Image Formation and Cameras

Computer Vision I

CSE 252A

Lecture 3

Announcements

- Assignment 0 is due on Tuesday
- Read Chapters 1 & 2 of Forsyth & Ponce
- Lecture notes on web page
- Waitlist
- Two slide decks that might be useful:
  - Linear Algebra Review
  - Probability and Random Variables Review
- (Subset of?) Final exam can be use for CSE MS Comprehensive Exam [Pending]

The course

- Part 1: The physics of imaging
- Part 2: Early vision
- Part 3: Reconstruction
- Part 4: Recognition

Image Formation: Outline

- Factors in producing images
- Projection
- Perspective/Orthographic Projection
- Vanishing points
- Projective Geometry
- Rigid Transformation and SO(3)
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance and Radiometry

Earliest Surviving Photograph

- First photograph on record, “la table service” by Nicephore Niepce in 1822.
- Note: First photograph by Niepce was in 1816.

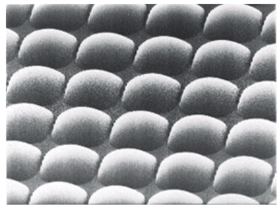
Compare to Paintings

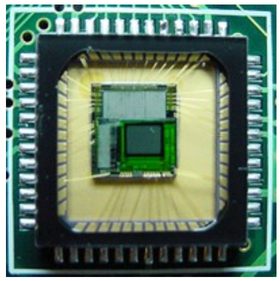
Willem Kalf, Mid 1600’s

Pedro Campos,

How Cameras Produce Images

- Basic process:
  - photons hit a detector
  - the detector becomes charged
  - the charge is read out as brightness
- Sensor types:
  - CCD (charge-coupled device)
    - high sensitivity
    - high power
    - cannot be individually addressed
    - blooming
  - CMOS
    - simple to fabricate (cheap)
    - lower sensitivity, lower power
    - can be individually addressed





Images are two-dimensional patterns of brightness values.

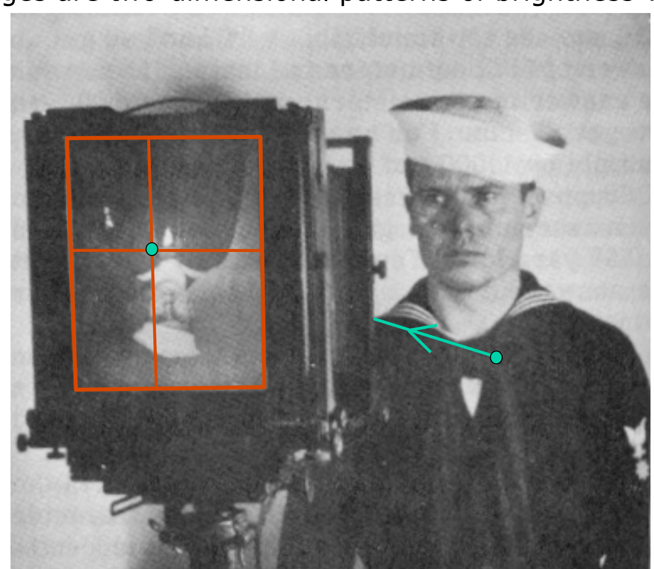


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

They are formed by the projection of 3D objects.

Effect of Lighting: Monet

CS252A, Fall 2018 Computer Vision I

Change of Viewpoint: Monet

Haystack at Chailly at sunrise (1865)

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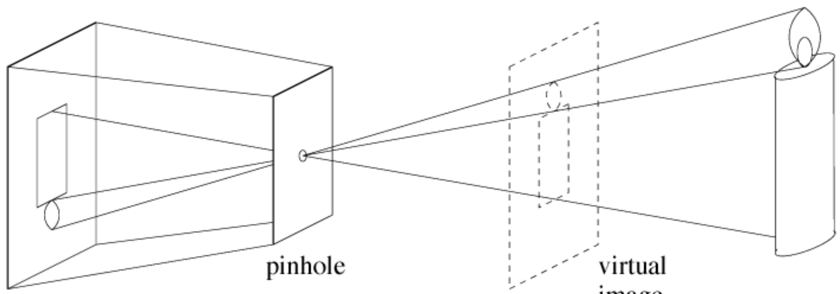
Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it



