ESSENTIAL MATRIX, LINEAR ESTIMATE

$$\underbrace{\underset{\sim}{x} = \underset{\sim}{K}[\underset{\sim}{R}|\underset{\sim}{t}]\underset{\sim}{X} \qquad \underset{\sim}{x}' = \underset{\sim}{K}'[\underset{\sim}{R}'|\underset{\sim}{t}']\underset{\sim}{X}}_{\text{IMAGE COORDINATES}}$$

$$\underset{\sim}{K}^{-1}\underset{\sim}{x} = [\underset{\sim}{R}|\underset{\sim}{t}]\underset{\sim}{X} \qquad \underset{\sim}{K}'^{-1}\underset{\sim}{x}' = [\underset{\sim}{R}'|\underset{\sim}{t}']\underset{\sim}{X}$$

$$\hat{\underset{\sim}{x}} = \hat{\underset{\sim}{P}}\underset{\sim}{X} \qquad \hat{\underset{\sim}{x}}' = \hat{\underset{\sim}{P}}'\underset{\sim}{X}$$

WHERE
$$\left.\begin{aligned} \hat{\underset{\sim}{x}} &= \underset{\sim}{K}^{-1}\underset{\sim}{x} \\ \hat{\underset{\sim}{x}}' &= \underset{\sim}{K}'^{-1}\underset{\sim}{x}' \end{aligned}\right\} \text{NORMALIZED COORDINATES}$$

$$\left.\begin{aligned} \hat{\underset{\sim}{P}} &= [\underset{\sim}{R}|\underset{\sim}{t}] \\ \hat{\underset{\sim}{P}}' &= [\underset{\sim}{R}'|\underset{\sim}{t}'] \end{aligned}\right\} \begin{array}{l}\text{NORMALIZED CAMERA} \\ \text{PROJECTION MATRICES}\end{array}$$

$$\hat{\underset{\sim}{l}}_i' = \underset{\sim}{E}\hat{\underset{\sim}{x}}_i \quad \forall i$$

$$\hat{\underset{\sim}{x}}_i'^T \hat{\underset{\sim}{l}}_i' = 0$$

$$\hat{\underset{\sim}{x}}_i'^T \underset{\sim}{E}\hat{\underset{\sim}{x}}_i = 0 \quad \forall i$$

$$\begin{bmatrix} \hat{x}_i' & \hat{y}_i' & \hat{w}_i' \end{bmatrix} \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} \begin{bmatrix} \hat{x}_i \\ \hat{y}_i \\ \hat{w}_i \end{bmatrix} = 0$$

$$\hat{x}_i\hat{x}_i' e_{11} + \hat{y}_i\hat{x}_i' e_{12} + \hat{w}_i\hat{w}_i' e_{13} + \hat{x}_i\hat{y}_i' e_{21} + \hat{y}_i\hat{y}_i' e_{22} + \hat{w}_i\hat{y}_i' e_{23} + \hat{x}_i\hat{w}_i' e_{31} +$$
$$\hat{y}_i\hat{w}_i' e_{32} + \hat{w}_i\hat{w}_i' e_{33} = 0$$

$$\underset{1\times 9}{\begin{bmatrix} \hat{x}_i\hat{x}_i' & \hat{y}_i\hat{x}_i' & \hat{w}_i\hat{x}_i' & \hat{x}_i\hat{y}_i' & \hat{y}_i\hat{y}_i' & \hat{w}_i\hat{y}_i' & \hat{x}_i\hat{w}_i' & \hat{y}_i\hat{w}_i' & \hat{w}_i\hat{w}_i' \end{bmatrix}} \underset{9\times 1}{\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \\ e_{21} \\ e_{22} \\ e_{23} \\ e_{31} \\ e_{32} \\ e_{33} \end{bmatrix}} = \underset{1\times 1}{0}$$

$$\underset{\sim}{a}_i^T \underset{\sim}{e} = 0$$

$$\underset{n\times 9}{\begin{bmatrix} a_1^T \\ a_2^T \\ \vdots \\ a_n^T \end{bmatrix}} \underset{9\times 1}{\underset{\sim}{e}} = \underset{n\times 1}{\underset{\sim}{0}} \qquad n \geq 8$$

$$\underset{n\times 9}{\underset{\sim}{A}}\ \underset{9\times 1}{\underset{\sim}{e}} = \underset{n\times 1}{\underset{\sim}{0}}$$, SOLVE FOR $\underset{\sim}{e}$

USING SINGULAR VALUE DECOMPOSITION (SVD)

$\underset{n\times 9}{A} = \underset{n\times n}{U}\,\underset{n\times 9}{\Sigma}\,\underset{9\times 9}{V^T}$ WHERE U AND V ARE ORTHOGONAL MATRICES

