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Asset pricing and ambiguity: Empirical evidence[☆]

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ABSTRACT

We introduce ambiguity in conjunction with risk to study the relation between risk, ambiguity, and expected returns. Distinguishing between ambiguity and attitudes toward ambiguity, we develop an empirical methodology for measuring the degree of ambiguity and for assessing attitudes toward ambiguity from market data. The main findings indicate that ambiguity in the equity market is priced. Introducing ambiguity alongside risk provides stronger evidence on the role of risk in explaining expected returns in the equity markets. The findings also indicate that investors' level of aversion to or love for ambiguity is contingent on the expected probability of favorable returns.

1. Introduction

In this paper, we propose looking at a dimension of uncertainty not accounted for by risk—the uncertainty of probabilities that make up risk, so-called ambiguity or Knightian uncertainty. Risk in equity markets means that future returns are realized with known probabilities, while ambiguity refers to situations where the probabilities associated with these realizations are not known or not uniquely assigned. We investigate the relation between risk, ambiguity, and expected return in the equity markets over time. To this end, we develop a new theoretically based empirical methodology for measuring the extent of ambiguity using stock return data. We show that, when introducing ambiguity in conjunction with risk, risk has a significant positive effect on the expected rate of return. This finding contrasts with prior puzzling results of a negative, or insignificant, relation between risk and expected return. We find that ambiguity, on average, has a significantly positive effect on expected returns.

An important consequence of these findings is that the risk-return relation cannot be studied without taking ambiguity into consideration. We find that the ambiguity premium, embedded in the equity premium, is contingent upon the expected probability of favorable returns and investors' attitudes toward ambiguity. Introducing the idea

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of probabilistic contingent ambiguity attitudes, we elicit investors' attitudes toward ambiguity and find that their aversion to ambiguity increases with the expected probability of favorable returns (gain), and that their love for ambiguity increases with the expected probability of unfavorable returns (loss).

Previous studies of ambiguity have focused mainly on the theoretical aspects of attitudes toward (aversion to) ambiguity, rather than on the actual measurement of ambiguity.[1] In general, these studies employ decision models that do not provide the necessary separations between risk and ambiguity and between tastes and beliefs that are crucial for the measurement of ambiguity. For this reason, in most of these studies, the models are not tested empirically. The idea of the decision-making model that we employ is that the preferences for ambiguity are applied exclusively to probabilities, such that the attitude toward ambiguity is defined as the attitude toward mean-preserving spreads in probabilities, analogous to the Rothschild-Stiglitz risk attitude toward mean-preserving spreads in outcomes. Based on this idea, we propose an empirical measure of ambiguity, which is independent of risk, attitudes toward risk, as well as attitudes toward ambiguity. Namely, we propose that the degree of ambiguity, denoted $\mho^2$ (mho^2), be measured by the expected volatility of probabilities, across the relevant outcomes. That is,

$$\mho^2[r] = \int \mathrm{E}[\varphi(r)]\mathrm{Var}[\varphi(r)]dr,$$

where $\varphi(\cdot)$ is a probability density function, $\mathrm{E}[\varphi(r)]$ is the expected probability of a given rate of return r, and $\mathrm{Var}[\varphi(r)]$ is the variance of the probability of r. The intuition of $\mho^2$ is that, as the degree of risk can be measured by the volatility of returns, so too can the degree of ambiguity be measured by the volatility of the probabilities of returns.

To illustrate the intuition behind $\mho^2$, consider the following binomial example of an asset with two possible future returns: $d = -10\%$ and $u = 20\%$. Assume that the probabilities, $\mathrm{P}(\cdot)$, of d and u are known, say $\mathrm{P}(d) = \mathrm{P}(u) = 0.5$. The expected return is thus 5%, and the standard deviation of the return (measuring the degree of risk) is 15%. In this case, since the probabilities are known, ambiguity is not present ($\mho = 0$), and investors face only risk. Assume next that the probabilities of d and u can be either $\mathrm{P}(d) = 0.4$ and $\mathrm{P}(u) = 0.6$, or alternatively $\mathrm{P}(d) = 0.6$ and $\mathrm{P}(u) = 0.4$, where these two alternative distributions are equally likely. Investors now face not only risk but also ambiguity. The degree of ambiguity, in terms of probabilities, is $\mho = \sqrt{\sum_i \mathrm{E}[\mathrm{P}(i)]\mathrm{Var}[\mathrm{P}(i)]} = \sqrt{2 \times 0.5 \times \left(0.5 \times (0.4 - 0.5)^2 + 0.5 \times (0.6 - 0.5)^2\right)} = 0.1$. Notice that the degree of risk, computed using the expected probabilities $\mathrm{E}[\mathrm{P}(d)] = \mathrm{E}[\mathrm{P}(u)] = 0.5$, has not changed.

[1] See, for example, Chen and Epstein (2002), Cao et al. (2005), and Epstein and Schneider (2008). For recent surveys, see Epstein and Schneider (2010) and Guidolin and Rinaldi (2013).

In this view, as opposed to other (arguable) measures of ambiguity that depend on the magnitude of outcomes, our measure, $\mho^2$, depends only on the probabilities of outcomes, regardless of their magnitude. Thereby, ambiguity is measured independently of risk. This measure of ambiguity can also be viewed as a nonlinear aggregate of the variance of the mean, the variance of the variance, and the variances of all other higher probability moments, through the variance of the probability density function. The measure $\mho^2$ is employed in explaining the early exercise of employee stock options (Izhakian and Yermack, 2017), the pricing of credit default swaps (Augustin and Izhakian, 2016), dividend payout policy (Dahya et al., 2017), and capital structure decisions (Izhakian et al., 2017). This risk-independent measure, $\mho^2$, is our centerpiece of the empirical tests.

Our point of departure is the underlying hypothesis that ambiguity is one of the determinants of the equity premium. In particular, that the risk-ambiguity-return relation is given by the following model

$$\mathbb{E}_t[r_{t+1}] - r_f = \underbrace{\gamma \frac{1}{2} \mathbb{V}\mathrm{ar}_t[r_{t+1}]}_{\text{Risk premium}} + \underbrace{\eta\left(1 - \mathrm{E}_t[\mathrm{P}_{t+1}]\right)\mathbb{E}_t\left[|r_{t+1} - \mathbb{E}_t[r_{t+1}]|\right]\mho_t^2[r_{t+1}]}_{\text{Ambiguity premium}}, \tag{1}$$

where r_f is the risk-free rate of return, r is the return on the market portfolio, $\mathbb{V}\mathrm{ar}_t[r_{t+1}]$ is the degree of risk, measured by the volatility of the market return, γ measures the aversion to risk of a representative investor or an aggregation of the risk aversion of investors, $\mho_t^2[r_{t+1}]$ measures the degree of ambiguity, and η measures the representative investor's attitude toward ambiguity, which is contingent on the expected cumulative probability of an unfavorable return, $\mathrm{E}_t[\mathrm{P}_{t+1}]$.[2] The expectation, the variance, and the ambiguity of the market return are conditional on the information available at the beginning of the return period, time t. The equity premium, defined in Eq. (1), has two separate components: a risk premium and an ambiguity premium. To see the intuition for the latter premium, recall that the conventional risk premium can be viewed as a premium that an investor is willing to pay to exchange a risky asset for a riskless one with an identical expected outcome. In a similar way, the ambiguity premium can be viewed as the premium that an investor is willing to pay to exchange a risky and ambiguous asset for a risky, but nonambiguous asset, with an identical expected outcome and identical risk.

The ambiguity premium, defined in Eq. (1), is a function of the investor's attitude toward ambiguity, and it is independent of her attitude toward risk. This property is of primary importance, not only from a theoretical perspective but also is essential for an empirical implementation. In addition to our empirical pricing questions, we explore the nature of attitudes toward ambiguity. In particular, to

[2] For example, a return higher than the risk-free rate can be considered favorable, while a return lower than the risk-free rate can be considered unfavorable (e.g., Barberis et al., 2001).

address the question of whether attitudes toward ambiguity change over time and what are the determinants of these changes, we focus on the subjective estimation of the perceived probabilities of a favorable return. These empirical questions have not previously been addressed in the literature, except for a few attempts using laboratory experiments.[3] To address this gap, we elicit attitudes toward ambiguity from the ambiguity premium. This requires that the ambiguity premium be attributed exclusively to ambiguity and attitudes toward it, such that ambiguity is measured from the data and the attitudes toward ambiguity are uniquely and endogenously determined by the empirical model. This is not possible if the ambiguity premium is also a function of risk attitudes.

It is important to note that to elicit attitudes toward ambiguity, it is necessary that the underpinning decision-making model distinguishes between risk and ambiguity and between tastes and beliefs. The classical models of decision-making under ambiguity, for example, do not provide these necessary separations. In the max-min expected utility (MEU) with multiple priors framework (Gilboa and Schmeidler, 1989), the set of priors reflects both information (beliefs) and aversion to ambiguity (love for ambiguity is not supported), which are not separable. Similarly, in the Choquet expected utility (CEU) framework (Gilboa and Schmeidler, 1989), capacities reflect both ambiguity and aversion to ambiguity, which again are not separable.

We test four hypotheses implied by the theoretical asset pricing model in Eq. (1), in line with prior behavioral and asset pricing studies. The first hypothesis is that, when ambiguity is introduced alongside risk, the latter has a positive and significant effect on the equity premium (i.e., a positive risk premium). The second hypothesis is that, for a high expected probability of favorable returns, the ambiguity premium is positive, implying ambiguity-averse behavior. The third hypothesis is that, for a high expected probability of unfavorable returns, the ambiguity premium is negative, implying ambiguity-loving behavior. The fourth hypothesis is that aversion to ambiguity increases with the expected probability of favorable returns, and love for ambiguity increases with the expected probability of unfavorable returns. This implies that the magnitude of the ambiguity premium increases with the expected probabilities. These hypotheses do not imply that the equity premium may be negative. We find that, even when the ambiguity premium is negative, it is smaller than the risk premium such that the equity premium as a whole remains positive.

To test our four hypotheses empirically, we use the model proposed in Eq. (1). Aligned with findings in the experimental literature (e.g., Cohen and Einav, 2007; Schechter, 2007), we assume constant relative risk aversion. Regarding attitudes toward ambiguity, in the experimental literature, individuals typically have asymmetric ambiguity preferences for unfavorable and favorable outcomes (e.g., Abdellaoui et al., 2011; Abdellaoui et al., 2005; Du and Budescu, 2005). Yet, the functional form of attitudes toward ambiguity has not been identified. Therefore, the empirical tests of our hypotheses are designed without imposing a particular functional structure on attitudes toward ambiguity and aim to elicit attitudes toward ambiguity from the data. As the model in Eq. (1) suggests, the regression tests account for ambiguity attitudes that are contingent upon the expected probability of favorable returns (gains) in time-series regressions of the risk-ambiguity-return relation. With this design in place, we explore the (nonlinear) relation between ambiguity and the market expected return. To conduct these investigations, we use the exchange-traded fund (ETF), Standard & Poor's Depositary Receipt (SPDR), on the Standard & Poor's (S&P) 500 Index, as a proxy for the market portfolio.

Our empirical findings support the hypotheses, showing that ambiguity significantly affects stock market returns. That is to say, investors act as if they consider the degree of ambiguity when they price financial assets. The findings provide strong evidence that individuals exhibit ambiguity aversion to favorable returns and love for ambiguity for unfavorable returns. Moreover, their ambiguity aversion increases with the expected probability of favorable returns, and their ambiguity loving increases with the expected probability of unfavorable returns. Introducing ambiguity, in our model, alongside risk (volatility) shows that the expected volatility of the market portfolio has a positive and significant effect on the portfolio's expected return. These findings provide evidence that support the classical theoretical risk-return relation.

We conduct robustness tests to verify that our findings are driven by ambiguity and related preferences and not by other potential risk factors. Among other tests, we control for skewness, kurtosis, volatility of the mean, and volatility of volatility. In addition, we test for the effect of investors' sentiment (Baker and Wurgler, 2006) and downside risk (Ang et al., 2006) alongside ambiguity. In all these tests, the effect and significance of ambiguity remain, while the other factors are mostly insignificant. We also conduct tests to verify that our results are derived from a set of probability distributions and could not have been derived from a (possible) single distribution due to sampling. Finally, we test for alternative models of decision-making under ambiguity, as well as for unstructured attitude toward risk.[4]

The rest of the paper is organized as follows. In Section 2, we discuss the related literature. Section 3 provides the theoretical framework. In Section 4, we discuss the data and develop the estimation methodology. In Section 5, there is a discussion of the regression tests and the empirical findings. Robustness texts are described in Section 6. We consider alternative models in Section 7. In Section 8, we provide the intuition behind our findings. We conclude in Section 9.

[3] These studies focus mainly on the differences between attitudes toward ambiguity concerning favorable and unfavorable outcomes and do not delve into studying the functional form and determinants of attitudes toward ambiguity (e.g., Baillon and Bleichrodt, 2015; Baillon et al., 2016).

[4] By "unstructured attitude toward risk," we mean that we do not impose a specific functional form over attitudes toward risk (e.g., constant relative risk aversion or constant absolute risk aversion).

2. Literature review

As early as Merton (1980), numerous studies have investigated the fundamental (linear) relation between the risk and return of the market portfolio. The findings are conflicting, ranging from a positive relation (e.g., French et al., 1987; Campbell and Hentschel, 1992; Guo and Whitelaw, 2006; Pástor et al., 2008) to a negative one (e.g., Black, 1976; Campbell, 1987; Nelson, 1991; Harvey, 2001). Over the years, many studies have attempted to reconcile the conflicting findings about the risk-return relation. Some have dealt with the econometric methodology employed to estimate the conditional variance (e.g., Glosten et al., 1993; Harvey, 2001), while others have proposed alternative risk measures (e.g., Ghysels et al., 2005). Another strand of the literature includes time-varying elements such as regime switching (e.g., Campbell and Cochrane, 1999; Whitelaw, 2000), time-varying risk aversion (e.g., Campbell and Cochrane, 1999), and investor sentiment (e.g., Yu and Yuan, 2011). One of the most comprehensive empirical investigations of the risk–return relation is by Welch and Goyal (2008), who find the variables suggested in previous studies are poor predictors of the equity premium.

The idea that ambiguity is a missing factor, in asset pricing models, that can affect asset prices and thus the risk-return relation has been studied mainly from a theoretical perspective, focusing on individuals' aversion to ambiguity. Recent theoretical studies have introduced ambiguity into pricing models, suggesting that ambiguity is one of the determinants of expected return (e.g., Epstein and Schneider, 2010; Ui, 2011; Izhakian and Benninga, 2011). In some studies, an attempt is made to calibrate the model to the data (e.g., Epstein and Schneider, 2008), while others contain proxies for ambiguity, such as disagreement among analysts (e.g., Anderson et al., 2009; Antoniou et al., 2015). Most empirical (behavioral) studies about ambiguity include data collected in controlled experiments and focus on individuals' aversion to ambiguity, not on the impact of ambiguity on financial decision-making. Market data are used in only a few studies to measure ambiguity; for example, Ulrich (2013) uses entropy of inflation and Williams (2015) uses the Volatility Index (VIX).

Driouchi et al. (2018) extract option implied ambiguity from the prices of put options written on the S&P 500 Index.[5] Focusing on the period prior to the 2008 subprime crisis, they infer the investors' attitude toward ambiguity from market data and show that aversion to ambiguity shifted during that period.[6] While these authors explain the lead-lag relation between options' implied volatility and realized volatility through the lens of ambiguity, we employ ambiguity (and risk) to explain the expected excess return on equities. In a follow-up paper, So et al. (2016) use the option implied ambiguity to explain the expected excess return across different countries during the 1990–2012 period. Using a different methodology, they find that ambiguity is an important determinant, in addition to risk, in explaining excess return. This finding, in general, is in line with our findings, although we differ by the direction of the effect of ambiguity. Also, our hypotheses differentiate between ambiguity and conditional attitudes toward it. Andreou et al. (2014) measure ambiguity using the dispersion of volume-weighted strike prices of the S&P 500 Index options. In our study, we deal with ambiguity and attitudes toward ambiguity in a different way. We elicit the investors' attitudes toward ambiguity and suggest that aversion to ambiguity is subject to the expected probability of favorable outcomes, for which we find strong support in the empirical findings.

In the literature concerning parameter uncertainty, the set of events and the nature of the probability distribution are known, but the parameters governing their distribution are unknown. In addition, the decision maker maximizes utility using posterior parameters that generate a set of priors à la (Gilboa and Schmeidler, 1989), which can be viewed as reflecting both information (beliefs) and aversion to ambiguity (e.g., Bawa et al., 1979; Coles and Loewenstein, 1988; Coles et al., 1995). Therefore, parameter uncertainty may be viewed as a special case of ambiguity in which the nature of the probability distributions is known. A related approach is model uncertainty. In this approach, an uncertainty about the true probability law governing the realization of states is assumed, and a decision maker, with her concerns about misclassification, looks for a robust decision-making rule (e.g., Hansen et al., 1999; Hansen and Sargent, 2001). Other studies use a model risk approach to empirically account for uncertainty about the true set of predictive variables. To account for such model misspecification, a Bayesian (predictive distribution) approach can be taken by assigning each set of variables (or model) a posterior probability (e.g., Pástor and Stambaugh, 1999; Pástor and Stambaugh, 2000; Cremers, 2002).

Our empirical findings further the understanding of the nature of attitudes toward ambiguity and thus are related to several behavioral studies. The behavioral literature shows that investors who face a high probability of losses typically tend to embrace ambiguity, while if they face a high probability of gains, they can exhibit ambiguity aversion. For example, Viscusi and Chesson (1999) find that people exhibit ambiguity aversion ("fear" effects) for small probabilities of loss and love for ambiguity ("hope" effects) for large probabilities of loss. Assuming risk neutrality, Maffioletti and Santoni (2005) find ambiguity seeking in individuals' trading behavior. Wakker et al. (2007) analyze health insurance information, concluding that individuals are ambiguity seeking. Other behavioral studies that find ambiguity-loving behavior when there is a relatively high probability of loss and ambiguity aversion when there is a relatively high probability of gain include Mangelsdorff and Weber (1994), Abdellaoui et al. (2005), and Du and Budescu (2005). Consistent with the findings in these studies, our empirical findings show that investors have asymmetric preferences for ambiguity. Moreover, we analyze

[5] In particular, Driouchi et al. (2018) extend Black and Scholes (1973) option pricing model by introducing ambiguity through Choquet-Brownian motions. Using this model, they measure implied ambiguity using the minimum absolute error between observed index options prices and the suggested model's intrinsic values.