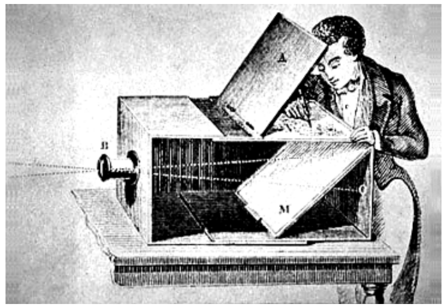
 Forsyth&Ponce

Camera Obscura

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays". --- Leonardo *Da Vinci*

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

Camera Obscura



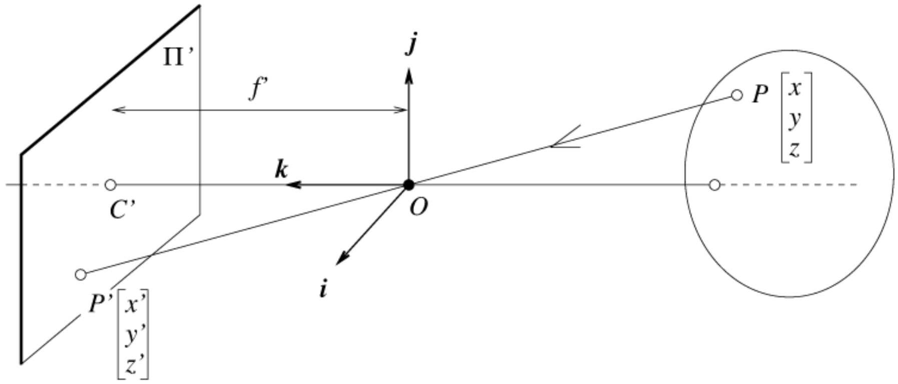
- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

Camera Obscura

Jetty at Margate England, 1898.

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

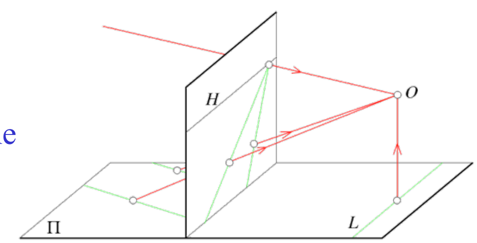
Purely Geometric View of Perspective



The projection of the point **P** on the image plane **Π'** is given by the point of intersection **P'** of the ray defined by **PO** with the plane **Π'**.

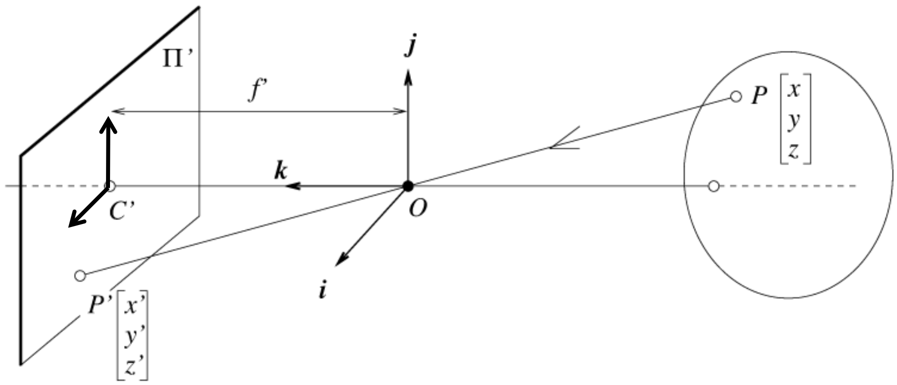
Geometric properties of projection

- 3-D points map to points
- 3-D lines map to lines
- Planes map to whole image or half-plane
- Polygons map to polygons



