. } x_1 = \frac{11}{2} + x_3 + s_1 - \frac{3}{2}s_2 + \frac{1}{2}s_3$

$x_2 = 4 + 2x_3 + s_1 - s_2$

$x_0 = \frac{11}{2} + s_1 + \frac{1}{2}s_2 - \frac{1}{2}s_3$

$\Rightarrow \text{Max } -x_0$

$\text{Max } \bar{z} = -40 - 12x_3 - 7s_1 + 5s_2 + 2x_0$

$\text{s.t. } x_1 = 14 + x_3 + s_1 - s_2 - x_0$

$x_2 = 4 + 2x_3 + s_1 - s_2$

$s_3 = 11 + 2s_1 + s_2 - 2x_0$

$\text{opt} = 0$

$\Rightarrow \text{Max } \bar{z} = -20 - 5x_2 - 2x_3 - 2s_1$

$\text{s.t. } x_1 = 10 + x_2 - x_3 - s_1$

$s_2 = 4 - x_2 + 2x_3 + s_1$

$s_3 = 15 + 3x_1 - 2x_2 + 4x_3$

$(x_1, x_2, x_3) = (10, 0, 0)$

$\text{Min } z = 20$

Because the b are negative, we can use 2-phase. And it also worked. The optimal solution is 20. But there were seven pivots here. It was tons of handwork and I had to take many time to make sure every plus-minus signs, fraction and process correct. So annoying.

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min $z = 2x_1 + 3x_2 + 4x_3$

s.t.

$x_1 - x_2 + x_3 \geq 10$

$x_1 - 2x_2 + 3x_3 \geq 6$

$3x_1 - 4x_2 + 5x_3 \geq 15$

$x_1, x_2, x_3 \geq 0$

$\Rightarrow$

Max $-z + 2x_1 + 3x_2 + 4x_3$

s.t.

$-x_1 + x_2 - x_3 + w_1 = -10$

$-x_1 + 2x_2 - 3x_3 + w_2 = -6$

$-3x_1 + 4x_2 - 5x_3 + w_3 = -15$

$x_1, x_2, x_3 \geq 0$

	z	x_1	x_2	x_3	w_1	w_2	w_3	RHS
	-1	2	$3 \; +\frac{8}{3}$	$4 \; -\frac{10}{3}$	$0 \; 0$	$0 \; 0$	$0 \; +\frac{2}{3}$	$0 \; -10$
w_1	0	-1	1	-1	1	0	0	-10
w_2	0	-1	2	-3	0	1	0	-6
w_3	0	-3	4	-5	0	0	1	-15
	-1	0	$\frac{17}{3} - \frac{2}{3}$	$\frac{2}{3} + \frac{4}{3}$	0	0	$\frac{2}{3} - \frac{2}{3}$	-10
w_1	0	0	$\frac{1}{3}$	$\frac{2}{3}$	1	0	$-\frac{1}{3}$	-5
w_2	0	0	$\frac{2}{3} + \frac{1}{3}$	$-\frac{4}{3} - \frac{2}{3}$	0	1	$-\frac{1}{3}$	$-1+5$
x_1	0	1	$-\frac{4}{3} + \frac{1}{3}$	$\frac{5}{3} - \frac{2}{3}$	0	0	$-\frac{1}{3}$	$5+5$
	-1	0	5	2	0	0	0	-20
w_3	0	0	1	-2	-3	0	1	15
w_2	0	0	1	-2	-1	1	0	4
x_1	0	1	-1	1	0	0	0	10

$\frac{2}{3} = \frac{10}{15}$

$\frac{4}{5} = \frac{12}{15}$

$17 >$

$(x_1, x_2, x_3, w_1, w_2, w_3) = (10, 0, 0, 0, 4, 15)$

opt $= 20$

It work undoubtedly. I thought the efficiency of dual simplex was greater than 2-phase. No matter the book told if the b were nnegative we could use the dual simplex.

primal problem

$\min\ z = 2x_1+3x_2+4x_3$

s.t. $x_1 - x_2 + x_3 \geq 10$

$x_1 - 2x_2 + 3x_3 \geq 6$

$3x_1 - 4x_2 + 5x_3 \geq 15$

$x_1, x_2, x_3 \geq 0$

$\Rightarrow$

dual problem.

$\max\ W = 10y_1 + 6y_2 + 15y_3$

s.t. $y_1 + y_2 + 3y_3 \leq 2$

$-y_1 - 2y_2 - 4y_3 \leq 3$

$y_1 + 3y_2 + 5y_3 \leq 4.$

$\max\ W - 10y_1 - 6y_2 - 15y_3$

S.t. $s_1 = 2 - y_1 - y_2 - 3y_3$

$s_2 = 3 + y_1 + 2y_2 + 4y_3$

$s_3 = 4 - y_1 - 3y_2 - 5y_3$

$\max\ -10 - 5y_1 - y_2 + 5s_1$

s.t. $y_3 = \frac{2}{3} - \frac{1}{3}y_1 - \frac{1}{3}y_2 - \frac{1}{3}s_1$

$s_2 = \frac{17}{3} - \frac{1}{3}y_1 + \frac{2}{3}y_2 - \frac{4}{3}s_1$

$s_3 = \frac{2}{3} + \frac{2}{3}y_1 - \frac{4}{3}y_2 + \frac{5}{3}s_1$

$\max\ -20 + 3y_2 + 15y_3 + 10s_1$

s.t. $y_1 = 2 - y_2 - 3y_3 - s_1$

$s_2 = 5 + y_2 - y_3 - s_1$

$s_3 = 2 - 2y_2 - 3y_3 + s_1$

$y_1 = 2 \Rightarrow w_1 = 0$

$y_2 = 0$

$y_3 = 0$

$s_1 = 0$

$s_2 = 5 \Rightarrow x_2 = 0$

$s_3 = 2 \Rightarrow x_3 = 0$

$x_1 = 10$

$10 = 6 + w_2 \quad w_2 = 4$

$30 = 15 + w_3 \quad w_3 = 15$

$(x_1, x_2, x_3, w_1, w_2, w_3) = (10, 0, 0, 0, 4, 15)$

Opt $= 20$

It worked if converting the problem into its dual problem. I consider it was the same concept in the dual simplex and the dual problem. There are some difference between this two way. In my opinon, the dual problem can let you know the shadow price clearly. However, we must convert the dual solution back to the primal. In the other hand we can know the optimal solution in the dual simplex immediately. Above all, it is my thought about the homework.