[6] They show that implied ambiguity can reduce biases in predicting the realized market variance.

aggregate preferences concerning ambiguity and contribute to the literature by allowing for the identification of the particular functional form of attitudes toward ambiguity.

3. The theoretical model

Knightian uncertainty has provided the basis for a rich body of literature in decision theory.[7] We distinguish between risk and ambiguity by using the theoretical framework of expected utility with uncertain probabilities (EUUP) proposed by Izhakian (2017). The main idea of EUUP is that preferences concerning ambiguity are applied exclusively to uncertain probabilities, such that ambiguity aversion is defined as aversion to mean-preserving spreads in probabilities. It is similar to Rothschild and Stiglitz (1970) approach, which is applied to outcomes to measure risk. Here this approach is applied to probabilities to measure ambiguity. Unlike other measures of ambiguity that are risk dependent and consider only the variance of a single moment of the distribution (i.e., the variance of the mean or the variance of the variance), our measure is risk independent and accounts for all higher moments of the return distribution.

3.1. The ambiguity premium

We employ EUUP to model two tiers of uncertainty: one with respect to outcomes and the other with respect to the probabilities of these outcomes. Under EUUP, there are two phases of the decision-making process: one for each of these tiers. In the first phase, the investor forms her perceived probabilities for all events that are relevant to her decision. In the second phase, she assesses the expected value of each alternative using her perceived probabilities and chooses accordingly. Ambiguity—the uncertainty about probabilities—dominates the first phase, while risk—the uncertainty about outcomes—dominates the second phase.

To formally define the uncertain return r, let $(\mathcal{S}, \mathcal{E}, \mathrm{P})$ be a probability space, where $\mathcal{S}$ is a state space, $\mathcal{E}$ is a σ-algebra of subsets of the state space (i.e., a set of events), $\mathrm{P} \in \mathcal{P}$ is a probability measure, and the set of probability measures $\mathcal{P}$ is convex. An algebra Π of measurable subsets of $\mathcal{P}$ is equipped with a probability measure, denoted ξ. The uncertain return is then given by the "uncertain" variable, $r : \mathcal{S} \to \mathbb{R}$. Denote by $\varphi(r)$ the marginal probability (density function or probability mass function) associated with the cumulative probability $\mathrm{P} \in \mathcal{P}$ of r. The expected marginal and cumulative probability of r, taken using the second-order probability measure ξ, are then respectively defined by

$$\mathrm{E}[\varphi(r)] \;\equiv\; \int_{\mathcal{P}} \varphi(r) d\xi \quad \text{and} \quad \mathrm{E}[\mathrm{P}(r)] \;\equiv\; \int_{\mathcal{P}} \mathrm{P}(r) d\xi, \tag{2}$$

and the variance of the marginal probability is defined by

$$\mathrm{Var}[\varphi(r)] \;\equiv\; \int_{\mathcal{P}} \Big(\varphi(r) - \mathrm{E}[\varphi(r)]\Big)^2 d\xi. \tag{3}$$

The expected return and the variance of return are then computed using the expected probabilities. That is,

$$\mathbb{E}[r] \;\equiv\; \int \mathrm{E}[\varphi(r)] r dr \quad \text{and}$$
$$\mathbb{V}\mathrm{ar}[r] \;\equiv\; \int \mathrm{E}[\varphi(r)]\Big(r - \mathbb{E}[r]\Big)^2 dr. \tag{4}$$

As usual, investor preferences concerning risk are modeled by a bounded, strictly increasing and twice-differentiable utility function $\mathrm{U} : \mathbb{R}_+ \to \mathbb{R}$. Risk aversion takes the form of a concave $U(\cdot)$, risk loving the form of a convex $U(\cdot)$, and risk neutrality takes the form of a linear $U(\cdot)$. Like Tversky and Kahneman (1992) cumulative prospect theory, we normalize $U(\cdot)$ to $\mathrm{U}\big(1 + r_f\big) = 0$, where the risk-free rate r_f is the reference point relative to which returns are classified as unfavorable (losses) or favorable (gains).[8] That is, any return lower than r_f is considered unfavorable, and any return higher than r_f is considered favorable. An alternative reference point could be a zero rate of return. However, it would be natural to assume that investors classify their investment opportunities relative to a neutral investment in a risk-free asset.[9]

As investors are sensitive to ambiguity, they do not compound the set of priors $\mathcal{P}$ and the prior ξ over $\mathcal{P}$ in a linear way (compounded lotteries), but instead they aggregate these probabilities in a nonlinear way, reflecting their ambiguity aversion. Preferences concerning ambiguity are defined by preferences over mean-preserving spreads in probabilities and modeled by a strictly increasing and twice-differentiable function over probabilities, $\Upsilon : [0, 1] \to \mathbb{R}$, called the outlook function. Similar to risk, ambiguity aversion takes the form of a concave $\Upsilon(\cdot)$, ambiguity loving takes the form of a convex $\Upsilon(\cdot)$, and ambiguity neutrality the form of a linear $\Upsilon(\cdot)$. In EUUP, ambiguity aversion is demonstrated when an investor prefers the expectation of an uncertain probability of each payoff over the uncertain probability itself.

Consider a decision to invest one unit of wealth, where future consumption is determined by the one-period (uncertain) return r, which is the only source of wealth. In

[7] Knight (1921) distinguishes the concept of uncertainty from risk by conditions under which the set of events that may occur is a priori unknown, and the odds of these events are also either not unique or are unknown. Roughly speaking, this concept can be viewed as underpinning two strands of literature. The first is the "unawareness" literature, where decision makers may not be aware of a subset of events (e.g., Karni and Vierø, 2013). The second is the ambiguity literature, where the set of events is perfectly known, but their probabilities are either not unique or are unknown (e.g., Gilboa and Schmeidler, 1989; Schmeidler, 1989). These two strands of literature can be viewed as overlapping when dealing with monetary outcomes (real numbers). In this case, the "uncertain"—risky and ambiguous—variable is defined by a measurable function from states into the real numbers such that there is no real monetary outcome that the decision maker is not aware of. It is possible that the decision maker is not aware of some states of nature, which may affect the uncertainty about the probabilities of some outcomes. But such uncertainty is already accounted for by ambiguity.

[8] The literature focuses on the implications of losses and gains for preferences (e.g., Barberis and Huang, 2001; Hirshleifer, 2001), while we focus on beliefs.

[9] We also test the model assuming that the zero rate of return is the reference point and the results are essentially the same.

EUUP, when the investor does not distort perceived probabilities, the expected utility of this investment opportunity can be approximated by[10]

$$\mathrm{W}(1+r) \approx \int_{r\le r_f} \mathrm{U}(1+r)\underbrace{\mathrm{E}[\varphi(r)]\left(1-\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(r)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(r)])}\mathrm{Var}[\varphi(r)]\right)}_{\text{Perceived probability of unfavorable return}}dr + \int_{r\ge r_f} \mathrm{U}(1+r)\underbrace{\mathrm{E}[\varphi(r)]\left(1+\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(r)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(r)])}\mathrm{Var}[\varphi(r)]\right)}_{\text{Perceived probability of favorable return}}dr, \tag{5}$$

where $\mathrm{P}(r)$ is the cumulative probability of return being lower than r. Notice that when investors are ambiguity neutral ($\Upsilon(\cdot)$ is linear), Eq. (5) collapses to the conventional expected utility (i.e., investors compound probabilities linearly). In contrast, when the investors are ambiguity averse ($\Upsilon(\cdot)$ is concave), they do not aggregate probabilities linearly, and the perceived probabilities are affected by the intensity of aversion to ambiguity. The advantage of EUUP is that ambiguity preferences are applied exclusively to probabilities, such that ambiguity aversion is an aversion to mean-preserving spreads in probabilities. Conceptually, the perceived probability of a given outcome can be viewed as the certain probability values the investor is willing to accept in exchange for its uncertain probability.

Using the notion of the volatility of probabilities in Eq. (5), the measure of ambiguity, defined in Izhakian (2018), is

$$\mho^2[r] = \int \mathrm{E}[\varphi(r)]\mathrm{Var}[\varphi(r)]dr. \tag{6}$$

The measure $\mho^2$ can be used both in a continuous space with infinitely many outcomes or in a discrete state space with finitely many outcomes. The main idea of $\mho^2$ is that ambiguity can be measured by the volatility of probabilities, just as the degree of risk can be measured by the volatility of returns (Rothschild and Stiglitz, 1970). In general, irrespective of the decision-theoretic framework, $\mho^2$ encompasses not only an ambiguous variance and an ambiguous mean but also the ambiguity of the higher moments of the probability distribution (i.e., skewness, kurtosis, etc.) through the uncertainty of probabilities. This measure, $\mho^2 \in [0,\infty)$, attains its minimum value, zero, only when all probabilities are known.

The key property of $\mho^2$ is risk independence, as it is a necessary property for studying the distinct role that ambiguity plays in pricing assets. Unlike other ambiguity measures, which are risk dependent and consider only the variance of a single moment of the distribution (e.g., the variance of the mean or the variance of the variance), $\mho^2$ is risk independent and accounts for the variance of all the moments of the outcome distribution.[11]

EUUP, modeled in Eq. (5), extends the Bayesian approach to probabilities by asserting that uncertain probabilities are subject to a prior probability. This concept of second-order beliefs (prior probability over probabilities) and preferences is not unfamiliar and has been employed in earlier decision theory models of ambiguity (e.g., Nau, 2006; Chew and Sagi, 2008). Some behavioral studies find evidence that decision makers act as if they have second-order preferences (e.g., Halevy, 2007; Hao and Houser, 2012). Furthermore, neural responses to second-order uncertainty (ambiguity) are associated with areas of the brain that are distinct from those supporting first-order uncertainty (e.g., Huettel et al., 2006; Bach et al., 2011).

The uncertainty premium of a risky and ambiguous consumption $1+r$ delivered by Eq. (5) is[12]

$$\mathcal{K} \approx \underbrace{-\frac{1}{2}\frac{\mathrm{U}''(1+\mathrm{E}[r])}{\mathrm{U}'(1+\mathrm{E}[r])}\mathbb{V}\mathrm{ar}[r]}_{\text{Risk premium}} \underbrace{-\mathbb{E}\left[\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(r)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(r)])}\right]\mathbb{E}\Big[|r-\mathbb{E}[r]|\Big]\mho^2[r]}_{\text{Ambiguity premium}}. \tag{7}$$

The uncertainty premium is required by investors in return for bearing the risk and ambiguity associated with holding the asset. It consists of two components: the risk premium and the ambiguity premium.

The model in Eq. (7) attains two separations. First, it distinguishes between the risk premium and the ambiguity premium. Second, within each premium, it distinguishes between the two sources of the premium: attitudes and beliefs. The risk premium is the Arrow-Pratt risk premium, where the expectation and the variance are taken with respect to expected probabilities. Independently, a higher risk, measured by $\mathbb{V}\mathrm{ar}[r]$, or a higher aversion to risk, measured by the coefficient of absolute (local) risk aversion $-\frac{\mathrm{U}''}{\mathrm{U}'}$, result in a greater risk premium. The ambiguity premium possesses attributes resembling those of the risk premium but with respect to probabilities rather than to outcomes. This premium separates ambiguity, measured by $\mho^2$, from attitudes toward ambiguity, measured by the coefficient of absolute (local) ambiguity aversion $-\frac{\Upsilon''}{\Upsilon'}$. Aversion to ambiguity ($-\frac{\Upsilon''}{\Upsilon'} > 0$) implies a positive ambiguity

[10] This functional representation is obtained by taking the Taylor expansion of the dual representation of EUUP, proposed by Izhakian (2017). The reminder of this approximation is of order $o\Big(\int \mathrm{E}\big[|\varphi(r)-\mathrm{E}[\varphi(r)]|^3\big]rdr\Big)$ as $\int |\varphi(r)-\mathrm{E}[\varphi(r)]|dr \to 0$, meaning that the accuracy of the approximation is equivalent to the accuracy of the cubic approximation, $o\big(\mathbb{E}\big[|r-\mathbb{E}[r]|^3\big]\big)$, in which the fourth and higher absolute central moments of outcomes are of strictly smaller order than the third absolute central moment as $|r-\mathbb{E}[r]| \to 0$ and are therefore negligible.

[11] The volatility of the volatility (stochastic volatility) or the volatility of the mean are sometimes used as measures of ambiguity. The measure of ambiguity $\mho^2$ is broader than either of these measures; it accounts for both, as well as for the volatility of all higher moments of the probability distribution (e.g., skewness and kurtosis), through the variance of probabilities. As opposed to the volatility of volatility and to the volatility of the mean, the measure $\mho^2$ is risk independent, as it does not depend on the magnitudes of outcomes but only on their probabilities. Furthermore, $\mho^2$ solves some major issues that arise from the use of only the volatility of volatility or only the volatility of mean as measures of ambiguity; for example, two equities with different degrees of ambiguity but constant volatility, or two equities with different degrees of ambiguity but constant mean.

[12] To prove this approximation, we assumed that returns and their probabilities are close to their expectation (Izhakian, 2018).

premium. Love for ambiguity ($-\frac{\Upsilon''}{\Upsilon'}<0$) implies a negative premium. Indifference to ambiguity ($-\frac{\Upsilon''}{\Upsilon'}=0$) implies a zero premium, which is also obtained when all probabilities are perfectly known (i.e., when $\mho^2=0$). A higher degree of ambiguity or a higher aversion to ambiguity result in a greater ambiguity premium. The ambiguity premium is also a function of the expected absolute deviation of returns from their expectation, $\mathbb{E}\big[|r-\mathbb{E}[r]|\big]$, which scales the ambiguity premium to the units of returns.

The unique functional representation of EUUP allows for the identification of the different components of each premium and the relation between these components. Previous attempts to extract the ambiguity premium using alternative models generate ambiguity premiums that are also a function of risk attitudes (e.g., Izhakian and Benninga, 2011; Ui, 2011), whereas in our model the ambiguity premium is solely a function of ambiguity and the attitudes toward it. As discussed earlier, this property is of primary importance for the empirical investigation of ambiguity attitudes. Recall that, besides the pricing questions we address, we explore the nature of ambiguity attitudes with a focus on the effect of the subjective estimation of the probability of a favorable return on investors ambiguity attitudes. We attempt to elicit investors' attitudes from the ambiguity premium.

3.2. The assumptions

The ambiguity measure $\mho^2$ is a centerpiece of our empirical tests. To estimate the degree of ambiguity from market data, the probability distributions of returns must be derived from the data. To this end, we assume that there is a representative investor whose set of priors is an aggregation of the sets of priors of all investors in the economy. We also assume that each subset of observed returns is the result of a realization of one prior out of this set of priors. In particular, every trading day is characterized by a different distribution of returns P, and the set $\mathcal{P}$ of these distributions over a month represents the representative investor's set of priors. Returns on the market portfolio are assumed to be normally distributed but not independently identically distributed.[13] That is, every $\mathrm{P}\in\mathcal{P}$ is normal, governed by a different mean μ and variance σ^2.[14] The degree of ambiguity is, therefore, measured by

$$\mho^2[r]=\int \mathrm{E}[\phi(r;\mu,\sigma)]\mathrm{Var}[\phi(r;\mu,\sigma)]dr, \tag{8}$$

where $\phi(\cdot)$ denotes the normal probability density function.

To simplify the functional form of the equity premium, defined in Eq. (7), we rely upon prior behavioral findings about risk and ambiguity attitudes, which assist us in forming our four hypotheses. The interactions among risk, ambiguity, and attitudes toward risk and ambiguity in determining the equity premium are dictated by Eq. (7). In stating our hypotheses, the prior behavioral findings assist us only in identifying the direction of the effect that investor's attitudes toward risk and toward ambiguity have. Similar to classical asset pricing theory (e.g., Arrow-Pratt's theory), the risk aversion determining the positive equity premium is not derived from the underlying (Von Neumann-Morgenstern) expected utility theory but from the behavioral findings of this theory.

We consider previous work and assume that the attitudes toward risk are of the constant relative risk aversion (CRRA) class (e.g., Chetty, 2006; Schechter, 2007; Cohen and Einav, 2007). However, there is no conclusive evidence about the functional representation of attitudes toward ambiguity. Therefore, no particular functional structure is imposed on Υ. Allowing the intensity of the effect of attitudes toward ambiguity to be determined endogenously, we examine the functional structure of ambiguity attitudes.

The uncertainty premium, defined in Eq. (7), can thus be simplified to

$$\mathcal{K}\approx\underbrace{\gamma\frac{1}{2}\mathbb{V}\mathrm{ar}[r]}_{\text{Risk premium}}+\underbrace{\mathbb{E}\big[\eta(1-\mathrm{E}[\mathrm{P}(r)])\big]\mathbb{E}\big[|r-\mathbb{E}[r]|\big]\mho^2[r]}_{\text{Ambiguity premium}}, \tag{9}$$

where γ is the coefficient of relative risk aversion, and $\eta(\cdot)=-\frac{\Upsilon''(\cdot)}{\Upsilon'(\cdot)}$ characterizes the ambiguity attitudes conditional on the expected cumulative probability $\mathrm{E}[\mathrm{P}(r)]=\mathrm{E}[\Phi(r;\mu,\sigma)]$, where $\Phi(\cdot)$ denotes the normal cumulative probability distribution. A positive (negative) γ implies aversion to (love for) risk, and a positive (negative) $\eta(\cdot)$ implies aversion to (love for) ambiguity. In this representation, while γ is constant across different returns, $\eta(\cdot)$ can vary across different expected return probabilities. Risk and ambiguity are measured separately and can have a different effect on excess returns.