- Important point to note: Angles & distances not preserved, nor are inequalities of angles & distances.
- Nor are ratios of angles or distances
- Degenerate cases:
  - line through focal point project to point
  - plane through focal point projects to a line

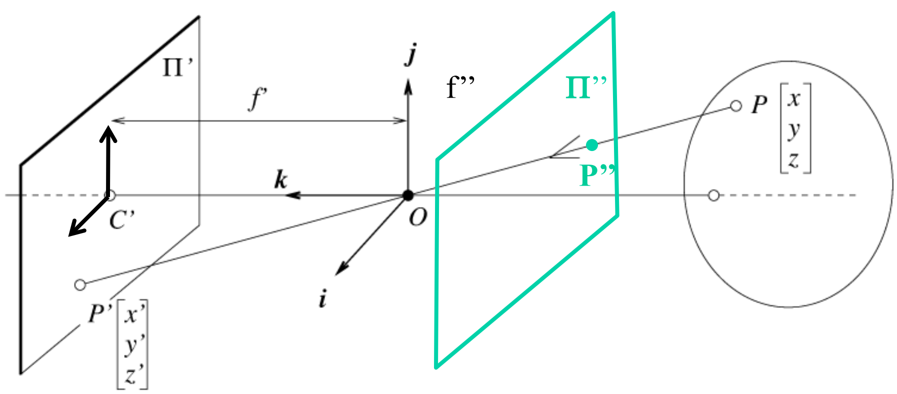
Equation of Perspective Projection



Cartesian coordinates:

- We have, by similar triangles, that for **P**=(x, y, z), the intersection of OP with **Π**' is (f' x/z, f' y/z, f')
- Establishing an image plane coordinate system at C' aligned with i and j, we get $(x, y, z) \rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$

Virtual Image Plane



- Virtual image plaen in front of optical center.
- Image is 'upright'

$$(x, y, z) \rightarrow (f''\frac{x}{z}, f''\frac{y}{z})$$

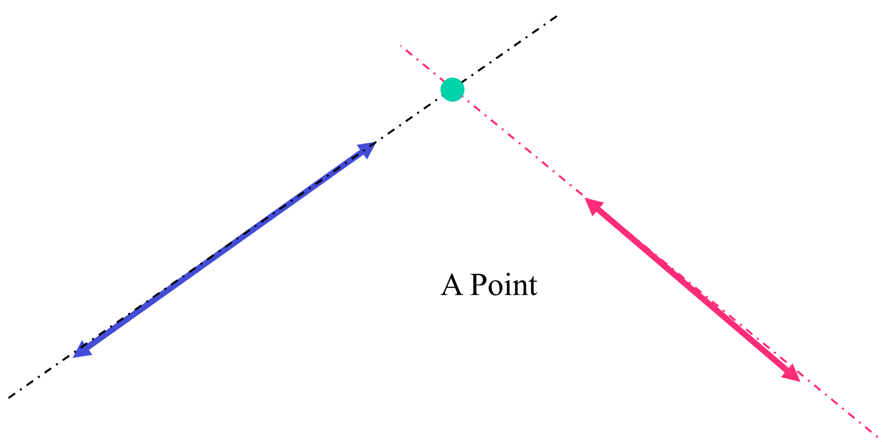
A Digression

Projective Geometry

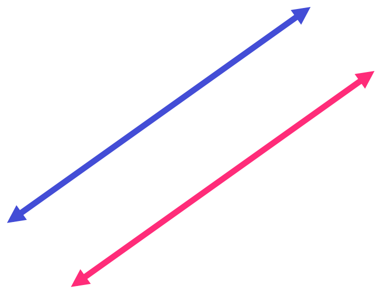
and

Homogenous Coordinates

What is the intersection of two lines in a plane?



