Lagrangian Modeling of a MIP

MIP track, week 4

Modeling a Mobile Inverted Pendulum (MIP)
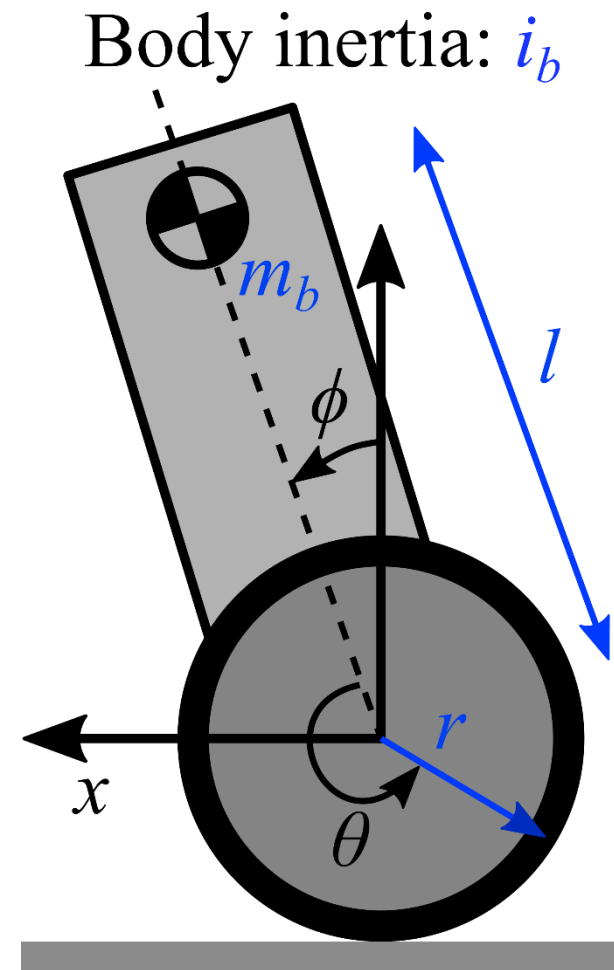
- Generalized coordinates

$$q := \begin{bmatrix} \theta \\ \phi \end{bmatrix}$$

Wheel/body angle

Body roll angle

- In the absence of slip $\quad x = (\theta + \phi)r$
- Assume massless wheel

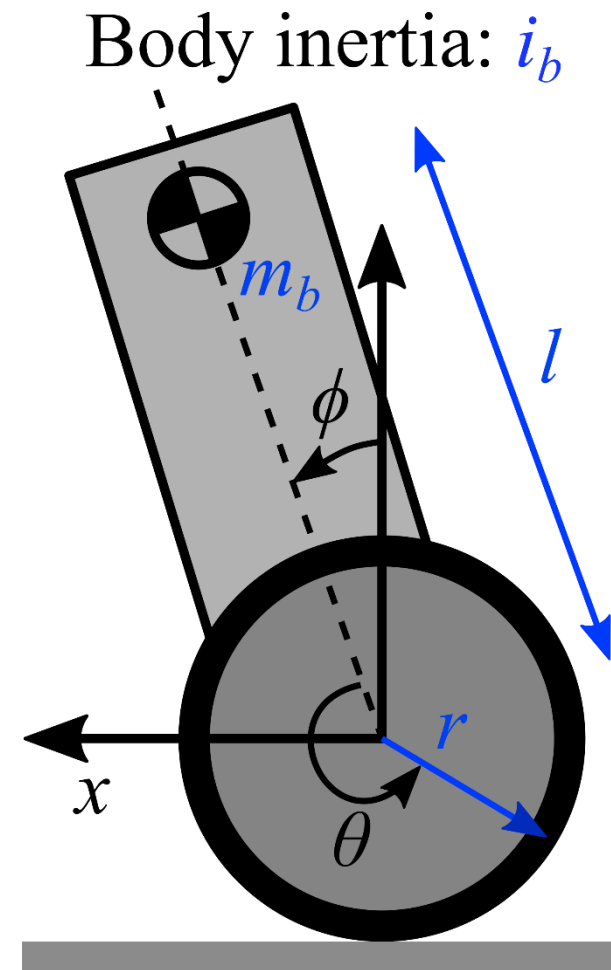Body inertia: i_b

m_b

ϕ

l

x

r

θ

Lagrangian Formulation

- Position of COM relative to drawn coord frame

$$p = r \begin{bmatrix} \theta + \phi \\ 0 \end{bmatrix} + l \begin{bmatrix} \sin \phi \\ \cos \phi \end{bmatrix}$$

- Kinetic energy $\quad T := \frac{1}{2} m_b \dot{p}^T \dot{p} + \frac{1}{2} i_b \dot{\phi}^2$

- Potential energy $\quad V := m_b g l \cos \phi$

- Lagrangian $\quad L := T - V$

- Equations of motion

$$(D_q - D_t D_{\dot{q}}) L = \begin{bmatrix} \tau \\ 0 \end{bmatrix}$$

Body inertia: i_b

m_b

l

ϕ

r

x

θ

Simulation

- Should get "natural" motions
- Ignore ground intersection