3.3. The hypotheses

We next turn to present our hypotheses, based on the model presented in Eq. (9). Our first hypothesis is standard in the asset pricing literature.

Hypothesis 1. When ambiguity is accounted for, the risk premium is positive, as investors typically exhibit risk aversion.

The next two hypotheses are related to investors' attitudes toward ambiguity, as reflected in the ambiguity premium defined in Eq. (9). These hypotheses rely on behavioral findings regarding individuals' attitudes toward ambiguity, which are found to be different for losses and gains (e.g., Abdellaoui et al., 2005; Wakker et al., 2007).

Hypothesis 2. Investors typically exhibit aversion to ambiguity when expecting favorable returns. Therefore, for a relatively high expected probability of favorable returns, the ambiguity premium is positive.

Hypothesis 3. Investors typically exhibit love for ambiguity when expecting unfavorable returns. Therefore, for a relatively high expected probability of unfavorable returns, the ambiguity premium is negative.

[13] In our robustness tests, we relax the assumption of normally distributed returns, elaborated on in Section 6.

[14] The volatility of μ and of σ^2 have been separately attributed to ambiguity (e.g., Cao et al., 2005; Faria and Correia-da Silva, 2014).

Hypotheses 2 are 3 are related to the logic in Dow and Werlang (1992). Assuming risk-neutral preferences, these authors show that, in the CEU and MEU frameworks, an investor buys an asset if the price is lower than the expected payoff (a positive ambiguity premium) and sells it if the price is higher than the expected payoff (a negative ambiguity premium), where the expectations are estimated using capacities (CEU) or the worst-case prior (MEU). Recall that in CEU (MEU), the capacities (the set of priors) do not distinguish between information (beliefs) and tastes (attitude) for ambiguity. Using our underlying framework, EUUP, we construct capacities from beliefs (information), captured by the set of priors, and tastes for ambiguity, captured by the outlook function. This construction allows us to identify the effect of beliefs on the expected payoff, and thus on the ambiguity premium, separately from the effect of tastes for ambiguity. Using Hypotheses 2 and 3, we refine the results of Dow and Werlang (1992) and conjecture that what determines why the premium is positive or negative is the investor's belief (information) about the expected probability of favorable (unfavorable) returns.

Our fourth hypothesis extends the discussion of ambiguity preferences. In Hypothesis 4, we conjecture that the intensity of attitudes toward ambiguity is subject to the expected probability of unfavorable returns (losses) and favorable returns (gains). Accordingly, the magnitude of the ambiguity premium is determined. This hypothesis is based on the behavioral findings of Mangelsdorff and Weber (1994), Viscusi and Chesson (1999), and Du and Budescu (2005).

Hypothesis 4. Aversion to ambiguity increases with the expected probability of favorable returns and love for ambiguity increases with the expected probability of unfavorable returns. Therefore, the higher the expected probability of favorable returns, the higher is the positive ambiguity premium. On the other hand, the higher the expected probability of unfavorable returns, the higher is the negative ambiguity premium.

A higher expected probability of favorable returns is not a sufficient condition for a higher expected excess return. To see this, note that the reference point r_f (the risk-free rate) must be lower than the expected return, $\mathbb{E}[r]$; otherwise, no rational investor would consider investing in the market. In this case, the expected probability of favorable returns can increase by a higher probability mass for returns in the range $r_f \leq r < \mathbb{E}[r]$, such that the expected return $\mathbb{E}[r]$ decreases. Furthermore, a critical difference between expected returns and expected probability of favorable returns is that the former is outcome dependent, which in turn implies risk dependence, while the latter is outcome independent. In Hypothesis 4 we conjecture that the ambiguity premium is increasing in the expected probability of favorable returns through the channel of attitude toward ambiguity (tastes), rather than through the channel of beliefs about outcomes (expected return). In particular, as the expected likelihood of favorable returns increases, the investor becomes more ambiguity averse and requires a higher premium.

4. Data and parameter estimation

To test our hypotheses empirically, we first propose a new method to measure the degree of ambiguity in the market using trading data.

4.1. Data

We use intraday data of the ETF SPDR taken from the Trade and Quote (TAQ) database. The SPDR is designed to track the S&P 500 Index. The stocks in the SPDR have the same weights as in the Index. Quarterly dividends are added to the SPDR every three months. The SPDR can be sold short like any other stock. The SPDR is selected to minimize the bias in return variances, since it is frequently traded and its bid-ask spread is minimal. We use the SPDR as a proxy for the market portfolio since the it trades continuously, while the S&P 500 contains illiquid stocks, so its values can be stale. The data cover the period from February 1993 to December 2016, 287 months in total.[15] Daily and monthly returns, adjusted for dividends, are obtained from the Center for Research in Security Prices (CRSP) database. VIX values are obtained from the Chicago Board Options Exchange (CBOE) website. The risk-free rate is the one-month Treasury bill rate of return, provided by Ibbotson Associates.

4.2. Estimating risk and ambiguity

Our first step is to estimate the time-series values of the monthly degree of risk. The monthly risk (volatility) is measured by the variance of daily rates of return. As in French et al. (1987), the variance of the returns is computed by applying the adjustment for non-synchronous trading, proposed by Scholes and Williams (1977).[16] Fig. 1 shows the time series of the volatility (in terms of standard deviation) for 1993–2016.

Our next step is to develop a method to estimate the time-series values of the monthly degree of ambiguity.[17] We take the prices of the SPDR every five minutes from 9:30 a.m. to 4:00 p.m. each day, which provides 79 prices for each day. If there was no trade at a specific time, we take the volume-weighted average of the closest trading prices within five minutes of that time stamp. By not including returns between the closing prices and the opening prices of the following day, we eliminate the impact of overnight price changes (new information) and dividend distributions. Using these prices, we compute the five-minute returns, which provides a maximum of 78 returns for each day. Observations with extreme price changes ($\pm 10\%$ log returns) within five minutes are omitted because many of them are probably due to erroneous orders that were cancelled by the exchange. When we include

[15] Under the ticker symbol SPY, the SPDR began trading on the American Stock Exchange (Amex) on January 29, 1993.

[16] Scholes and Williams (1977) suggest that the volatility of returns takes the form $\sigma_t^2 = \sum_{i=1}^{N_t} (r_{t,i} - \mathrm{E}[r_{t,i}])^2 + 2\sum_{i=2}^{N_t} (r_{t,i} - \mathrm{E}[r_{t,i}])(r_{t,i-1} - \mathrm{E}[r_{t,i-1}])$.

[17] We focus on one-month intervals; however, the same procedure can be applied to shorter or longer periods than one month.

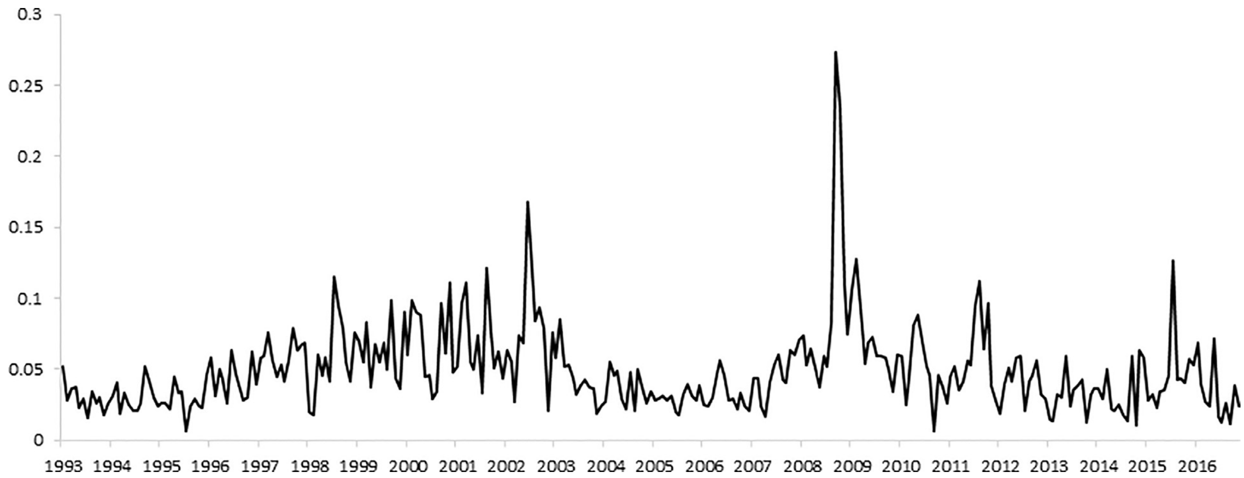

Fig. 1. Time series of volatility.
The figure illustrates the realized volatility between February 1993 and December 2016. The x-axis shows the timeline. The y-axis shows the volatility in terms of standard deviation. The standard deviation, in monthly terms, is computed from daily returns of SPDR adjusted for dividends.

these observations in the model, the effect of ambiguity is even more significant than when we exclude these observations.

The choice of five-minute intervals is dictated by the measure of ambiguity. To perform meaningful time-series tests, in our 24-year period (February 1993 to December 2016), we need to use monthly observations. To obtain a statistically meaningful monthly measure of ambiguity, a daily estimate of a probability distribution is needed. This, in turn, requires intraday observations. The decision to compute returns using five-minute intervals is motivated by Andersen et al. (2001), who show that this time interval is sufficient to minimize microstructure effects.[18]

For each day in our sample period there are between 33 and 78 observations. We use these observations to compute the normalized (by the number of intraday observations) daily mean and variance of the return, denoted μ and σ^2, respectively. This variance is also computed by applying the adjustment for nonsynchronous trading, proposed by Scholes and Williams (1977).[19] Based on the assumption that the intraday returns are normally distributed, we construct the set of priors $\mathcal{P}$, where each prior P within the set $\mathcal{P}$ is defined by a pair of μ and σ. Importantly, in our approach, the set of priors $\mathcal{P}$ that underlies ambiguity is endogenously derived. To the best of our knowledge, all other empirical and experimental studies, except for Hey et al. (2010), take the set of priors to be exogenously given or designed by the experimenter.

Given the set $\mathcal{P}$ of (normal) probability distributions, we first compute for each day (prior) the cumulative probability of favorable returns (gain), $\mathrm{P}\left(r \geq r_f\right) = 1 - \Phi\left(r_f; \mu, \sigma\right)$, where any return greater than the risk-free rate is considered favorable.[20] For each month, there are 20 to 22 different gain probabilities. Their expectation, $\mathrm{E}[\mathrm{P}(r \geq r_f)]$, is computed assuming that the daily ratios of the sample mean and standard deviation, $\frac{\mu}{\sigma}$, are student's-t distributed, which assigns lower weights to values of $\frac{\mu}{\sigma}$ that deviate from the monthly mean of $\frac{\mu}{\sigma}$. This method implies that the cumulative probabilities of favorable returns, $\mathrm{P}(r \geq r_f)$, are uniformly distributed over the month. See, for example, 2010, 21, Proposition 1.27 (Kendall and Stuart, 2010). It is consistent with the assumption that the representative investor does not have any information indicating which probability distributions is more likely and thus acts as if she assigns an equal weight to each one. The expected probabilities of favorable returns are used to estimate ambiguity attitudes.

To compute the monthly degree of ambiguity, specified in Eq. (8), we represent each daily return distribution by a histogram. To this end, we divide the range of daily returns, from −6% to +6%, into 60 intervals (bins), each of width 0.2%. For each day, we compute the probability of the return being in each bin. In addition, we compute the probability of the return being lower than −6% and higher than +6%. We then compute the mean and the variance of the probabilities for each of the 62 bins separately. Then, we estimate the degree of ambiguity of each month using the following discrete form

$$\mho^2[r] = \frac{1}{w(1-w)} \times \begin{pmatrix} \mathrm{E}\left[\Phi(r_0; \mu, \sigma)\right]\mathrm{Var}\left[\Phi(r_0; \mu, \sigma)\right] \\ + \sum_{i=1}^{60} \mathrm{E}\left[\Phi(r_i; \mu, \sigma) - \Phi(r_{i-1}; \mu, \sigma)\right] \\ \times \mathrm{Var}[\Phi(r_i; \mu, \sigma) - \Phi(r_{i-1}; \mu, \sigma)] \\ + \mathrm{E}[1 - \Phi(r_{60}; \mu, \sigma)]\mathrm{Var}[1 - \Phi(r_{60}; \mu, \sigma)] \end{pmatrix},$$

where $r_0 = -0.06$, $w = r_i - r_{i-1} = 0.002$, and $\frac{1}{w(1-w)}$ scales the weighted-average volatilities of probabilities to the bin

[18] To ensure that our findings are not derived by the selection of 5-minute intervals, we also test our model using 10, 15, and 20-minute intervals; the results are essentially the same.

[19] We also test our model without the Scholes-Williams correction for nonsynchronous trading. The results are essentially the same.

[20] We also test our model assuming that a positive return is considered favorable, i.e., $\mathrm{P}(r \geq 0)$. The results are essentially the same.

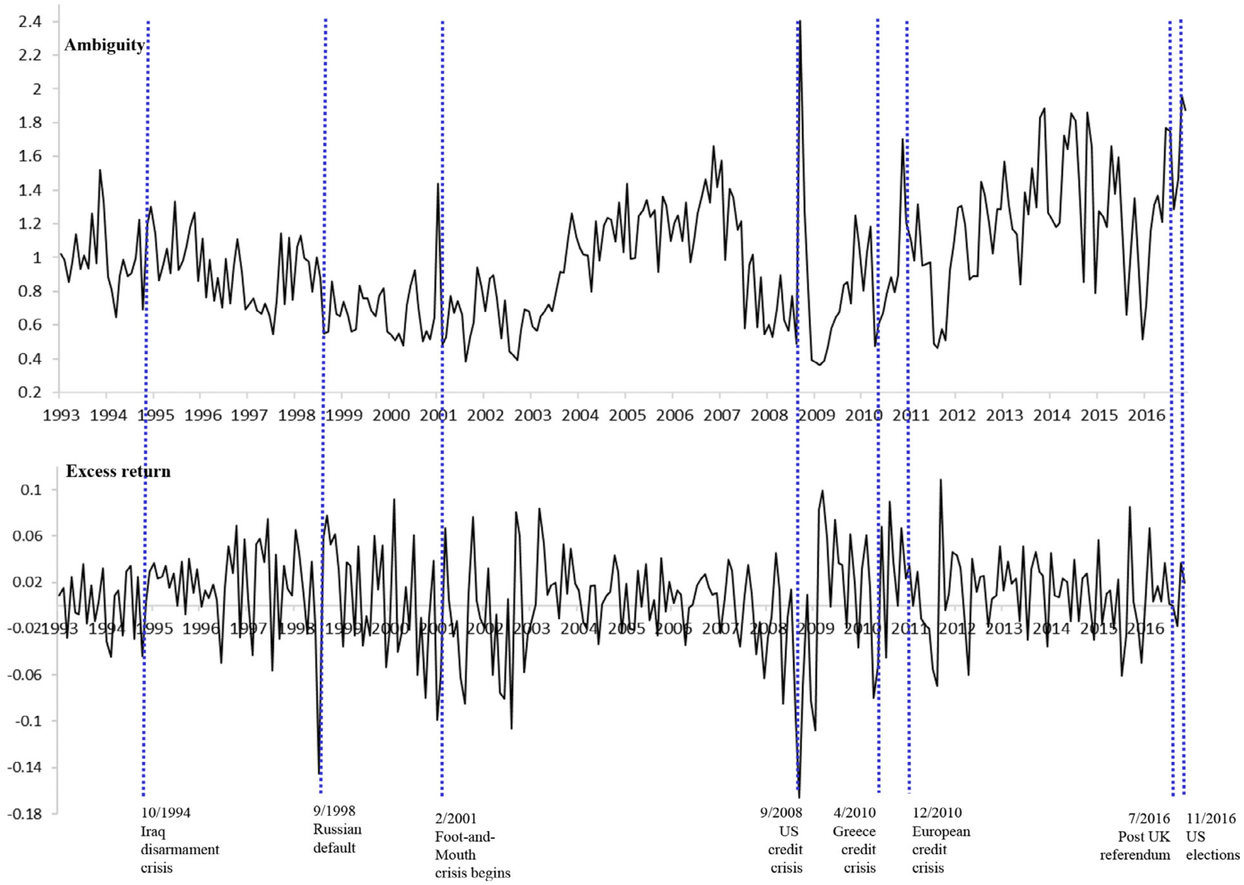

Fig. 2. Time series of market ambiguity and excess return.
The figure depicts the realized market ambiguity and excess return for the period between February 1993 and December 2016. The exchange-traded fund SPDR is the proxy of the market. The x-axis shows the timeline. The y-axis of the upper plot shows the monthly level of ambiguity $\mho$, measured by the standard deviation of the daily probabilities of returns over the month. Probabilities of returns are based on the daily mean and variance of returns on the SPDR, computed from five-minute returns, taken from the TAQ database and normalized to daily terms. The lower plot depicts the monthly, adjusted for dividends, excess return on the SPDR. The dotted vertical lines denote special events that had a significant impact on monthly excess returns.

size.[21] This scaling, which is analogous to Sheppard's correction, is tested to verify that it minimizes the effect of the bin size on the values of $\mho^2$. As with the expected probabilities, the variance of the probabilities is computed assuming that the daily ratios $\frac{\mu}{\sigma}$ are student's-*t* distributed. As in the risk literature, since expected future returns (and therefore expected probabilities) cannot be observed, realized returns must be used to construct realized probability distributions.

We next examine the time-series of ambiguity. The upper plot in Fig. 2 shows the time-series of the monthly degree of ambiguity $\mho$ for 1993–2016, and the lower plot shows the monthly excess returns on the market. We observe only a few months that contain big downward moves in the market. The obvious one is September–October 2008, during the recent financial crisis. Notice that crisis times, typified by negative or relatively low returns, are accompanied by relatively high levels of ambiguity. For example, in September 2008, the excess return was −16%, while ambiguity rose to 2.4, about 2.5 times the average ambiguity of 0.98 over the sample period. Another example is the "foot-and-mouth crisis" that began in February 2001 in the United Kingdom and was seemingly the reason for the ambiguity level jumping to 1.4 at that time.