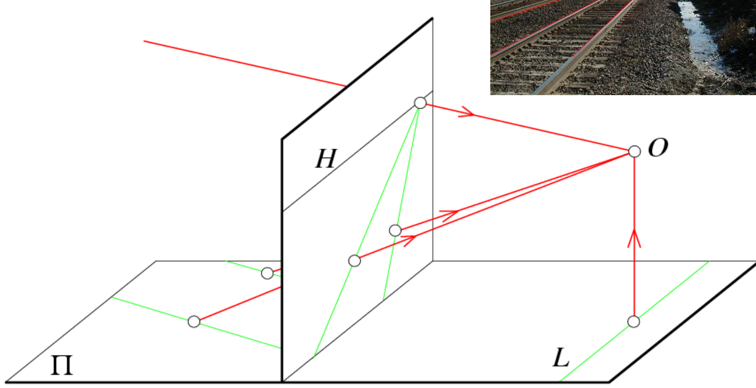
Do two lines in the plane always intersect at a point?



No, Parallel lines don't meet at a point.

Can the perspective image of two parallel lines meet at a point?

YES



Projective geometry provides an elegant means for handling these different situations in a unified way and **homogenous coordinates** are a way to represent entities (points & lines) in projective spaces.

Projective Geometry

- Axioms of Projective Plane
  1. Every two distinct points define a line
  2. Every two distinct lines define a point (intersect at a point)
  3. There exists three points, A,B,C such that C does not lie on the line defined by A and B.
- Different than Euclidean (affine) geometry
- Projective plane is "bigger" than affine plane – includes "line at infinity"

Homogenous coordinates
A way to represent points in a projective space

- Use three numbers to represent a point on a projective plane

Why? The projective plane has to be bigger than the Cartesian plane so that it contains the point where parallel lines interest.

How: Add an extra coordinate

e.g., (x,y) -> (x,y,1)

Impose equivalence relation

$$(x,y,z) \approx \lambda^*(x,y,z)$$

such that $\lambda \neq 0$

i.e., $(x,y,1) \approx (\lambda x, \lambda y, \lambda)$

- Points at infinity – zero for last coordinate e.g., (x,y,0)

- Why do this?
  - Possible to represent points "at infinity"
    - Where parallel lines intersect
    - Where parallel planes intersect
  - Possible to write the action of a perspective camera as a matrix

Homogenous coordinates
A way to represent points in a projective space

Use three numbers to represent a point on a projective plane

Add an extra coordinate

e.g., (x,y) -> (x,y,1)

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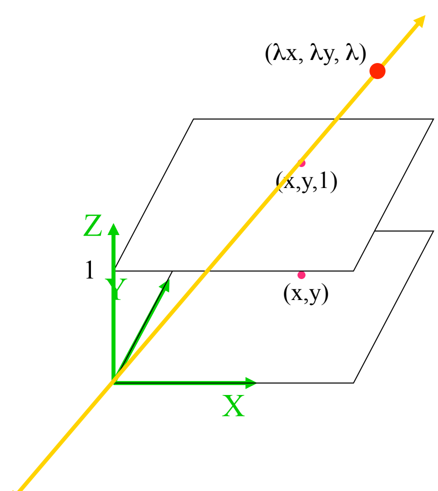


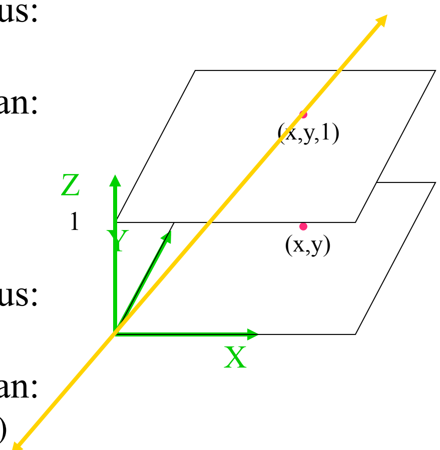
Conversion
Euclidean -> Homogenous -> Euclidean