AND $\Sigma = diag(\sigma_1, \sigma_2, \ldots, \sigma_{8 \text{ or } 9})$ ← SMALLEST

WHERE $\sigma_i \geq \sigma_{i+1}$

COLUMNS OF U ARE LEFT SINGULAR VECTORS
COLUMNS OF V ARE RIGHT SINGULAR VECTORS
} CORRESPONDING TO SINGULAR VALUES

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e IS RIGHT SINGULAR VECTOR CORRESPONDING TO SMALLEST SINGULAR VALUE (I.E, e IS THE LAST COLUMN OF V)

"RESHAPE" $\underset{9\times 1}{e} \longrightarrow E = \begin{bmatrix} e_1 & e_2 & e_3 \\ e_4 & e_5 & e_6 \\ e_7 & e_8 & e_9 \end{bmatrix}$

ENFORCE CONSTRAINTS

$$\underset{3\times 3}{E} = \underset{3\times 3}{U}\,\underset{3\times 3}{\Sigma}\,\underset{3\times 3}{V^T}$$

$$E = U\, diag(\sigma_1, \sigma_2, \sigma_3)\, V^T$$

$$\sigma_1 = 1$$
$$\sigma_2 = 1$$
$$\sigma_3 = 0$$

$$E = U\, diag(1, 1, 0)\, V^T$$

FUNDAMENTAL MATRIX, LINEAR ESTIMATE

$$\underset{\sim}{x}_i'^T \underset{\sim}{F} \underset{\sim}{x}_i = 0 \quad \forall i$$

$$\begin{bmatrix} x_i x_i' & y_i x_i' & w_i x_i' & x_i y_i' & y_i y_i' & w_i y_i' & x_i w_i' & y_i w_i' & w_i w_i' \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

$$\underset{1\times 9}{\underset{\sim}{a}_i^T} \; \underset{9\times 1}{\underset{\sim}{f}} = \underset{1\times 1}{0}$$

$$\underset{n\times 9}{\begin{bmatrix} a_1^T \\ a_2^T \\ \vdots \\ a_n^T \end{bmatrix}} \; \underset{9\times 1}{\underset{\sim}{f}} = \underset{n\times 1}{\underset{\sim}{0}}$$

$\underset{\sim}{A} \underset{\sim}{f} = \underset{\sim}{0}$, SOLVE FOR $\underset{\sim}{f}$ (USING SVD)

$$\underset{\sim}{A} = \underset{\sim}{U} \underset{\sim}{\Sigma} \underset{\sim}{V}^T$$

$\underset{\sim}{f}$ IS THE LAST COLUMN OF $\underset{\sim}{U}$

$$\underset{\sim}{F} = \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix}$$

$$\underset{3\times 3}{\underset{\sim}{F}} = \underset{3\times 3}{\underset{\sim}{U}} \; \underset{3\times 3}{\underset{\sim}{\Sigma}} \; \underset{3\times 3}{\underset{\sim}{V}^T}$$

$$\underset{\sim}{F} = \underset{\sim}{U}\, diag(\sigma_1, \sigma_2, \sigma_3) \underset{\sim}{V}^T$$

$$\sigma_3 = 0$$

$$\underset{\sim}{F} = \underset{\sim}{U}\, diag(\sigma_1, \sigma_2, 0) \underset{\sim}{V}^T$$

DATA NORMALIZE POINTS IN IMAGE 1

CALCULATE MEAN AND VARIANCE OF EACH COORDINATE: $\mu_{\tilde{x}}, \mu_{\tilde{y}}$, AND $\sigma^2_{\tilde{x}}\ \sigma^2_{\tilde{y}}$

SCALE $s = \frac{\sqrt{2}}{\sigma} = \sqrt{\frac{2}{\sigma^2}}$ WHERE $\sigma^2 = \sigma^2_{\tilde{x}} + \sigma^2_{\tilde{y}}$ TOTAL VARIANCE

$$\underset{\sim}{T}_{DN} = \begin{bmatrix} s & 0 & -\mu_{\tilde{x}} s \\ 0 & s & -\mu_{\tilde{y}} s \\ 0 & 0 & 1 \end{bmatrix}$$ DATA NORMALIZING TRANSFORMATION

$\underset{\sim}{x}_{DN} = \underset{\sim}{T}_{DN} \underset{\sim}{x}$ TRANSFORM POINTS

SIMILARLY, DATA NORMALIZE POINTS IN IMAGE 2

$\underset{\sim}{x}'_{DN} = \underset{\sim}{T}'_{DN} \underset{\sim}{x}'$ TRANSFORM POINTS

ESTIMATE DATA NORMALIZED FUNDAMENTAL MATRIX $\underset{\sim}{F}_{DN}$ FROM $\underset{\sim}{x}_{DN} \leftrightarrow \underset{\sim}{x}'_{DN}$