Fig. 2 shows that during some years, there is a relatively high average level of ambiguity, accompanied by a relatively low rate of excess return (e.g., 2004–2007, 2014, and 2016). Fig. 1 shows that during these years the risk, measured by return volatility, was relatively low on average. As the expected rate of return compensates for both risk and ambiguity, in these years, the effect of lower risk on expected return dominates the effect of ambiguity. This raises the question of what might explain the relatively high ambiguity in periods of relatively "high" prices (low rates of return). A glimpse of this phenomenon, high prices accompanied by high ambiguity, can be observed in 2016, when important political and economic events occurred (e.g., the UK Brexit decision and the US elections). In this period, the "feeling" in the financial markets was of high

[21] We find that this scaling improves the pervious scaling $\frac{1}{w \ln \frac{1}{w}}$ for the bin size, used in Izhakian and Yermack (2017), in the sense that it reduces the sensitivity of the estimated $\mho^2$ to the bin size.

Table 1
Summary statistics.
Descriptive statistics are reported for the sample of returns between February 1993 and December 2016.
Panel A reports summary statistics of the daily parameters employed to compute probabilities of returns. N is the number of days in the sample. n denotes the five-minute returns in different five-minute time intervals. The mean return, μ, is the daily average five-minute SPDR (Ticker: SPY) return, in daily terms, where intraday returns are computed using prices taken from the TAQ database. σ is the daily standard deviation of five-minute returns, in daily terms. Probabilities of favorable returns, P, are based on the daily mean, μ, and variance, σ^2, of the return, assuming a normally distributed return. A return is considered favorable if it is greater than the risk-free rate.
Panel B reports summary statistics of the dependent and main independent variables. The monthly risk-free rate of return, r_f, is the one-month Treasury bill rate of return, taken from Ibbotson Associates. The market return, r, is the monthly return, adjusted for dividends, of the exchange-traded fund SPDR, taken from the CRSP database. The volatility, $\sqrt{\nu}$, is the standard deviation of the daily return, adjusted for dividends in monthly terms. The absolute deviation ϑ is the average absolute daily deviation of returns from the monthly average daily return. The mean probability, $\overline{\mathrm{P}}$, is the average daily probability of favorable returns over a month. Ambiguity, $\mho$, is the standard deviation of the daily probabilities of returns over the month.
Panel C reports the cross-correlations among the main estimates in the regression tests. Volatility is the variance of daily returns and ambiguity is the variance of daily probabilities, as used in the regression tests. p-values are in parentheses.

		N	Mean	Median	Min	Max	Std. dev.	Skewness	Kurtosis
Panel A: Daily descriptive statistics									
n	Num. of obs.	6025	70.783	78.000	3.000	78.000	16.974	−2.246	3.518
μ	Mean return	6025	0.000	0.000	−0.094	0.101	0.011	0.032	7.871
σ	Std. dev.	6025	0.008	0.007	0.001	0.067	0.005	2.940	16.094
μ/σ	Mean/Std	6025	0.073	0.057	−19.003	13.915	1.356	−0.234	16.062
P	Probability	6025	0.512	0.517	0.000	1.000	0.316	−0.037	−1.309
Panel B: Monthly descriptive statistics									
r_f	Risk-free rate	287	0.002	0.002	0.000	0.006	0.002	0.187	−1.564
r	Return	287	0.008	0.013	−0.165	0.109	0.041	−0.648	1.246
$r-r_f$	Excess return	287	0.006	0.010	−0.166	0.109	0.041	−0.641	1.250
$\sqrt{\nu}$	Volatility	287	0.049	0.043	0.007	0.273	0.030	2.847	15.091
VIX	VIX	257	0.059	0.054	0.030	0.175	0.024	1.692	4.405
ϑ	Avg. absolute dev.	287	0.043	0.037	0.012	0.222	0.026	2.791	13.154
$\overline{\mathrm{P}}$	Mean Prob.	287	0.512	0.507	0.358	0.691	0.064	0.204	−0.332
$\mho$	Ambiguity	287	0.986	0.958	0.361	2.405	0.353	0.622	0.367
Panel C: Cross-correlations									
			$r-r_f$	ν	ϑ	$\overline{\mathrm{P}}$	$\mho^2$		
		$r-r_f$	1						
		ν	−0.347	1					
			(< 0.0001)						
		ϑ	−0.382	0.831	1				
			(< 0.0001)	(< 0.0001)					
		$\overline{\mathrm{P}}$	0.648	−0.153	−0.275	1			
			(< 0.0001)	(0.0093)	(< 0.0001)				
		$\mho$	0.056	0.027	−0.327	0.244	1		
			(0.3423)	(0.6506)	(< 0.0001)	(< 0.0001)			

uncertainty, while risk measures like the VIX were very low. However, the measure $\mho^2$ indicates high levels of ambiguity, which can reflect the perception in the financial markets. In Section 4.3, we further investigate this pattern, observed for example in 2004–2007, when a sustained period of high prices is accompanied by relatively high ambiguity.

In other studies, different methodologies are employed to extract ambiguity from trading data. For example, for 2004–2007, Andreou et al. (2014) and So et al. (2016) find a relatively low degree of ambiguity.[22] One possible reason for this difference compared to our paper may be the nature of the measure used. The measures used in these papers are outcome related. The former measures ambiguity using the volume-weighted dispersion of option strike prices, and the latter measures implied ambiguity by the minimum absolute error between observed index options prices and the suggested model intrinsic values. We take a different approach with our ambiguity measure and suggest an outcome-independent measure that relies only on probabilities.

4.3. Descriptive statistics

The building blocks for measuring the degree of ambiguity in the market are the daily means and variances, computed from the five-minute returns. Panel A of Table 1 provides descriptive statistics of the intraday returns. The statistics are reported in daily terms. The average number of five-minute return observations is 70.78 per day. The probabilities of the favorable returns, P, are computed using the ratio $\frac{\mu}{\sigma}$, which ranges between −19 and 13.9, with an average of 0.073. The source of the variation in this ratio shows that it is driven by the variations in both μ and σ. Over the entire sample, the standard deviation of μ, in terms of the daily return, is 1.1%, while the standard deviation of σ is 0.5%. It is important to emphasize that the mean of the realized returns, measured over short intervals, is a poor proxy for the annual expected return (i.e., its standard error is very large). In our context,

[22] For 2006–2007, Driouchi et al. (2018)'s method also demonstrates a relatively low degree of ambiguity.

however, daily probabilities are extracted from the ratio of $\frac{\mu}{\sigma}$, whose distribution gives very little weight to extreme observations. Indeed, the estimated probability values appear to be very reasonable. On average, the probabilities of favorable returns in our sample is 0.51 as would be expected, and their standard deviation is about 0.32.

Panel B of Table 1 provides descriptive statistics of the variables. The dependent variable is the monthly return on the SPDR, r, which serves as a proxy for the return on the market portfolio, minus the risk-free rate, r_f, which is the one-month Treasury bill rate. The monthly return on the market, r, is computed using the opening price on the first trading day of the month and the closing price on the last trading day of the month and is adjusted for monthly dividends. On average, between 1993 and 2016, this return is 0.79% (about 9.56% annually). On average, the monthly risk-free rate is 0.21% (about 2.56% annually), and the monthly excess return, $r - r_f$, is 0.58% (about 6.99% annually). The distribution of $r - r_f$ is somewhat negatively skewed. The positive excess kurtosis, 1.2, indicates that the returns has tails.

Our measure of risk, denoted $\sqrt{\nu}$, is the monthly standard deviation, computed from daily returns and reported on a monthly basis. Across all 287 months, on average, the risk is about 5.0% (about 17.2% annually). The monthly average absolute deviation of returns from the average return $\mathbb{E}\Big[|r - \mathbb{E}[r]|\Big]$, denoted ϑ, is also computed from daily returns and reported on a monthly basis. On average, ϑ (across all 287 months) is about 4.3% (about 15.01% annually). Over the period between 1993 and 2016, the degree of ambiguity $\mho$ ranges between 0.361 and 2.4. The average ambiguity is 0.986. There are periods of relatively low ambiguity, like 1999–2000, and periods of relatively high ambiguity, like 2004–2007, as observed in Fig. 2. The phenomenon of relatively high ambiguity in periods of relatively "high" prices (low rates of return), as in 2004–2007, requires further investigation. A plausible explanation may be that in periods of sustained "high" prices investors‘ concern of a "correction" (i.e., a large price drop following a price run up) increases and manifests itself in relatively high ambiguity (the uncertainty of the probability) of a price drop. To test this conjecture empirically, we regress ambiguity against price run ups, measured by the number of months with a positive excess return in the preceding three months. We repeat this test for the preceding 4 to 12 months. In all these regression tests, ambiguity is significantly positively related to price run ups. For example, in the 3-month run-up regression, the t-statistics of the slope coefficient is 4.27, in the 6-month run-up regression the t-statistics of the slope coefficient is 5.28, and the 12-month run-up regression the t-statistics of the slope coefficient is 4.53.[23] As the focus of this paper is to study the effect of ambiguity on the equity premium, we leave the thorough exploration of the determinants of ambiguity to future research.

Panel C of Table 1 provides the cross-correlations among the main variables. For risk we use the variance of returns, rather than the standard deviation, and for ambiguity we use the variance of probabilities, rather than the standard deviation. As expected, the mean probability of favorable returns is positively and significantly correlated with the excess return (0.648). The excess return and the volatility are negatively and significantly correlated as observed in some studies (e.g., Nelson, 1991; Harvey, 2001). However, the lack of correlation between risk and ambiguity potentially provides independent evidence of their effect on excess return.

5. Empirical methodology and results

We turn now to develop the empirical methodology for testing our hypotheses. We first estimate the expected risk and ambiguity. Then, we design the empirical tests of the model in Eq. (9) and conduct the regressions analysis.

5.1. Estimating expected ambiguity and volatility

The fundamental hypothesis is that expected ambiguity, in addition to expected volatility (risk), is a determinant of the expected return. Based on the model in Eq. (9), we suggest in Hypotheses 1–4 that the effect of ambiguity on returns is subject to the investor's attitude toward ambiguity, which in turn is contingent on the expected probability of favorable returns. Thus, to conduct the empirical tests, the expectations of the following four variables need to be estimated: the volatility (ν), the average absolute deviation of returns from the expected return (ϑ), the probability of favorable returns (P), and the degree of ambiguity ($\mho^2$).

To select an appropriate estimation model, we first examine the autocorrelations in Table 2. The first-order autocorrelation of the volatility is large, the second-order is lower, while the decay beyond the fifth order is relatively slow. This behavior is indicative of a nonstationary integrated moving average (e.g., French et al., 1987). To address the issue of nonstationarity, we examine the changes in the natural logarithm of the standard deviation. These changes have a negative first-order autocorrelation, implying that the residuals have at least one lagged effect. The autocorrelation of the average absolute deviation, ϑ, is strongly positive for the first lag and decays consistently over the next eight lags; however, the changes in its natural logarithm are negative, but meaningful, only for the first-order autocorrelation. Examining the first-order autocorrelation of the changes in the natural logarithm of the probabilities of favorable returns shows that it is highly negative and significant only for the first-order autocorrelation.[24] Similarly, the ambiguity time series is nonstationary, while the autocorrelation of the changes in the natural logarithm of ambiguity is also highly negative and significant only for the first lag. We then conduct a Box-Pierce test of thes four variables, and in each case the null hypothesis that the observations are independently distributed is rejected.

[23] To ensure that our results are not driven by price run ups, in our main regression tests that follow, we control also for investors' sentiment.

[24] By definition, probabilities are bounded between zero and one. As expected, the negative autocorrelations of changes in their natural logarithm indicate that they follow a mean-reverting process; otherwise, the probabilities of this variable would deviate from the [0, 1] range.

Table 2
Autocorrelations.
Autocorrelations are reported for the monthly parameters employed in the empirical tests using monthly observations between February 1993 and December 2016, 287 months in total. The volatility, ν, is the variance of the daily returns of the SPDR, adjusted for dividends, in monthly terms. The absolute deviation ϑ is the average absolute daily deviation of returns from the monthly average daily return. The mean probability, $\overline{\mathrm{P}}$, is the average daily probability of favorable returns over a month. A return is considered favorable if it is greater than the risk-free rate, where returns are assumed to be normally distributed. Probabilities are based on the daily mean and variance of returns computed from the five-minute returns, taken from the TAQ database, given in daily terms. Ambiguity, $\mho^2$, is the variance of the daily probabilities of returns over the month. *p*-values are in parentheses.

		ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}
ν	Volatility	0.529	0.177	0.127	0.151	0.152	0.094	0.058	0.079	0.086	0.084
		(<0.0001)	(0.0027)	(0.0321)	(0.0112)	(0.0105)	(0.1157)	(0.3350)	(0.1883)	(0.1515)	(0.1642)
$\ln\frac{\nu_t}{\nu_{t-1}}$	Volatility change	−0.416	−0.023	−0.004	−0.070	−0.007	0.003	0.049	0.001	−0.027	0.073
		(<0.0001)	(0.7008)	(0.9483)	(0.2429)	(0.9087)	(0.9652)	(0.4113)	(0.9882)	(0.6534)	(0.2242)
ϑ	Absolute dev.	0.733	0.571	0.470	0.400	0.357	0.294	0.253	0.245	0.212	0.194
		(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(0.0004)	(0.0012)
$\ln\frac{\vartheta_t}{\vartheta_{t-1}}$	Absolute dev. change	−0.355	−0.040	−0.017	−0.059	0.032	−0.012	−0.047	0.021	0.072	−0.087
		(<0.0001)	(0.5050)	(0.7739)	(0.3201)	(0.5913)	(0.8412)	(0.4383)	(0.7275)	(0.2348)	(0.1475)
$\overline{\mathrm{P}}$	Mean prob.	0.097	0.131	0.116	0.004	0.146	0.131	0.061	0.146	0.196	0.037
		(0.1025)	(0.0276)	(0.0508)	(0.9444)	(0.0140)	(0.0281)	(0.3123)	(0.0146)	(0.0010)	(0.5377)
$\ln\frac{\overline{\mathrm{P}}_t}{\overline{\mathrm{P}}_{t-1}}$	Mean prob. change	−0.520	0.034	0.049	−0.148	0.095	0.031	−0.091	0.019	0.125	−0.129
		(<0.0001)	(0.5637)	(0.4107)	(0.0128)	(0.1108)	(0.6088)	(0.1314)	(0.7584)	(0.0369)	(0.0319)
$\mho^2$	Ambiguity	0.554	0.434	0.398	0.384	0.354	0.314	0.286	0.284	0.273	0.258
		(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)
$\ln\frac{\mho^2_t}{\mho^2_{t-1}}$	Ambiguity change	−0.331	−0.076	−0.014	−0.045	0.020	−0.007	−0.035	0.018	−0.066	0.021
		(<0.0001)	(0.2005)	(0.8125)	(0.4565)	(0.7377)	(0.9102)	(0.5563)	(0.7680)	(0.2740)	(0.7265)

The estimated autocorrelations in Table 2 suggest that an autoregressive moving average (ARMA) model is appropriate for estimating each of these variables: expected volatility, expected absolute deviation, expected probability, and expected ambiguity. Using the formal link between realized and conditional volatilities, as provided by Andersen et al. (2003), we estimate the expected volatilities by substituting the realized volatilities, ν, for the latent monthly volatilities. To do so, for each month, $\widehat{\ln\sqrt{\nu_t}}$ is computed using the coefficients estimated by the time-series ARMA(p, q) model

$$\ln\sqrt{\nu_t} = \psi_0 + \epsilon_t + \sum_{i=1}^{p}\psi_i\cdot\ln\sqrt{\nu_{t-i}} + \sum_{i=1}^{q}\theta_i\cdot\epsilon_{t-i}, \tag{10}$$

with the minimal corrected Akaike information criterion (AICC). This model uses the natural logarithm of volatility, $\ln\sqrt{\nu}$, to avoid negative expected volatility estimates and because $\sqrt{\nu}$ is skewed. The expected volatility is then calculated as

$$\nu^E_{t+1} = \mathbb{E}_t[\nu_{t+1}] = \exp\left(2\widehat{\ln\sqrt{\nu_t}} + 2\mathbb{V}\mathrm{ar}[u_t]\right),$$

where $\mathbb{V}\mathrm{ar}[u_t]$ is the minimal predicted variance of the error term. For every month t, using the data from the 30 preceding months, i.e., from month $t-30$ to month $t-1$, the regression in Eq. (10) is estimated for each $p = 1, \ldots, 10$ and $q = 1, \ldots, 10$; in total $p\times q = 100$ combinations.[25] The coefficients that attain the minimal AICC (i.e., the highest-quality model) are then used to estimate the expected volatility. Similarly, we estimate the expected absolute deviation, ϑ, using its monthly realized values, to obtain ϑ^E_t.

Expected ambiguity is also estimated with ARMA(p, q), using a method similar to the one used to estimate the expected volatilities.[26] Using realized ambiguity, the parameter $\widehat{\ln\mho_t}$ is estimated using the coefficients of the time-series model

$$\ln\mho_t = \psi_0 + \epsilon_t + \sum_{i=1}^{p}\psi_i\cdot\ln\mho_{t-i} + \sum_{i=1}^{q}\theta_i\cdot\epsilon_{t-i} \tag{11}$$

that attains the minimal AICC out of the $p\times q = 100$ combinations of coefficients obtained from this regression. The expected ambiguity is then calculated as

$$\left(\mho^2_{t+1}\right)^E = \mathbb{E}_t\left[\mho^2_{t+1}\right] = \exp\left(2\widehat{\ln\mho_t} + 2\mathbb{V}\mathrm{ar}[u_t]\right).$$

To estimate the expected probability of unfavorable returns, the parameter $\widehat{\ln Q_t}$, where $Q_t = \frac{\overline{\mathrm{P}}_t}{1-\overline{\mathrm{P}}_t}$, is estimated using the coefficients of

$$\ln Q_t = \psi_0 + \epsilon_t + \sum_{i=1}^{p}\psi_i\cdot\ln Q_{t-i} + \sum_{i=1}^{q}\theta_i\cdot\epsilon_{t-i} \tag{12}$$

that attain the minimal AICC out of the $p\times q = 100$ combinations of coefficients of this regression. In this model, we use the natural logarithm of the transformed probability, $\ln Q = \ln\left(\frac{\overline{\mathrm{P}}}{1-\overline{\mathrm{P}}}\right)$, to avoid estimated probabilities that are negative or are greater than one. The expected probability

[25] We also estimate the expected values using an extended window of all preceding month observations; the results are essentially the same.