In 2-D

- Euclidean -> Homogenous:
  (x, y) -> λ (x,y,1)
- Homogenous -> Euclidean:
  (x, y, z) -> (x/z, y/z)

In 3-D

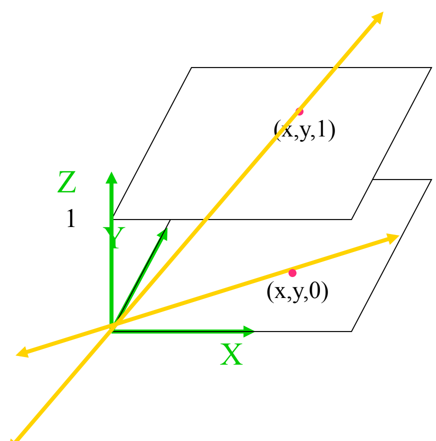
- Euclidean -> Homogenous:
  (x, y, z) -> λ (x,y,z,1)
- Homogenous -> Euclidean:
  (x, y, z, w) -> (x/w, y/w, z/w)



Points at infinity

Point at infinity – last coordinate is zero (x,y,0) and equivalence relation (x,y,0) ≈ λ*(x,y,0)

No corresponding Euclidean point (you'd divide by zero).



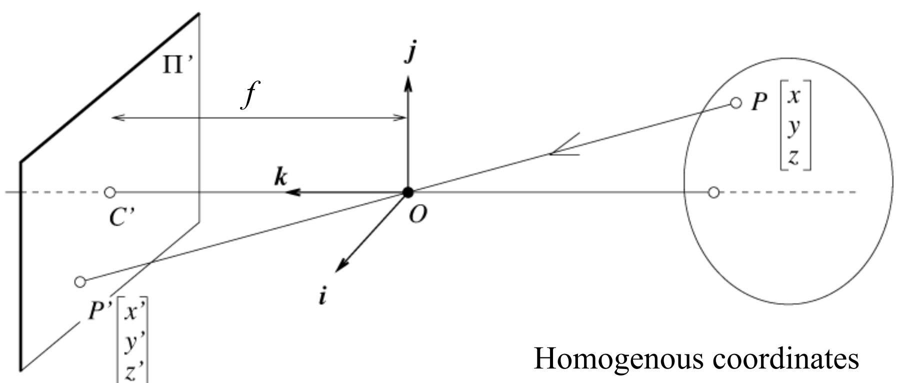
Projective transformation

- 3 x 3 linear transformation of homogenous coordinates

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{21} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- Points map to points
- Lines map to lines
- Points in X can map to points at infinity in U. e.g., $u_3=0$

The equation of projection



Cartesian coordinates:

$$(x,y,z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Homogenous coordinates and camera matrix

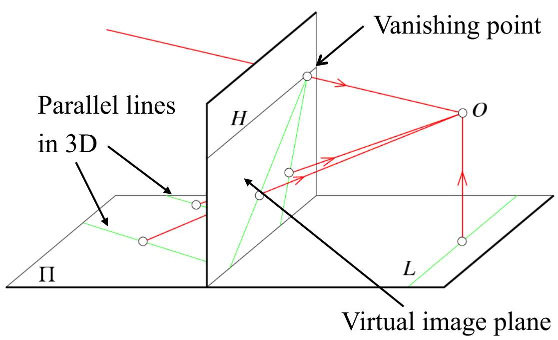
$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

End of the Digression

In a perspective image, parallel lines meet at a point, called the vanishing point

