矩阵论

Second Assignment

Student Name: 杨澍　　Student ID: 202218019427012

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1. **Question 2.2.2:**

   Solve: 因为

$$Ax = \lambda x$$

所以有

$$A^m x = \lambda^m x$$

即有

$$\begin{aligned} ||A^m x|| &= ||\lambda^m x|| \\ &= |\lambda|^m ||x|| \end{aligned}$$

且有 $||A^m x|| \leq ||A^m|| ||x||$ 所以

$$|\lambda| \leq \sqrt[m]{||A^m||}$$

2. **Question 2.2.5:**

   Solve

$$\begin{aligned} ||A||_S &= \max \frac{||Ax||_S}{||x||_S} \\ &= \max \frac{||SAx||_2}{||Sx||_2} \\ &= \max \frac{||SAS^{-1}y||_2}{||y||_2} \\ &= ||SAS^{-1}||_2 \end{aligned}$$

3. **Question 2.2.7: Solve**

$$\begin{aligned} ||x||_V &= ||yx^T||_F \\ &= [\sum_{i=1}^{m} \sum_{j=1}^{n} y_i^2 x_j^2]^{1/2} \\ &= [\sum_{i=1}^{m} y_i^2 \sum_{j=1}^{n} x_j^2]^{1/2} \\ &= ||y||_2 ||x||_2 \end{aligned}$$

因为$||y||_2$是常数，所以$||x||_V$是C^n上的向量范数且

$$\begin{aligned} ||Ax||_V &= ||y||_2 ||Ax||_2 \\ &\leq ||y||_2 ||A||_F ||x||_2 \\ &= ||A||_F ||x||_V \end{aligned}$$

4. **Question 2.3.1: Solve**

$$||A^{-1}B|| \leq ||A^{-1}||||B|| \leq -1$$

因此

$$I + A^{-1}B = I - (-A^{-1}B)$$

为可逆矩阵，故，$A(1 + A^{-1}B) = A + B$也为可逆矩阵