[26] Andersen et al. (2003) provide the theoretical framework for integrating high-frequency intraday data into the measurement of daily volatility. They show that long-memory Gaussian vector autoregression for the logarithmic daily realized volatilities performs admirably. We apply the same approach to ambiguity, since we use probabilities that are estimated from intraday data.

Table 3
Expected values.
Expected values are reported for the monthly observations between July 1995 and December 2016, 257 months in total.
Panel A reports summary statistics of the estimated expected values of the following factors: the expected volatility, ν^E, the expected absolute deviation, ϑ^E, the expected probability of favorable returns, $\overline{\mathrm{P}}^E$, and the expected ambiguity, $\mho^E$. For each month t, the expected values are estimated based only on their realized values from month $t-30$ to month $t-1$ and using the ARMA(p, q) model with the minimal AICC out of the $n \times m = 100$ combinations of the coefficients. The volatility, ν, is the variance of the daily returns of the SPDR, adjusted for dividends, in monthly terms. The absolute deviation ϑ is the average absolute daily deviation of returns from the monthly average daily return. The mean probability, $\overline{\mathrm{P}}$, is the average daily probability of favorable returns over a month. A return is considered favorable if it is greater than the risk-free rate, where returns are assumed to be normally distributed. Probabilities are based on the daily mean and variance of returns computed from the five-minute returns, taken from the TAQ database. Ambiguity, $\mho^2$, is the variance of the daily probabilities of returns over the month.
Panel B reports the cross-correlations of the estimated expected values. These serve as the main independent variables in the empirical pricing tests. p-values are in parentheses.

Panel A: Descriptive statistics of forecasted variables

	N	Mean	Median	Min	Max	Std. dev.	Skewness	Kurtosis
$\sqrt{\nu^E}$	257	0.047	0.044	0.018	0.221	0.020	3.399	23.130
ϑ^E	257	0.042	0.036	0.020	0.201	0.020	3.418	19.693
VIX	257	0.062	0.059	0.030	0.175	0.024	1.665	4.366
$\overline{\mathrm{P}}^E$	257	0.514	0.513	0.453	0.572	0.026	−0.078	−0.865
$\mho^E$	257	0.948	0.936	0.466	1.623	0.262	0.309	−0.795

Panel B: Cross-correlations of expected values of the variables

	ν^E	ϑ^E	VIX	$\overline{\mathrm{P}}^E$	$(\mho^2)^E$
ν^E	1				
ϑ^E	0.850 (< 0.0001)	1			
VIX	0.783 (< 0.0001)	0.908 (< 0.0001)	1		
$\overline{\mathrm{P}}^E$	−0.305 (< 0.0001)	−0.458 (< 0.0001)	−0.322 (< 0.0001)	1	
$(\mho^2)^E$	−0.314 (< 0.0001)	−0.537 (< 0.0001)	−0.451 (< 0.0001)	0.570 (< 0.0001)	1

is then calculated as

$$\mathrm{P}^E_{t+1} = \mathbb{E}_t[\mathrm{P}_{t+1}] = \frac{\exp\left(\widehat{\ln Q_t} + \frac{1}{2}\mathbb{V}\mathrm{ar}[u_t]\right)}{1+\exp\left(\widehat{\ln Q_t} + \frac{1}{2}\mathbb{V}\mathrm{ar}[u_t]\right)}.$$

Note that the estimates of expected volatility, expected absolute deviation, expected probability, and expected ambiguity are all out-of-sample estimates.

Panel A of Table 3 reports descriptive statistics of the estimated expectation of volatility, absolute deviation, the probability of favorable returns, and ambiguity. Each expected variable is the out-of-sample fitted value from the relevant ARMA model described above. Comparing the statistics of their realized values in Table 1 to their expectations, we find that the dispersion of the expected values is less than the dispersion of thier realized values. The differences between the minimum and maximum values, as well as the variance, the skewness, and the kurtosis of the estimated expected values, are all lower than those of the realized values. The lower dispersion implies that the time series of the expected values of all four variables—volatility, absolute deviation, probability, and ambiguity—are smoother than the related realized time series. Panel B of Table 3 reports the cross-correlations among the expected values. The results show that the expected volatility is significantly correlated with the VIX, which is also highly correlated with the expected absolute deviation. Although expected volatility and expected ambiguity are negatively correlated, the value −0.314 is relatively low; it can affect the significance of both variables when used in the regression tests of our model.

5.2. Empirical design

We next design the empirical tests of the risk-ambiguity-return relation, using the estimated expected volatility, expected absolute deviation, expected probability, and expected ambiguity. In particular, the attitudes toward ambiguity, given by

$$\mathbb{E}\Big[\eta(1-\mathrm{E}[\mathrm{P}(r)])\Big] = \mathbb{E}\left[\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(r)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(r)])}\right] = \int \mathrm{E}[\varphi(r)]\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(r)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(r)])}dr, \quad (13)$$

determines the ambiguity–return relation in Eq. (9). Ideally, we would like to compute this expression directly from market data. However, the functional form of $\Upsilon(\cdot)$ is unknown, so we are constrained in extracting it from the data. Therefore, to elicit the functional form of $\eta(\cdot)$, we consider only two subsets of returns: unfavorable and favorable. Accordingly, the empirical effect of investors' attitudes toward ambiguity on the ambiguity-return relation is determined by

$$\mathbb{E}\Big[\eta(1-\mathrm{E}[\mathrm{P}(r)])\Big] = \eta\left(\mathrm{P}^E\right), \quad (14)$$

where P^E is the expected probability of favorable returns.[27]

To elicit the asymmetric attitudes toward ambiguity, which can be nonlinearly contingent on expected probabilities, as proposed by in Eq. (9), we use the following design. The expected probabilities of favorable returns range from 0.453 to 0.572 (see Table 3). Winsorizing the very few outlier values provides the range [0.46,0.56] of expected probabilities. This range is divided into ten equal intervals (bins) of 0.01 each, indexed by i. For example, $i = 1$ denotes the probability bin [0.46,0.47]. The few values lower than 0.46 are indexed as $i = 1$, while the few values higher than 0.56 are indexed as $i = 10$. The decision to use a ten-bin resolution and not a higher one is dictated by the number of observations, 257 (of expected values). To have meaningful statistical results, a minimum number of observations is required for each bin; a smaller number of bins will not allow us to clearly identify the shape of $\Upsilon(\cdot)$.

Next, a dummy variable, D, is constructed for each probability bin. If the expected probability of favorable returns in a given month t falls into bin i, the dummy variable $D_{i,\,t}$ is assigned the value one; otherwise it is zero. The empirical model is then given by

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \sum_{i=2}^{10} \eta_i \cdot \left(D_{i,t} \times \mathrm{P}_i^E \times \left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \epsilon_t, \tag{15}$$

where P_i^E is the midpoint of probability bin i. It should be noted that, unlike the attitude toward risk that is constant, the attitude toward ambiguity can vary with the expected probability of favorable returns. The model in Eq. (15) is a discrete model in which attitudes toward ambiguity only have a finite number of values. We also examine a continuous version of the model, given by

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \eta_s \cdot \left(\mathrm{P}_t^E \times \left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \epsilon_t. \tag{16}$$

In this model, we assume that ambiguity attitudes are linearly contingent on the expected probability of favorable returns P_t^E. Since all the estimates of the explanatory variables are out of sample, the first 30 months are dropped and we use only the data that start at August 1995 in the regressions.

To interpret the discrete model, Eq. (15), we can write the coefficient of ambiguity attitude as $\eta\left(\mathrm{P}_i^E\right) = \hat{\eta} + \hat{\eta}_i$. This expression represents the investor's attitude toward ambiguity, conditional on the expected probability of favorable returns, P^E, being in probability bin i. A negative value of $\eta\left(\mathrm{P}_i^E\right)$, determined by $\hat{\eta} + \hat{\eta}_i$, implies ambiguity-loving behavior and results in a negative ambiguity premium; a positive value implies ambiguity-averse behavior and results in a positive ambiguity premium. Furthermore, a greater $|\hat{\eta}_i|$ in the range of low probabilities of favorable returns implies an increasing love for ambiguity; a higher $\hat{\eta}_i$ in the range of high probabilities of favorable returns implies an increasing aversion to ambiguity.

5.3. Findings

We first examine the model using ordinary least squares (OLS). The dependent variable in all the regression tests is the monthly excess return on the SPDR. We use the Newey-West estimator to deal with potential autocorrelation and heteroskedasticity in the error terms.[28] Panel A of Table 4 reports the coefficients of the regressions that test the discrete model in Eq (15).

In the first regression, we examine the risk-return relation excluding ambiguity. In this univariate regression, the only explanatory variable is the expected volatility, a proxy for risk, which has a negative coefficient, a result consistent with previous studies (e.g., Nelson, 1991; Harvey, 2001). We introduce expected ambiguity in the subsequent regressions. We first use the unconditional expected ambiguity as a sole explanatory variable. Though the coefficient is positive, as hypothesized, it is insignificant. We next use the conditional attitude toward ambiguity terms, specified in Eq. (15), without the risk term. The results in Panel A of Table 4 show that all ambiguity coefficients, except for $\hat{\eta}_5$, are significant at the 5% level. In our fourth regression, we incorporate the risk factor, expected volatility, as a main explanatory variable in conjunction with the contingent ambiguity terms. We find that the coefficient of expected volatility is positive and highly significant. Thus, we cannot reject Hypothesis 1 that risk does have a positive effect on the equity premium when it is introduced in conjunction with ambiguity. To verify that our estimates are not biased, we conduct Amihud and Hurvich (2004) test for biases in the estimated coefficients.[29,30] In this test, none of the estimated slope coefficients in the residuals regression is found to be significant, indicating that our estimates in the regression test in Eq. (15) are not biased.

Panel B of Table 4 provides a summary of the estimates of attitudes toward ambiguity. It also shows the level of aversion to (love for) ambiguity, contingent on the expected probability of favorable returns, computed for each probability bin i by $\eta\left(\mathrm{P}_i^E\right) = \hat{\eta} + \hat{\eta}_i$. Fig. 3 depicts the probabilistically contingent coefficients of ambiguity attitudes in the fourth regression.

To interpret these findings, recall that η_1 is associated with probabilities of favorable returns (gains) in the range [0.46,0.47], η_2 with probabilities in the range [0.47,0.48], etc. The evidence is that the coefficient of attitude toward ambiguity associated with probabilities of gains greater than 0.49 (i.e., probabilities of losses lower than 0.51) is positive. On the other hand, this coefficient is negative for probabilities of gains lower than 0.49. The trend line

[27] Note that, since every $\mathrm{P} \in \mathcal{P}$ is additive, the expected probability of favorable returns satisfies $\mathrm{P}^E = 1 - \mathrm{E}[\mathrm{P}(r)]$, where $\mathrm{P}(r)$ is a cumulative probability of unfavorable returns.

[28] We also test our models using (White, 1980) standard errors and without standard errors correction, and the results (t-stats) are qualitatively the same.

[29] We thank Yakov Amihud for suggesting this test.

[30] In particular, we regress the residuals obtained from the model in Eq. (15) against the residuals obtained from the following regression tests: $\nu_t^E = \alpha + \beta \cdot \nu_{t-1}^E + \epsilon_t$ and $\left(\mho_t^2\right)^E \times \vartheta_t^E = \alpha + \beta \cdot \left(\mho_{t-1}^2\right)^E \times \vartheta_{t-1}^E + \epsilon_t$.

Table 4
Main OLS regression tests.
The table reports the results of the tests of the theoretical discrete model. The estimated expected values are for the period between August 1995 and December 2016, 257 monthly observations in total.
Panel A reports the results obtained using OLS regressions defined by

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \textstyle\sum_{i=2}^{10} \eta_i \cdot \left(D_{i,t} \times \mathrm{P}_i^E \times \left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \epsilon_t .$$

All results utilize the Newey-West estimator. The estimated expected value of each variable at time t is the out-of-sample fitted value of an ARMA model over its realized values from month $t-30$ to month $t-1$. All expectations are estimated using the out of sample ARMA(p, q) model with the minimal AICC. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends and given in monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2\right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$, is the variance of the daily probabilities of returns over the month. Probabilities of returns are based on the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database given in daily terms. The expected probability of favorable returns, P^E, is estimated from the monthly averages of the daily probabilities of favorable returns. A return is considered favorable if it is greater than the risk-free rate, where returns are assumed to be normally distributed. The dummy variable D_i is assigned the value one if the expected probability of favorable returns, P^E, in that month falls into the range i of probabilities, and zero otherwise. $\hat{\theta}$ is the coefficient of the univariate regression of expected ambiguity.
Panel B reports the estimated coefficients of ambiguity attitude, calculated as $\eta\left(\mathrm{P}_i^E\right) = \hat{\eta} + \hat{\eta}_i$ using the estimated coefficients of the regression tests in Panel A. t-statistics are in parentheses.

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\eta}_2$	$\hat{\eta}_3$	$\hat{\eta}_4$	$\hat{\eta}_5$	$\hat{\eta}_6$	$\hat{\eta}_7$	$\hat{\eta}_8$	$\hat{\eta}_9$	$\hat{\eta}_{10}$	N	R^2	Adj. R^2
Panel A: Regression results																
1	0.006	−0.098												257	0.000	−0.004
	(2.360)	(−0.160)														
2	0.005		0.001											257	0.000	−0.004
	(0.690)		(0.160)													
3	0.012			−1.255	1.676	1.788	2.310	1.451	2.902	2.373	2.697	2.256	2.725	257	0.067	0.029
	(1.130)			(−3.310)	(2.170)	(2.690)	(2.700)	(1.800)	(4.130)	(3.520)	(3.760)	(3.350)	(3.840)			
4	0.005	3.400		−1.446	1.762	1.176	2.542	1.853	3.325	2.897	3.159	2.806	3.275	257	0.095	0.055
	(0.420)	(3.580)		(−3.970)	(2.290)	(1.670)	(2.790)	(2.200)	(4.640)	(4.050)	(4.240)	(3.990)	(4.460)			
Panel B: Coefficients of ambiguity attitude																
P^E				−0.460	−0.470-	−0.480	−0.490	−0.500	−0.510	−0.520	−0.530	−0.540	−0.550			
				0.470	0.480	0.490	0.500	0.510	0.520	0.530	0.540	0.550	0.560			
3				−1.255	0.421	0.533	1.055	0.196	1.647	1.118	1.442	1.001	1.470			
4				−1.446	0.316	−0.270	1.096	0.407	1.879	1.451	1.713	1.360	1.829			

in Fig. 3 shows that ambiguity-averse behavior is demonstrated for probabilities of favorable returns greater than 0.485, while ambiguity-loving behavior is demonstrated for probabilities of favorable returns lower than 0.485. These findings provide support for Hypotheses 2 and 3 and are consistent with the findings of other behavioral studies (e.g.,Mangelsdorff and Weber, 1994; Abdellaoui et al., 2005; Du and Budescu, 2005). However, unlike these studies, in our model, investors' attitudes toward ambiguity are contingent on the probability of returns, rather than on the returns themselves.

The confidence intervals in Fig. 3 imply that the coefficients of ambiguity aversion associated with adjacent probability ranges are not necessarily significantly different from each other. One of the reasons is the small number of observations available for each probability range. In total, we have 257 months for which the data to compute expected ambiguity were available. Thus, on average, we have 25.7 observations associated with each probability interval (there are ten intervals). This small number of observations generates high standard errors and wide confidence intervals. Still, testing the null hypothesis that two coefficients of ambiguity aversion associated with different probability ranges are equal reveals that in many cases the coefficients are indeed significantly different. For example, pairwise hypothesis tests (not reported) show that each coefficient, η_2, η_3, and η_5, is significantly different from η_6, η_7, η_8, η_9, and η_{10}. Thus, in general, the ambiguity aversion coefficients associated with low probability ranges are significantly different from those associated with high probability ranges.

The decrease in the coefficient of ambiguity attitude from its highest value of 1.829 to its lowest value of −1.446 indicates that aversion to ambiguity decreases with the expected probability of unfavorable returns and turns to love for ambiguity when this probability exceeds 0.51. This implies that investors' attitudes toward ambiguity are not of the constant absolute class.[31] There is no conclusive evidence in the behavioral literature on whether investors' attitudes toward ambiguity are of the constant relative class or the constant absolute class. One of our goals is to narrow this gap by providing some evidence about the shape of ambiguity attitudes. To test whether investors' ambiguity attitudes are of the former class, using the same regression format as in Eq. (15), we replace the independent variables representing expected ambiguity with $\frac{\left(\mho_t^2\right)^E}{\mathrm{P}_t^E}$. That is, expected ambiguity normalized by the expected probability of favorable returns. The coefficients (not reported) show the same pattern as in the regressions with the nonnormalized ambiguity, implying that investors' ambiguity attitudes are not of the constant relative

[31] Constant absolute ambiguity attitude means that, given a return, when the range of its possible probabilities is shifted linearly, the ambiguity attitude remains unchanged. Constant relative ambiguity attitude means that ambiguity attitude is not sensitive to a positive probability scaling Izhakian (2017).