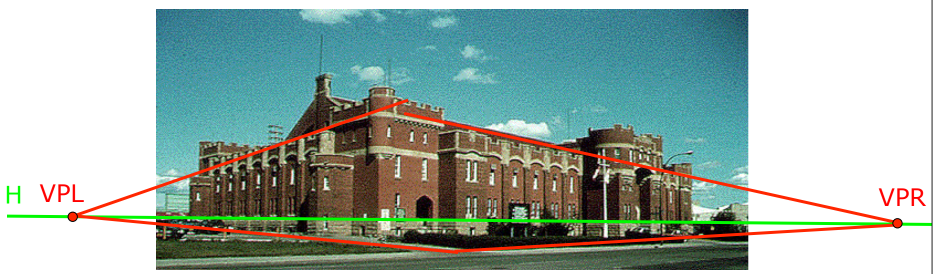
Doesn't need to be near the center of the image

Parallel lines meet in the image

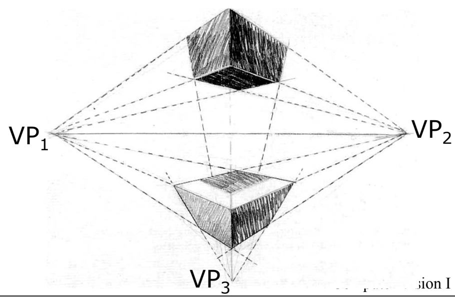


- Vanishing point location: Intersection of 3-D line through O parallel to given line(s)
- A single line can have a vanishing point

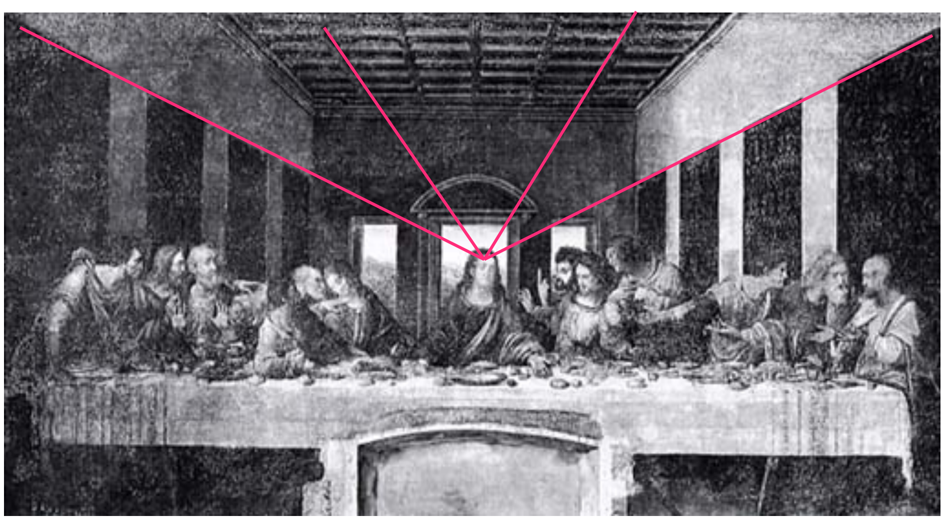
Vanishing points



- A scene can have more than one vanishing point
- Different 3-D directions correspond different vanishing points



Vanishing Points



Vanishing Point

- In the **projective plane**, parallel lines meet at a point at infinity.
- The vanishing point is the perspective projection of that point at infinity, resulting from multiplication by the camera matrix.

What is a Camera?

- An mathematical expression that relates points in 3D to points in an image for different types of physical cameras or imaging situations

Simplified Camera Models

Perspective Projection

↓

Affine Camera Model

↓

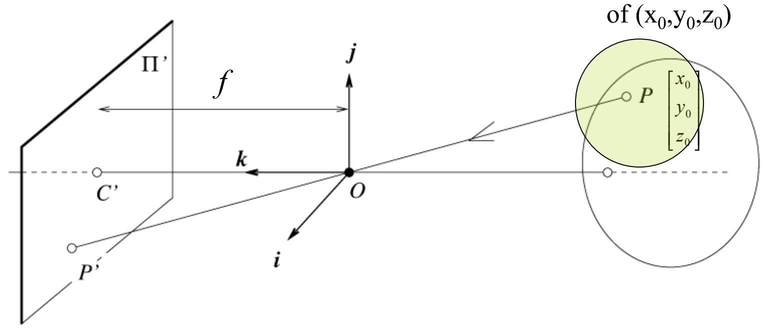
Scaled Orthographic Projection

↓

Orthographic Projection

Affine Camera Model

Appropriate for in neighborhood of (x_0,y_0,z_0)



- Take perspective projection equation, and perform Taylor series expansion about some point (x_0,y_0,z_0)
- Drop terms that are higher order than linear
- Resulting expression is the Affine Camera Model

- Perspective

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{z}\begin{bmatrix} x \\ y \end{bmatrix}$$

- Perform a Taylor series expansion about (x_0, y_0, z_0)

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{z_0}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \frac{f}{z_0^2}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}(z - z_0) + \frac{f}{z_0}\begin{bmatrix} 1 \\ 0 \end{bmatrix}(x - x_0)$$

$$+\frac{f}{z_0}\begin{bmatrix} 0 \\ 1 \end{bmatrix}(y - y_0) + \frac{f}{2}\frac{2}{z_0^3}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}(z - z_0)^2 + \cdots$$

- Dropping higher order terms and regrouping.

$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{f}{z_0}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} f/z_0 & 0 & -fx_0/z_0^2 \\ 0 & f/z_0 & -fy_0/z_0^2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{Ap} + \mathbf{b}$$

Affine camera model

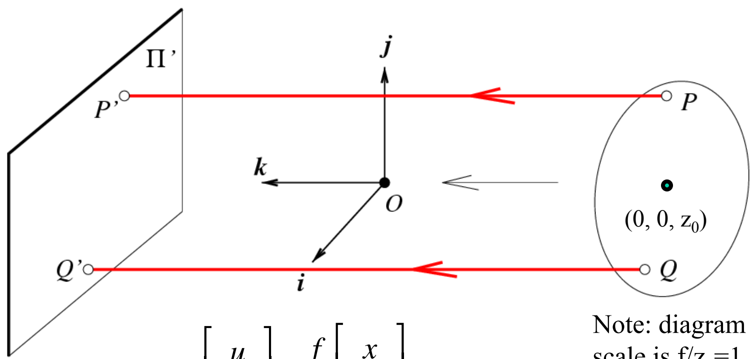
$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{f}{z_0}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} f/z_0 & 0 & -fx_0/z_0^2 \\ 0 & f/z_0 & -fy_0/z_0^2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{Ap} + \mathbf{b}$$

Rewrite affine camera model
in terms of Homogenous Coordinates

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \approx \begin{bmatrix} f/z_0 & 0 & -fx_0/z_0^2 & fx_0/z_0 \\ 0 & f/z_0 & -fy_0/z_0^2 & fy_0/z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scaled orthographic projection

Starting with Affine Camera Model, take Taylor series about $(x_o, y_0, z_0) = (0, 0, z_0)$ – a point on the optical axis



Note: diagram depicts scale is f/z_0=1

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{z_0}\begin{bmatrix} x \\ y \end{bmatrix}$$

f/z_0 is the scale

The projection matrix for scaled orthographic projection

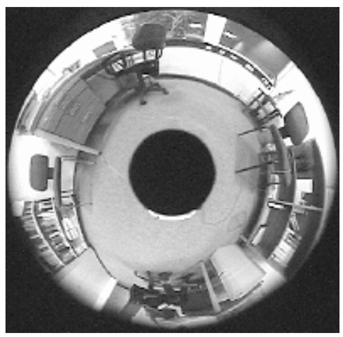
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} f/z_0 & 0 & 0 & 0 \\ 0 & f/z_0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

- Parallel lines project to parallel lines
- Ratios of distances are preserved under orthographic projection

Other camera models

- Generalized camera – maps points lying on rays and maps them to points on the image plane.

Omnicam (hemispherical)



Light Probe (spherical)

Some Alternative "Cameras"