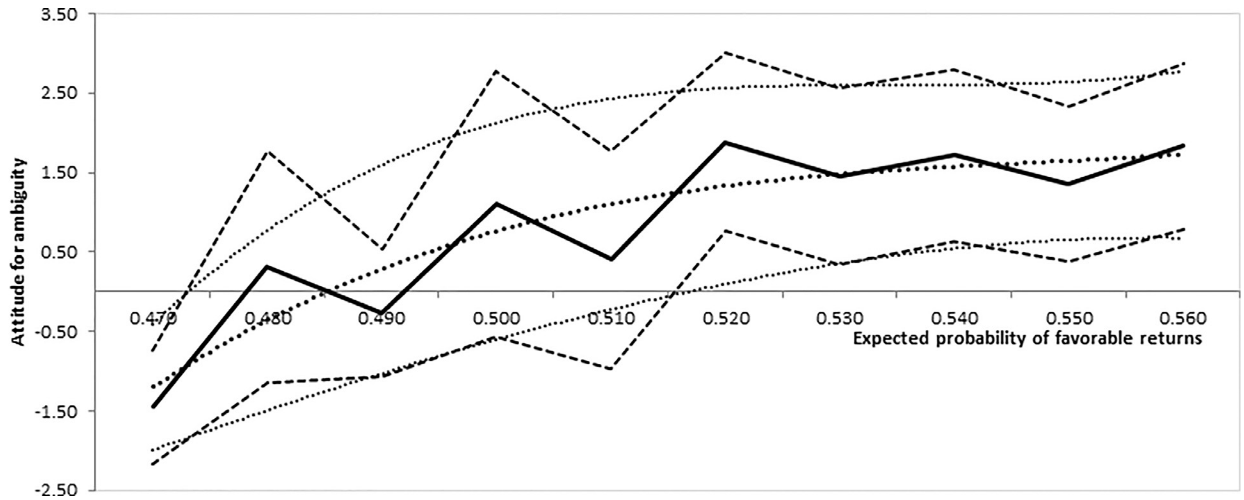

Fig. 3. Ambiguity attitudes.
The figure depicts ambiguity attitudes contingent upon the expected probability of favorable returns. The x-axis shows the expected probability of favorable returns. The y-axis shows the coefficient of the ambiguity attitude, η. The solid broken line depicts the ambiguity attitude, and the smooth line is created by a polynomial of third degree. The dotted broken line depicts the 5% confidence interval of ambiguity attitudes. The monthly probabilities of favorable are based on the daily mean and variance of returns on the SPDR, computed from five-minute returns, taken from the TAQ database, given in daily terms. A return is considered favorable if it is greater than the risk-free rate, where returns are assumed to be normally distributed.

class either. The conclusion is, consistent with Hypothesis 4, that investors' relative ambiguity aversion increases in the expected probability of favorable returns, and investors' relative ambiguity loving increases in the probability of unfavorable returns.

Hypothesis 4 and the related findings imply an inverse S-shape in the perceived probabilities. These findings coincide with et al. (Abdellaoui et al., 2011), who attribute this shape to different sources of uncertainty, which translate the subjective probabilities into the willingness to bet. They show that these probabilities depend "not only on the person but also on the source of uncertainty" (page 720). More importantly, our findings support the likelihood insensitivity proposed by Abdellaoui et al. (2011) (page 701). Likelihood insensitivity means "a lack of sensitivity to intermediate changes in likelihood, so that all intermediate likelihoods are moved in the direction of 50-50," suggesting that "decisions will not be influenced much by the updating of probabilities after receipt of new information." Aligned with the likelihood insensitivity phenomenon, our findings indicate that, for the intermediate range of probabilities, investors do not exhibit a clear sensitivity to ambiguity.

In summary, we find that ambiguity explains at least some of the variation in the equity premium; however, the combined effect of risk and ambiguity is more powerful. As can be observed in Panel A of Table 4, the effect of expected volatility on expected return in the univariate OLS regressions is insignificantly negative, aligned with French et al. (1987). However, when ambiguity is introduced alongside volatility, the latter becomes positive and significant.

The degree of relative risk aversion can be elicited, for example, using the values of the coefficients in Table 4. The value, $2 \times \hat{\gamma} = 6.80$, is in the upper part of the range of values reported in many behavioral studies (e.g., Chetty, 2006; Booij et al., 2009) and empirical asset pricing studies (e.g., Brown and Gibbons, 1985; French et al., 1987; Bliss and Panigirtzoglou, 2004).

We use the Newey-West, and alternatively the White corrections, in the OLS regressions. We repeat our tests using weighted least squares (WLS). In the WLS regressions, the weights are inversely proportional to the sum of the estimated risk and the estimated ambiguity.[32] We report the findings in Table 5. Panel A of Table 5 shows that the effect of expected volatility, by itself, is positive but insignificant. The effect of expected ambiguity is significant in all but one of the ten probability bins, as in the OLS regressions. When expected ambiguity is added to expected volatility (risk), the effect of expected volatility becomes positive and significant, while the effect of expected ambiguity remains almost the same as in the previous regression. The findings of the WLS regressions are not qualitatively different form our results in Table 4 and provide support for Hypotheses 1–4.

Although our findings are in line with previous behavioral findings, our main concern is that they can be affected by the division of the probability space into ten bins. Therefore, we examine a continuous (with respect to ambiguity attitude) version of the model in Eq. (16). Although the pattern of the coefficient of ambiguity attitude, $\eta(P^E)$ in Table 4, in the discrete model does not indicate a clear linear relation between ambiguity attitudes and the expected probabilities of favorable returns, this relation can be viewed as roughly positively monotone.

As in our previous tests, we test the model using both OLS and WLS regressions. Panel A of Table 6 presents the

[32] French et al. (1987) use one divided by the monthly standard deviation of the daily returns, $\frac{1}{\sqrt{\nu_t^E}}$, as weights in the WLS regression tests. Similarly, here we use $\frac{1}{\sqrt{\nu_t^E+\mho_t^E}}$ as weights.

Table 5
Main WLS regression tests.
The table reports the results of the tests of the theoretical discrete model. The estimated expected values are for the period between August 1995 and December 2016, 257 monthly observations in total.
Panel A reports the results obtained using WLS regressions defined by

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \textstyle\sum_{i=2}^{10} \eta_i \cdot \left(D_{i,t} \times \mathrm{P}_i^E \times \left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \epsilon_t,$$

where the weights are inversely proportional to the sum of the estimated risk and the estimated ambiguity. All results utilize the Newey-West estimator. The estimated expected value of each variable at time t is the fitted value of an ARMA model over its realized values from month $t-30$ to month $t-1$. All expectations are estimated using the out of sample ARMA(p, q) model with the minimal AICC. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends, in monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2 \right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$ is the variance of the daily probabilities of returns over the month. Probabilities of returns are based on the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database, given in daily terms. The expected probability of favorable returns, P^E, is estimated from the monthly averages of the daily probabilities of favorable returns. A return is considered favorable if it is greater than the risk-free rate, where returns are assumed to be normally distributed. The dummy variable D_i is assigned the value one if the expected probability of favorable returns, P^E, in that month falls into the range i of probabilities, and zero otherwise. $\hat{\theta}$ is the coefficient of the univariate regression of expected ambiguity.
Panel B reports the estimated coefficients of ambiguity attitude, calculated as $\eta\left(\mathrm{P}_i^E\right) = \hat{\eta} + \hat{\eta}_i$ using the estimated coefficients of the regression tests in Panel A.
t-statistics are in parentheses.

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\eta}_2$	$\hat{\eta}_3$	$\hat{\eta}_4$	$\hat{\eta}_5$	$\hat{\eta}_6$	$\hat{\eta}_7$	$\hat{\eta}_8$	$\hat{\eta}_9$	$\hat{\eta}_{10}$	N	R^2	Adj. R^2
Panel A: Regression results																
1	0.005	0.459												257	0.001	−0.003
	(1.450)	(0.490)														
2	0.006		0.000											257	0.000	−0.004
	(0.700)		(0.020)													
3	0.011			−1.244	1.785	1.772	2.422	1.301	2.993	2.446	2.746	2.260	2.745	257	0.065	0.027
	(0.990)			(−2.930)	(2.050)	(2.310)	(2.650)	(1.460)	(3.890)	(3.220)	(3.470)	(3.090)	(3.530)			
4	0.002	4.114		−1.480	1.817	1.043	2.630	1.763	3.475	3.070	3.282	2.920	3.400	257	0.102	0.062
	(0.220)	(3.770)		(−3.720)	(2.080)	(1.330)	(2.740)	(1.890)	(4.520)	(3.850)	(4.030)	(3.840)	(4.290)			
Panel B: Coefficients of ambiguity attitude																
P^E				−0.460	−0.470	−0.480	−0.490	−0.500	−0.510	−0.520	−0.530	−0.540	−0.550			
				0.470	0.480	0.490	0.500	0.510	0.520	0.530	0.540	0.550	0.560			
3				−1.244	0.541	0.528	1.178	0.057	1.749	1.202	1.502	1.016	1.501			
4				−1.480	0.337	−0.437	1.150	0.282	1.994	1.590	1.802	1.440	1.920			

OLS results, and Panel B presents the WLS results. These results are similar to those of the discrete model with 10 probability bins (Tables 4 and 5). Expected volatility is positive and significant when expected ambiguity is included. The coefficients of expected ambiguity in both panels are significant and indicate an increasing aversion to ambiguity in the expected probability of favorable returns. The coefficient of expected volatility, however, is somewhat lower (2.44 in the OLS regression, and 3.20 in the WLS regression), bringing the values more into the acceptable range of risk aversion coefficients reported in previous studies. In Panel B, when expected volatility is included, investors exhibit aversion to ambiguity when the expected probability of favorable returns $\mathrm{P}^E > \frac{6.25}{11.31} = 0.55$. As expected, the explanatory power of the tests is smaller than the discrete tests. This model is less precise because we impose linear (in the expected probability of favorable returns) attitudes toward ambiguity. Similar to the discrete model, to verify that our estimates are not biased, we conduct Amihud and Hurvich (2004) test for biases in the estimated coefficients.[33] In this test, none of the estimated slope coefficients in the residuals regression is found to be significant, indicating that our estimates in the regression test in Eq. (16) are not biased.

6. Robustness tests

Our findings provide support for the theoretical model and the related hypotheses. In this section, we examine whether the findings can be due to other, unaccounted for, risk factors or to our methodology.

6.1. VIX as expected volatility

In the previous tests, we estimated expected volatility using historical observations. An alternative approach is to use the forward-looking volatility implied by options on the S&P 500 index, given by the VIX. One of the main reasons that we rerun our tests using the VIX is because sometimes it is used as a proxy for ambiguity (e.g., Williams, 2015). Thus, we want to rule out the possibility that our ambiguity measure $\mho^2$ captures the same aspects of uncertainty as the VIX. We use the VIX value, at the beginning of month t, as a measure of the expected volatility for that month. We then estimate the following model

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot VIX_{t+1} + \eta \cdot \left(\left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \eta_s \cdot \left(\mathrm{P}_t^E \times \left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \epsilon_t. \tag{17}$$

[33] In particular, we regress the residuals obtained from the model in Eq. (16) against the residuals obtained from the following regression tests: $\nu_t^E = \alpha + \beta \cdot \nu_{t-1}^E + \epsilon_t, \left(\mho_t^2 \right)^E \times \vartheta_t^E = \alpha + \beta \cdot \left(\mho_{t-1}^2 \right)^E \times \vartheta_{t-1}^E + \epsilon_t$, and $\mathrm{P}_t^E \times \left(\mho_t^2 \right)^E \times \vartheta_t^E = \mathrm{P}_{t-1}^E \times \left(\mho_{t-1}^2 \right)^E \times \vartheta_{t-1}^E + \epsilon_t$.

Table 6
Continuous ambiguity attitudes.
The table reports the results from testing the theoretical continuous model. The tests cover the period between August 1995 and December 2016, 257 monthly observations in total.
Panel A reports the results obtained using OLS regressions defined by

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \eta_s \cdot \left(\mathrm{P}_t^E \times \left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \epsilon_t .$$

All results utilize the Newey-West estimator. The estimated expected value of each variable at time t is the fitted value of an ARMA model over its realized values from month $t-30$ to month $t-1$. All expectations are estimated using the out-of-sample ARMA(p, q) model with the minimal AICC. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends, in monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2\right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$, is the variance of the daily probabilities of returns over the month. Probabilities of returns are based on the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database, given in daily terms. The expected probability of favorable returns, P^E, is estimated from the monthly averages of the daily probabilities of favorable returns. A return is considered favorable if it is greater than the risk-free rate, where returns are assumed to be normally distributed. $\hat{\theta}$ is the coefficient of the univariate regression of expected ambiguity.
Panel B reports the results form testing the above model using WLS regressions.
t-statistics are in parentheses.

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\eta}_s$	N	R^2	Adj. R^2
Panel A: OLS								
1	0.006	−0.098				257	0.000	−0.004
	(2.370)	(−0.160)						
2	0.005		0.001			257	0.000	-0.004
	(0.700)		(0.170)					
3	0.012			−2.656	4.786	257	0.024	0.016
	(2.010)			(-2.640)	(2.300)			
4	0.013	2.448		−5.070	9.089	257	0.045	0.033
	(2.210)	(2.970)		(−3.550)	(3.360)			
Panel B: WLS								
1	0.005	0.459				257	0.001	−0.003
	(1.450)	(0.490)						
2	0.006		0.000			257	0.000	−0.004
	(0.710)		(0.020)					
3	0.012			−3.085	5.639	257	0.021	0.013
	(1.730)			(−2.600)	(2.290)			
4	0.011	3.207		−6.250	11.316	257	0.052	0.041
	(1.770)	(3.310)		(−4.070)	(3.880)			

We report the results in Table 7. In both OLS and WLS tests, the main findings regarding the effect of risk (expected volatility) and expected ambiguity are maintained, although they are weaker. Expected volatility, proxied by the VIX, has a positive effect on excess returns, but it is insignificant, unlike our previous measure of risk. This could be due to the relatively low volatility of the VIX, 2.4%, during the sample period. Interestingly, the coefficients of expected volatility using VIX are significantly smaller than those using realized volatility (Table 4). These coefficients (1.22 and 1.83) imply degrees of risk aversion ($2 \times \hat{\gamma} = 2.44$ and 3.66) that are more in line with behavioral-experimental findings. Thus, the VIX can be considered a better proxy of expected volatility than other proxies based on past data. The findings regarding ambiguity are very similar to our earlier findings for Eq. (16) in Table 6. In particular, the coefficients of ambiguity are of the same magnitude and show the same pattern—increasing in the expected probabilities of favorable returns.[34] These findings alleviate our concern that our previous results could have been due to a latent relation between the estimates of expected volatility and expected ambiguity.

6.2. Is ambiguity a proxy for other factors?

We next investigate whether our measure is a proxy for other known factors that can affect returns (e.g., skewness, kurtosis, volatility of volatility, etc.). We first examine the correlations between expected volatility, expected ambiguity, and the expectations of other moments of the empirical distribution. We start by considering the third and fourth moments (i.e., excess skewness and excess kurtosis) computed from the daily returns of the SPDR and adjusted for dividends, as additional factors.[35] Their expected values are estimated using the same methodology as used for all

[34] We also test the discrete model of Eq. (15), where VIX_t is taken to be the expected volatility (not reported). Again, the coefficients of ambiguity are of the same magnitude and show the same pattern as in the discrete model using estimated expected volatility.

[35] We also test the model using skewness and kurtosis, computed from intraday data, taking the expected values of their monthly averages as additional factors.

Table 7
VIX regression tests.
The table reports the results from testing the theoretical continuous model. The tests cover the period between August 1995 and December 2016, 257 monthly observations in total.
Panel A reports the results obtained using OLS regressions defined by

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot VIX_{t+1} + \eta \cdot \left(\left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \eta_s \cdot \left(\mathrm{P}_t^E \times \left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \epsilon_t .$$

All results utilize the Newey-West estimator. VIX_{t+1} is the value of VIX at the beginning of month $t+1$. The estimated expected value of each variable at time t is the fitted value of an ARMA model over its realized values from month $t-30$ to month $t-1$. All expectations are estimated using the out-of-sample ARMA(p, q) model with the minimal AICC. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2 \right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$ is the variance of the daily probabilities of returns over the month. Probabilities of returns are based on the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database, given in daily terms. The expected probability of favorable returns, P^E, is estimated from the monthly averages of the daily probabilities of favorable returns. A return is considered favorable if it is greater than the risk-free rate, where returns are assumed to be normally distributed. $\hat{\theta}$ is the coefficient of the univariate regression of expected ambiguity.
Panel B reports the results from testing the above model using WLS regressions.
t-statistics are in parentheses.

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\eta}_s$	N	R^2	Adj. R^2
Panel A: OLS								
1	0.007	−0.167				257	0.000	−0.004
	(1.630)	(−0.140)						
2	0.005		0.001			257	0.000	−0.004
	(0.700)		(0.170)					
3	0.012			−2.656	4.786	257	0.024	0.016
	(2.010)			(−2.640)	(2.300)			
4	0.006	1.224		−5.040	9.516	257	0.040	0.027
	(1.060)	(0.980)		(−3.380)	(3.230)			
Panel B: WLS								
1	0.004	0.424				257	0.001	−0.003
	(0.670)	(0.300)						
2	0.005		0.001			257	0.000	−0.004
	(0.530)		(0.160)					
3	0.010			−4.024	7.645	257	0.026	0.017
	(1.350)			(−2.730)	(2.480)			
4	0.004	1.834		−5.992	11.351	257	0.044	0.032
	(0.580)	(1.310)		(−3.400)	(3.260)			

other variables. As functions of returns, skewness and kurtosis change with the magnitude of the return, while ambiguity is outcome independent (and thus risk independent) as it is exclusively a function of probabilities. As shown in Panel A of Table 8, the correlation of expected skewness with expected ambiguity is insignificant. The correlation of expected kurtosis with expected ambiguity is significant, although its value 0.2 is rather low. Also, the correlations of expected skewness and expected kurtosis with expected volatility are insignificant.

We next introduce into the continuous model of Eq. (16) the other factors (e.g., expected skewness and expected kurtosis) that can affect the expected returns, alongside expected ambiguity. The same tests are repeated for the discrete model (not reported) using Eq. (15), where for both models we run both OLS and WLS regressions. The regression results in Panel B show that expected skewness and expected kurtosis have no effect on expected returns. Their coefficients are negligible and insignificant. Furthermore, introducing these factors does not affect the significance of expected ambiguity. These findings strengthen our claim that ambiguity is not derived by sampling the same (skewed or leptokurtic) probability distribution over the month.

Another concern regarding our methodology for estimating ambiguity is that the measure $\mho^2$ can be an outcome of time-varying risk or volatility innovation (e.g., Brandt and Kang, 2004). To address this concern, we test our discrete (not reported) and continuous models by adding expected volatility of the volatility (*VolVol*) into the regression. *VolVol* is the variance of the daily variances, computed from intraday data, and its expectation is estimated similarly to those of the other factors. Panel A in Table 8 shows that expected ambiguity and expected *VolVol* are not correlated (−0.11). Although expected *VolVol* is highly correlated with expected volatility (0.92), the inclusion of expected *VolVol* in the regressions shows that its effect is insignificant and has a negligible effect on the coefficients of expected ambiguity.[36] Therefore, we can rule

[36] It does affect the significance of expected volatility, since it is highly correlated with it (both are outcome dependent).

Table 8
Robustness tests.
The table reports the findings of the robustness tests of the theoretical model. The tests cover the period between August 1995 and December 2016, 257 monthly observations in total.
Panel A reports the cross-correlations among the expected values of the uncertainty factors. The estimated expected value of each variable at time t is the fitted value of an ARMA model over its realized values in the months from month $t-30$ to month $t-1$. All expectations are estimated using the out-of-sample ARMA(p, q) model with the minimal AICC. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends, in monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2\right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$ is the variance of the daily probabilities of returns over the month. Probabilities of favorable returns are based on the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database given in daily terms. The expected probability of favorable returns, P^E, is estimated from monthly averages of daily probabilities of favorable returns. A return is considered favorable if it is greater than the risk-free rate, where returns are assumed to be normally distributed. The expected skewness $Skew^E$ is estimated from the realized skewness of daily returns over the month. The expected kurtosis $Kurt^E$ is estimated from the realized kurtosis of daily returns over the month. The expected volatility of the mean $VolM^E$ is estimated from the volatility of the mean, computed as the variance of daily mean returns, which are computed from five-minute returns given in daily terms. The expected volatility of volatility $VolV^E$ is estimated from the volatility of volatility, computed as the variance of daily returns, which are computed from five-minute returns given in daily terms. p-values are in parentheses.
Panel B reports the results from testing the OLS regressions defined by

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E\right) + \eta_s \cdot \left(\mathrm{P}_t^E \times \left(\mho_t^2\right)^E \times \vartheta_t^E\right) + \beta_1 \cdot Skew_t^E + \beta_2 \cdot Kurt_t^E + \beta_3 \cdot VolM_t^E + \beta_4 \cdot VolV_t^E + \epsilon_t.$$

All results utilize the Newey-West estimator. t-values are in parentheses.
Panel C reports the results form testing the WLS regressions. t-values are in parentheses

Panel A: Cross-correlations

	$r-r_f$	ν^E	ϑ^E	P^E	$\left(\mho^2\right)^E$	$Skew^E$	$Kurt^E$	$VolM^E$	$VolV^E$
$r-r_f$	1								
ν^E	−0.103	1							
	(0.0978)								
ϑ^E	−0.139	0.850	1						
	(0.0257)	(< 0.0001)							
P^E	0.217	−0.305	−0.458	1					
	(0.0005)	(< 0.0001)	(< 0.0001)						
$\left(\mho^2\right)^E$	0.021	−0.314	−0.537	0.570	1				
	(0.7425)	(< 0.0001)	(< 0.0001)	(< 0.0001)					
$Skew^E$	0.053	0.006	0.025	−0.114	−0.062	1			
	(0.3926)	(0.9188)	(0.6897)	(0.0687)	(0.3224)				
$Kurt^E$	0.060	0.009	−0.026	0.378	0.212	−0.065	1		
	(0.3380)	(0.8870)	(0.6820)	(< 0.0001)	(0.0006)	(0.2975)			
$VolM^E$	−0.134	0.872	0.861	−0.327	−0.324	0.011	0.060	1	
	(0.0318)	(< 0.0001)	(< 0.0001)	(< 0.0001)	(< 0.0001)	(0.8665)	(0.3339)		
$VolV^E$	−0.135	0.922	0.741	−0.195	−0.119	−0.020	0.058	0.873	1
	(.0299)	(< 0.0001)	(< .0001)	(0.0016)	(.0558)	(0.7470)	(0.3541)	(< 0.0001)	

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\eta}$	$\hat{\eta}_s$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	N	R^2	Adj. R^2
Panel B: OLS regressions											
1	0.012	2.461	−4.987	8.901	−0.022				257	0.053	0.038
	(2.190)	(3.110)	(−3.530)	(3.320)	(−1.300)						
2	0.012	2.423	−4.954	8.851		0.001			257	0.045	0.030
	(2.210)	(2.990)	(−3.210)	(2.990)		(0.260)					
3	0.013	2.797	−4.840	8.673			−17.117		257	0.045	0.030
	(2.250)	(2.180)	(−3.260)	(3.100)			(−0.320)				
4	0.014	1.940	−5.162	9.206				128890.8	257	0.045	0.030
	(1.840)	(0.750)	(−3.750)	(3.500)				(0.250)			
Panel C: WLS regressions											
1	0.011	3.187	−5.987	10.762	−0.030				257	0.065	0.050
	(1.760)	(3.510)	(−4.070)	(3.840)	(−1.600)						
2	0.011	3.181	−6.125	11.059		0.001			257	0.052	0.037
	(1.760)	(3.420)	(−3.730)	(3.480)		(0.210)					
3	0.011	2.976	−6.397	11.583			11.155		257	0.052	0.037
	(1.740)	(2.410)	(−3.740)	(3.610)			(0.200)				
4	0.013	2.690	−6.385	11.499				143229.1	257	0.052	0.037
	(1.420)	(1.010)	(−4.160)	(3.950)				(0.240)			

out the possibility that the results are driven by the volatility of the volatility.

Since the volatility of the mean (*VolMean*), the variance of the daily means, has been used as a proxy for ambiguity in some studies (e.g., Cao et al., 2005; Garlappi et al., 2007), we examine the possibility that ambiguity is derived from expected *VolMean*. As can be seen in Panel A of Table 8, expected ambiguity and expected *VolMean* are negatively correlated, −0.32, but still too low to be considered a substitute for expected ambiguity. The inclusion of expected *VolMean* in the regressions shows that its effect on the coefficients of expected ambiguity and on their significance is negligible.

6.3. Additional tests

To address the concern that all probabilities within a month are derived from the same distribution, we conduct the following simulation. Assuming that monthly returns are determined by a single probability distribution, we compute the mean and variance using all intraday returns during the month. We then randomly form 22 groups of 78 return observations each. For each group, we compute the mean and the variance. Based on these 22 mean-variance pairs, we compute the expected ambiguity for each month and rerun the regression in Eq. (15). We repeat this procedure 1000 times, and generally the simulations do not present the pattern of ambiguity attitudes obtained in our previous regressions. Furthermore, in most cases the results are not statistically significant.

Next, we use a nonparametric test to examine whether all daily return distributions over a month are identical or are the same as the monthly return distribution. For each month, we conduct two-sample Kolmogorov-Smirnov tests among all pairs of 20–22 daily probability distributions, where the null hypothesis is that, in every pair, the two distributions are identical. We conduct 61,485 tests, and in 58,640 of these tests, the null hypothesis is rejected at the 5% level. Second, for each month, we conduct a one-sample Kolmogorov-Smirnov test between each daily probability distribution and a reference probability distribution defined by all intraday observations during the month. We conduct 6082 tests, and in 5851 of these tests, the null hypothesis is rejected at the 5% level. These findings rule out the possibility that there is a single unique prior that investors follow, rather than a set of priors, as suggested by the multiple prior ambiguity approach.

Additional robustness tests are described next. As seen in Table 1, returns are negatively skewed, which can be caused by negative shocks. To account for this skewness, we relax the assumption that intraday returns are normally distributed, and execute tests assuming that the returns obey an elliptical distribution with parameters estimated from the data.[37] Elliptical distributions are uniquely determined by the mean and variance and can be skewed and at the same time platykurtic or leptokurtic. Thus, each distribution accounts for the third and fourth moments of the distribution (i.e., skewness and kurtosis). The results are qualitatively the same.

The skewness of the return distribution can affect expected returns through two channels: risk and ambiguity. The former is reflected in more "volatile" return distributions, i.e., higher outcome-dependent (risk-dependent) measures. The latter is reflected in a more "volatile" set of priors, i.e., a higher outcome-independent (risk-independent) ambiguity measure, $\mho^2$. To investigate which effect dominates, we extract the daily (skewed) return distributions using a skewed elliptical distribution and use them to compute expected ambiguity. We then include this expected ambiguity in the regressions alongside expected skewness, computed from daily returns. Skewness is found to be insignificant. Next, we include this expected ambiguity in the regressions alongside expected average skewness, estimated from the monthly average of daily skewness computed from intraday returns. Again, expected skewness is insignificant. These findings indicate that the effect of skewness through the ambiguity channel dominates its effect through the risk channel and can explain spikes in the ambiguity premium when unusually negative returns occur.

Next, we examine our model when downside risk is included as an additional factor (e.g., Ang et al., 2006).[38] We conduct these tests with different cutoffs for downside risk, and the results are qualitatively the same. In addition, we test for the effect of investors' sentiment (Baker and Wurgler, 2006) alongside ambiguity, and again the results are qualitatively the same. Another concern is that our results are driven by our proxy for the market return—the ETF SPDR. We thus test our model using another proxy for the market portfolio—a value-weighted portfolio of all the stocks quoted in the TAQ database. The results are qualitatively the same as those obtained using the SPDR.

Finally, a question can be raised regarding the methodology for estimating ambiguity, whether $\mho^2$ captures variations in a singleton information set (a single probability distribution) across trading days over a month, generated only by the new information that might be obtained overnight. To address this concern, we run all the tests omitting all trading transactions that occur during the first half-hour of the trading day. Berry and Howe (1994) show that the flow of public information to financial markets peaks between 4:30 and 5:00 p.m., after the market has closed. This implies that the most significant impact of the information flow (on bid-ask spread, volatility, and trading volume) occurs in the first half-hour of the following trading day. In addition, "price discovery" (price formation) happens mainly in this part of the trading day (e.g., Lockwood and Linn, 1990; Heston et al., 2010; Pagano et al., 2013). The results of the tests excluding these transactions are essentially the same, ruling out the possibility that our findings are due only to changes in a single information set.

To rule out the possibility that our ambiguity measure $\mho^2$ only captures the changes in a single probability distribution caused by news clustered on Mondays and Fridays, we run all the tests omitting these two days. Chang et al. (1998) and Steeley (2001) show that the main effect of macroeconomic news occurs on these days (e.g., the unemployment rate and nonfarm payroll are announced on Fridays). Dyl and Maberly (1988), Schatzberg and Datta (1992), and Damodaran (1989) show that firm-specific an-

[37] Particular forms of the elliptical distribution include the normal distribution, student-*t* distribution, logistic distribution, exponential power distribution, and Laplace distribution (e.g., Owen and Rabinovitch, 1983).

[38] Our measure of ambiguity is different than downside risk. While downside risk is outcome dependent (risk dependent), measured by the variance of returns that are lower than a given threshold, ambiguity is risk independent, measured by the variance of probabilities, independent of returns.

nouncements are typically clustered on weekends such that their main effect is on Mondays. As before, our results are essentially the same.

Overall, we could not find any evidence that our ambiguity measure is a proxy for some other known factors.

7. Alternative models

In this section, we consider alternative models of decision-making under risk and ambiguity, as well as alternative risk attitudes.

$$DRE = \max_{\Phi_a,\Phi_b\in\mathcal{P}} \left(\begin{array}{l} \Phi(r_0;\mu_a,\sigma_a)\ln\dfrac{\Phi(r_0;\mu_a,\sigma_a)}{\Phi(r_0;\mu_b,\sigma_b)} \\ +\displaystyle\sum_{i=1}^{60}\big(\Phi(r_i;\mu_a,\sigma_a)-\Phi(r_{i-1};\mu_a,\sigma_a)\big)\ln\dfrac{\Phi(r_i;\mu_a,\sigma_a)-\Phi(r_{i-1};\mu_a,\sigma_a)}{\Phi(r_i;\mu_b,\sigma_b)-\Phi(r_{i-1};\mu_b,\sigma_b)} \\ +\big(1-\Phi(r_{60};\mu_a,\sigma_a)\big)\ln\dfrac{1-\Phi(r_{60};\mu_a,\sigma_a)}{1-\Phi(r_{60};\mu_b,\sigma_b)} \end{array} \right).$$

$$DDS = \max_{\Phi_a,\Phi_b\in\mathcal{P}} \left(\begin{array}{l} \big|\Phi(r_0;\mu_a,\sigma_a)-\Phi(r_0;\mu_b,\sigma_b)\big| \\ +\displaystyle\sum_{i=1}^{60}\big|(\Phi(r_i;\mu_a,\sigma_a)-\Phi(r_{i-1};\mu_a,\sigma_a))-(\Phi(r_i;\mu_b,\sigma_b)-\Phi(r_{i-1};\mu_b,\sigma_b))\big| \\ +\big|(1-\Phi(r_{60};\mu_a,\sigma_a))-(1-\Phi(r_{60};\mu_b,\sigma_b))\big| \end{array} \right).$$

7.1. Max-Min expected utility with multiple priors

The point of departure of this paper is a functional representation of preferences concerning risk and ambiguity. Note that different classes of preferences representation may not provide the same decision rule and may provide different orderings by ambiguity, resulting in different pricing schemes. In the EUUP, model we employ the set of priors captures only beliefs (information), in isolation from tastes for ambiguity and risk. This approach is different than the MEU (Gilboa and Schmeidler, 1989) approach in which the set of priors captures both beliefs and tastes. Thus, although the "width" of the set of priors in MEU and the variance of probabilities in EUUP are strongly related, in MEU the "width" of the set cannot be attributed exclusively to ambiguity, independent of aversion to ambiguity. Therefore, MEU does not allow for the elicitation of aversion to ambiguity, as it is not distinguishable from beliefs. In addition, since MEU does not support love for ambiguity, identifying the conditions where investors can act as ambiguity lovers is not possible in MEU settings.

In the MEU framework, decisions are made based only on the worst prior within the set of priors and disregard the width of the set of priors. An extension of the MEU model, the α-MEU (Ghirardato et al., 1998), accounts for the width of the prior set by considering the worst and the best priors. Yet, the information between these two priors is disregarded. Nevertheless, we next explore whether the width of the set of priors is priced. In such a framework, attitudes toward ambiguity are independent of expected probabilities, since to compute expected probabilities a second-order prior has to be defined (as in EUUP).

We estimate the width of the set of priors similarly to the estimation conducted for the ambiguity measure, as detailed in Section 4.2. We examine three alternative estimates of the width:

1. The maximum relative entropy (RE) among all the pairs within the set of priors, computed for each month by the discrete form[39]
2. The maximum Kolmogorov-Smirnov (KS) statistic among all the pairs within the set of priors, computed for each month by the discrete form[40]
$$DKS = \max_{\Phi_a,\Phi_b\in\mathcal{P}}\left[\max_{i\in 0,\ldots,60}\big|\Phi(r_i;\mu_a,\sigma_a)-\Phi(r_i;\mu_b,\sigma_b)\big|\right].$$
3. The maximum aggregate distance (DS) among all the pairs within the set of priors, computed for each month by the discrete statistic form

For each of these divergence indices, the expected value is estimated using the same methodology as is used for all other variables. The out-of-sample expected value of each divergence index is then added to the regression that includes the expected risk and the ambiguity measures. Recall that under the MEU settings, only aversion to ambiguity is supported, and aversion to ambiguity is not a function of expected probabilities as in the EUUP model. Accordingly, the expected value of each of the divergence indices is added to the regression without an interaction term with expected probabilities. The findings are reported in Table 9.

Panel A of Table 9 reports the cross-correlations between the expected measure of ambiguity, $\mho^2$, and the expected value of each of the three divergence indices, *DER*, *DKS*, and *DDS*. The correlation between $\mho^2$ and each of these divergence indices is weak and insignificant. Panel B

[39] Relative entropy (Kullback-Leibler divergence) measures the distance of one prior from a reference prior. This divergence index, $D_{KL}(\mathrm{P}\,|\,\mathrm{Q}) \equiv \sum_i \mathrm{P}(x_i)\ln\frac{\mathrm{P}(x_i)}{\mathrm{Q}(x_i)}$, is limited to only two probability distributions and is asymmetric. For this reason, we look at all the possible distribution pairs and pick the maximum as a proxy for the width of the set.

[40] The KS statistic is defined by $D_{KS}(\mathrm{P},\mathrm{Q}) \equiv \max_i |\mathrm{P}(x_i) - \mathrm{Q}(x_i)|$.

Table 9
Max-Min expected utility with multiple priors.
The table reports the findings of the tests of the alternative theoretical model. The tests cover the period between August 1995 and December 2016, 257 monthly observations in total.
Panel A reports the cross-correlations among the expected values of the uncertainty factors.The estimated expected value of each variable at time t is the fitted value of an ARMA model over its realized values in the months from month $t-30$ to month $t-1$. All expectations are estimated using the out-of-sample ARMA(p, q) model with the minimal AICC. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends, in monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2\right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$ is the variance of the daily probabilities of returns over the month. Probabilities of favorable returns are based on the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database given in daily terms. The expected probability of favorable returns, P^E, is estimated from monthly averages of daily probabilities of favorable returns. A return is considered favorable if it is greater than the risk-free rate, where returns are assumed to be normally distributed. The expected maximum relative entropy DRE^E is estimated from the realized maximum relative entropy of daily return distributions over the month. The expected maximum Kolmogorov-Smirnov statistic DKS^E is estimated from the realized maximum Kolmogorov-Smirnov statistic of daily return distributions over the month. The expected maximum distribution distance DRE^E is estimated from the realized maximum distribution distance of daily return distributions over the month. p-values are in parentheses.
Panel B reports the results from testing the OLS regressions defined by

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot ((\mho_t^2)^E \times \vartheta_t^E) + \eta_s \cdot (\mathrm{P}_t^E \times (\mho_t^2)^E \times \vartheta_t^E) + +\beta_1 \cdot DRE_t^E + \beta_2 \cdot DKS_t^E + \beta_3 \cdot DDS_t^E + \epsilon_t.$$

All results utilize the Newey-West estimator. t-values are in parentheses.
Panel C reports the results from using the WLS regressions. t-values are in parentheses.

Panel A: Cross-correlations

	$r-r_f$	ν^E	ϑ^E	P^E	$\left(\mho^2\right)^E$	DRE^E	DKS^E	DDS^E
$r-r_f$	1							
ν^E	−0.103	1						
	(.0978)							
ϑ^E	−0.139	0.850	1					
	(0.0257)	(< 0.0001)						
P^E	0.217	−0.305	−0.458	1				
	(0.0005)	(< 0.0001)	(< 0.0001)					
$\left(\mho^2\right)^E$	0.021	−0.314	−0.537	0.570	1			
	(0.7425)	(< 0.0001)	(< 0.0001)	(< 0.0001)				
DRE^E	0.060	−0.124	−0.192	0.156	0.042	1		
	(0.3379)	(0.0458)	(0.0019)	(0.0118)	(0.4971)			
DKS^E	−0.038	0.109	0.081	−0.230	−0.112	0.312	1	
	(0.5411)	(0.0792)	(0.1927)	(0.0002)	(0.0729)	(< 0.0001)		
DDS^E	−0.027	0.061	0.020	−0.206	−0.103	0.385	0.959	1
	(0.6615)	(0.3307)	(0.7436)	(0.0009)	(0.0984)	(< 0.0001)	(< 0.0001)	

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\eta}$	$\hat{\eta}_s$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	N	R^2	Adj. R^2
Panel B: OLS regressions										
1	0.003	0.651			0.000			258	−0.003	−0.011
	(0.870)	(0.740)			(1.680)					
2	0.048	0.411				−0.042		258	−0.005	−0.013
	(0.380)	(0.530)				(−0.320)				
3	0.007	0.426					0.000	258	−0.006	-0.014
	(0.060)	(0.540)					(0.000)			
4	0.008	2.530	−5.075	9.111	0.000			258	0.048	0.033
	(1.070)	(3.070)	(−3.520)	(3.360)	(1.760)					
5	−0.030	2.457	−5.187	9.316		0.044		258	0.045	0.030
	(−0.220)	(2.980)	(−3.410)	(3.220)		(0.310)				
6	−0.091	2.492	−5.350	9.646			0.054	258	0.046	0.031
	(−0.670)	(3.070)	(−3.580)	(3.390)			(0.760)			
Panel C: WLS regressions										
1	0.004	1.299			0.000			257	−0.009	−0.017
	(1.170)	(1.050)			(1.180)					
2	0.093	1.052				−0.091		257	−0.007	−0.015
	(0.590)	(0.950)				(−0.550)				
3	0.030	1.073					−0.012	257	−0.009	-0.017
	(0.200)	(0.950)					(−0.150)			
4	0.009	3.276	−6.216	11.251	0.000			257	0.053	0.038
	(1.170)	(3.380)	(−4.050)	(3.870)	(1.180)					
5	−0.027	3.212	−6.340	11.494		0.039		257	0.052	0.037
	(−0.160)	(3.340)	(−3.900)	(3.720)		(0.230)				
6	−0.098	3.238	−6.492	11.799			0.057	257	0.053	0.038
	(−0.600)	(3.430)	(−4.100)	(3.910)			(0.670)			

reports the OLS results, and Panel C reports the WLS results. In all the regressions, the effect of each of the three divergence indices is insignificant. These findings do not indicate that investors act as if they follow MEU or the α-MEU decision criteria.

Our findings show that expected probabilities can play an important role in determining the ambiguity premium. These findings can be interpreted more broadly with respect to other models of decision-making under ambiguity in multiple priors paradigm besides the EUUP model.[41] For example, although in the MEU framework expected probabilities are not formally defined, since a second-order belief has to be defined, yet expected probabilities can affect pricing. A "shift" in expected probabilities over time may be a result of a "shift" of the set of priors, independent of the decision theoretical model employed. Therefore, econometrically, changes in expected probabilities can be used as a statistical measure for changes in the set of priors, establishing a connection between the multiple prior approach (MEU framework) and the statistic measure used econometrically.

7.2. Subjective expected utility

The EUUP framework can be viewed as an extension of the subjective expected utility (SEU) framework (Savage, 1954) by applying a two-stage SEU: first to probabilities and then to outcomes. In the special case of an ambiguity-neutral attitude, investors reduce this two-stage "lottery" linearly, ending up with the classical SEU representation. In the pricing model in Eq. (1), we decompose the equity premium into the risk and ambiguity premium. The former is a function of the variance of outcomes computed using the expected probabilities, which is equivalent to the volatility of outcomes (risk) computed using a two-stage lottery. The latter is a function of the volatility of probabilities (lottery of probability distributions). Through the lens of SEU, when testing the effect of the expected variance of outcomes alongside the expected variance of the probabilities, the effect of the former should be significantly positive and the effect of the latter not significantly different from zero.

Our results (Tables 4–7) show that the lottery of probability distributions—ambiguity—significantly affects pricing. This means that the hypothesis that the effect of the volatility of the probabilities is not a pricing factor is rejected, implying that SEU behavior is not supported by the data. Our findings coincide with Halevy (2007), who finds that, while facing two-stage objective lotteries, individuals may not reduce compound (objective) lotteries and exhibit ambiguity-averse behavior.[42]

7.3. Unstructured risk attitudes

In this section, we relax our assumption of CRRA and repeat the analysis. Similar to our discrete model in Eq. (15), no particular functional structure is imposed on attitudes toward risk, allowing for the intensity of the effect of risk attitude to be determined endogenously. We examine whether unstructured attitudes toward risk can explain our findings.

In EUUP, formed in Eq. (5), attitudes toward ambiguity are captured by the outlook function, Υ, applied to expected probabilities, while attitudes toward risk are captured by the utility function, U, applied to outcomes (wealth). In a representative investor economy, the wealth of the investor can be normalized relative to the initial wealth. Thus, we can take the aggregate return, w_t, relative to the beginning of our sample as an estimate of the investor's wealth at time t.[43]

We test a discrete model in which ambiguity attitudes may be nonlinearly contingent upon wealth. Analogous to the model in Eq. (15), we use the following design. The (relative) wealth range is from 0.0 to 1.659 (not reported). Winsorizing the few outlier values provides the range [0.1,1.6] of wealth. This range is divided into ten equal intervals (bins) of 0.15 each, indexed by j. For example, $j = 1$ denotes the wealth bin [0.1,0.25]. The few values lower than 0.1 are indexed as $j = 1$, and the few values higher than 1.6 are indexed as $j = 10$. The decision to use a ten-bin resolution is due to the number of observations, 257 (of expected values).

Next, a dummy variable, C, is constructed for each wealth bin. If the wealth in a given month t falls into bin j, the dummy variable $C_{j,\,t}$ is assigned the value one; otherwise it is zero. The model is then given by

$$
\begin{aligned}
r_{t+1} - r_{f,t+1} &= \alpha + \gamma \cdot \nu_t^E + \sum_{j=2}^{10} \gamma_j \cdot \left(C_{j,t} \times w_j \times \nu_t^E \right) + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E \right) \\
&\quad + \sum_{i=2}^{10} \eta_i \cdot \left(D_{i,t} \times \mathrm{P}_i^E \times \left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \epsilon_t, \qquad (18)
\end{aligned}
$$

where w_j is the midpoint of wealth bin j. Note that risk attitudes can vary with the wealth. Thus, this is a discrete model in the sense that risk attitudes may obtain only a finite number of values.

To interpret the discrete model, we can write the coefficients of risk attitude as $\gamma\left(w_j\right) = \hat{\gamma} + \hat{\gamma}_j$. This expression represents the investor's attitude toward risk, conditional upon the wealth w being in wealth bin j. A negative value of $\gamma(w)$, determined by $\hat{\gamma} + \hat{\gamma}_j$, implies risk-loving behavior and results in a negative risk premium. On the other hand, a positive value implies risk-averse behavior and results in a positive risk premium. Furthermore, a greater $\hat{\gamma}_j$ in the range of high wealth implies an increasing aversion to risk. On the other hand, a lower $\hat{\gamma}_j$ in the range of high wealth implies a decreasing aversion to risk.

The results of these regression tests are reported in Table 10. With respect to attitude toward risk, we couldn't identify a consistent pattern, and the coefficients are mostly insignificant. With respect to attitudes toward ambiguity, as with CRRA, the same pattern is sustained, where all coefficients except for one are statistically significant.

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[41] We thank the anonymous referee for this interpretation.

[42] Segal's 1987 theory supports these findings.

[43] The wealth in the first observation serves as a numeraire. Thus, w_t is taken to be log aggregate return for the period 0 to t.

Table 10
Unstructured risk attitude.
The table reports the results of the tests of the theoretical discrete model. The estimated expected values are for the period between August 1995 and December 2016, 257 monthly observations in total.
Panel A reports the results from testing the OLS regressions defined by

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \sum_{j=2}^{10} \gamma_j \cdot \left(C_{j,t} \times w_j \times \nu_t^E\right) + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E\right) + \sum_{i=2}^{10} \eta_i \cdot \left(D_{i,t} \times \mathrm{P}_i^E \times \left(\mho_t^2\right)^E \times \vartheta_t^E\right) + \epsilon_t,$$

All results utilize the Newey-West estimator. The estimated expected value of each variable at time t is the out-of-sample fitted value of an ARMA model over its realized values from month $t-30$ to month $t-1$. All expectations are estimated using the out of sample ARMA(p, q) model with the minimal AICC. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends and given in monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2\right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$ is the variance of the daily probabilities of returns over the month. Probabilities of returns are based on the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database given in daily terms. The expected probability of favorable returns, P^E, is estimated from the monthly averages of the daily probabilities of favorable returns. A return is considered favorable if it is greater than the risk-free rate, where returns are assumed to be normally distributed. The dummy variable C_j is assigned the value one if the wealth w in that month falls in the range j of wealth, and zero otherwise. The dummy variable D_i is assigned the value one if the expected probability of favorable returns, P^E, in that month falls in the range i of probabilities, and zero otherwise.
Panel B reports the results from testing the above model using WLS regressions. t-statistics are in parentheses.

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\gamma}_2$ / $\hat{\eta}$	$\hat{\gamma}_3$ / $\hat{\eta}_2$	$\hat{\gamma}_4$ / $\hat{\eta}_3$	$\hat{\gamma}_5$ / $\hat{\eta}_4$	$\hat{\gamma}_6$ / $\hat{\eta}_5$	$\hat{\gamma}_7$ / $\hat{\eta}_6$	$\hat{\gamma}_8$ / $\hat{\eta}_7$	$\hat{\gamma}_9$ / $\hat{\eta}_8$	$\hat{\gamma}_{10}$ / $\hat{\eta}_9$	$\hat{\eta}_{10}$	N	R^2	Adj. R^2
Panel A: OLS															
1	0.006	4.299	32.318	15.657	0.152	−5.548	−4.415	-4.664	−7.998	0.130	−1.443		257	0.056	0.018
	(1.090)	(0.500)	(1.640)	(1.070)	(0.010)	(−0.550)	(−0.550)	(−0.680)	(−1.280)	(0.020)	(−0.340)				
2	0.012		−1.255	1.676	1.788	2.310	1.451	2.902	2.373	2.697	2.256	2.725	257	0.067	0.029
	(1.130)		(−3.310)	(2.170)	(2.690)	(2.700)	(1.800)	(4.130)	(3.520)	(3.760)	(3.350)	(3.840)			
3	0.008	1.526	45.766	26.075	6.078	2.315	0.186	−0.974	−3.791	0.767	1.174				
	(0.660)	(0.140)	(1.620)	(1.520)	(0.450)	(0.190)	(0.020)	(−0.110)	(−0.500)	(0.090)	(0.180)				
			−1.443	1.812	1.073	2.467	1.844	3.291	2.635	3.088	2.728	3.184	257	0.144	0.072
			(−4.020)	(2.360)	(1.530)	(2.670)	(2.310)	(4.750)	(3.700)	(4.040)	(3.770)	(4.130)			
Panel B: WLS															
1	0.005	5.311	33.054	13.954	−0.083	−6.491	−5.244	−5.319	−9.203	−1.527	−0.925		257	0.075	0.037
	(0.840)	(0.580)	(1.640)	(0.910)	(−0.010)	(−0.590)	(−0.610)	(−0.730)	(−1.390)	(−0.250)	(−0.210)				
2	0.011		−1.244	1.785	1.772	2.422	1.301	2.993	2.446	2.746	2.260	2.745	257	0.065	0.027
	(0.990)		(−2.930)	(2.050)	(2.310)	(2.650)	(1.460)	(3.890)	(3.220)	(3.470)	(3.090)	(3.530)			
3	0.006	2.912	49.417	25.159	5.490	1.078	−0.662	−1.662	−4.677	−1.577	1.226				
	(0.490)	(0.270)	(1.630)	(1.450)	(0.390)	(0.090)	(−0.070)	(−0.190)	(−0.610)	(−0.190)	(0.180)				
			−1.451	1.856	0.974	2.522	1.753	3.404	2.734	3.214	2.814	3.287	257	0.162	0.091
			(−3.720)	(2.130)	(1.240)	(2.550)	(1.970)	(4.580)	(3.430)	(3.790)	(3.580)	(3.950)			

These findings alleviate our concern that our previous results in Table 4 could have been due to more flexible ambiguity attitudes relative to risk attitudes.

8. The risk-ambiguity-return relation

Asset pricing theory, including our theoretical model, predicts a positive risk-return relation. However, broadly speaking, the multitude empirical tests of this fundamental relation struggle to find consistent supporting evidence. This also includes our basic tests of the risk-return relation. Our investigation, which attempts to explain the returns exclusively by risk, show an insignificant negative relation, in contrast to what theory suggests. When we introduce ambiguity into the pricing model, the effect of risk becomes positive and significant. This important result raises the question of why the risk premium is positive in the excess return regression only when ambiguity is included.

To address this question, we focus on the risk-ambiguity relation. From the theoretical perspective, the degree of ambiguity can be affected by two elements: the shape of each distribution within the set and the variety of these distributions. Suppose that the shape of each distribution within the set changes such that each distribution becomes riskier, in the sense that higher probability mass is assigned to the tail outcomes. In this case, naturally, the risk increases. In contrast, if each distribution becomes "flatter," the volatility of the probability of each outcome can decrease such that ambiguity decreases.[44] In this case, we expect a negative risk-ambiguity relation. Suppose now that the shape of the probability distributions within the set of priors remains unchanged and only the dispersion across the priors changes.[45] In this case, risk can increase, since the average probability of tail outcomes increases. At the same time, ambiguity increases such that we expect a positive risk-ambiguity relation. Using this intuition, in regular times, when the distributions within the set of priors are relatively stable (i.e., a relatively low dispersion across the probability distributions), a negative risk-ambiguity relation can be found. On the other hand, in highly uncertain (ambiguous) times, one can find a positive risk-ambiguity relation.

[44] To understand this intuition, consider the effect of increasing variance on a normal distribution. In the most extreme case, when the variance goes to infinity, the distribution goes to a uniform one

[45] This can be obtained, for example, by a linear shifting of the probability distributions within the set of priors.

The consequences of these conjectures can be observed in the correlation between risk and ambiguity. Panel C of Table 1 shows that the correlation between risk and ambiguity is insignificant at 0.026. This value, however, is misleading since this correlation (like other statistics) is time varying. Measuring the correlation over the entire sample between 1993 and 2016 ignores the significant positive and negative correlations over subperiods. For example, for the period of 2004–2007, the correlation between risk and ambiguity is negative at −0.581 and significant. On the other hand, for the period around the financial crisis (2008–2010), the correlation between risk and ambiguity was positive at 0.69 and significant. With regard to our results, the relevant correlation is between expected ambiguity and expected risk. For the period of 2004–2007, this correlation is negative at −0.676 and significant, and for the period around the financial crisis (2008–2010) it is positive at 0.37.

The varying risk-ambiguity relation over different time periods can provide an explanation for the question of why when omitting ambiguity from the regression tests that explain excess return, risk has an insignificant negative coefficient, but when ambiguity is introduced, risk is positive and significant. Clearly, a further exploration of the risk-ambiguity relation is required. We leave this for future research.

9. Conclusion

The basic tenet in asset pricing theory is the risk-return relation, which has been tested a multitude of times using a variety of models and factors. The results have been mixed at best. One possibility is that ambiguity is an important missing factor that can restore the risk-return relationship. In this study, we introduce ambiguity into the traditional risk-return relation. Our results show that the excess return on the market as a whole, known as the equity premium, is determined by two distinct factors: ambiguity and risk. Risk is measured in a variety of ways; for example, using a rate of return volatility or alternatively implied volatility. Ambiguity is measured by the volatility of the uncertain probabilities of returns. To this end, we introduce an empirical methodology for measuring the degree of ambiguity and for eliciting probabilistically contingent preferences directly from market data.

We present four hypotheses to examine the effects of risk, ambiguity, and ambiguity attitudes on the excess returns. Consistent with the classical asset pricing paradigm, one would expect excess return and risk to be positively related. Regarding ambiguity, many behavioral experiments find an aversion to ambiguity for gains and a love for ambiguity for losses, although the particular form of ambiguity attitudes has yet to be determined. We find that in the case of a high expected probability of gains, the effect of ambiguity is positive and highly significant, while for a high expected probability of losses, it is negative and highly significant. Furthermore, our findings indicate that aversion to ambiguity increases with the expected probability of gains, while love for ambiguity increases with the expected probability of losses. When we include ambiguity in the pricing model, the effect of risk is positive and significant, while its effect is insignificant when ambiguity is not accounted for. The positive equity premium in this instance contains a premium for risk and a premium for ambiguity.

The empirical methodology we propose to estimate ambiguity can be employed in other economic and financial studies. It has the potential to add clarification for some anomalies that previously could not be fully explained.

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