THEN, DATA DENORMALIZE RESULTING $\underset{\sim}{F}_{DN}$

$$\underset{\sim}{x}'^{T}_{DN} \underset{\sim}{F}_{DN} \underset{\sim}{x}_{DN} = 0$$

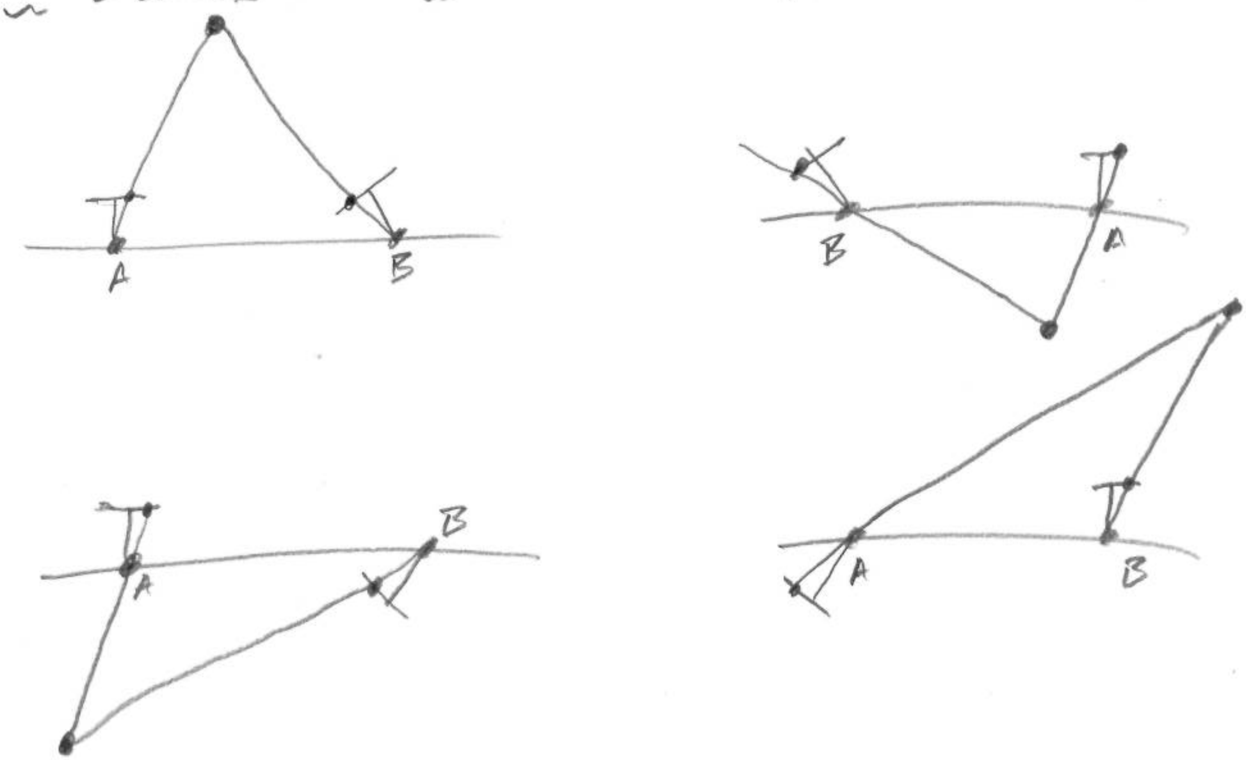
$$(\underset{\sim}{T}'_{DN} \underset{\sim}{x}')^T \underset{\sim}{F}_{DN} \underset{\sim}{T}_{DN} \underset{\sim}{x} = 0$$

$$\underset{\sim}{x}'^T \underset{\sim}{T}'^T_{DN} \underset{\sim}{F}_{DN} \underset{\sim}{T}_{DN} \underset{\sim}{x} = 0$$

$\underset{\sim}{x}'^T \underset{\sim}{F} \underset{\sim}{x} = 0$ WHERE $\underset{\sim}{F} = \underset{\sim}{T}'^T_{DN} \underset{\sim}{F}_{DN} \underset{\sim}{T}_{DN}$

DECOMPOSE ESSENTIAL MATRIX

$\hat{P} = [I \mid 0]$ AND $\hat{P}' = [R \mid t]$ NEED ONE 2D POINT CORRESPONDENCE



$E = U D V^T$ WHERE $D = diag(1,1,0)$ TO SCALE

TWO CHOICE OF R

$R_1 = U Z V^T$ OR $R_2 = U Z^T V^T$ WHERE $Z = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

IF $det(R_1) < 0$, $R_1 = -R_1$

IF $det(R_2) < 0$, $R_2 = -R_2$

TWO CHOICE OF t

$t_1 = u_3$ (LAST COLUMN OF U) OR $t_2 = -t_1$

FOUR SOLUTIONS

$\hat{P}' = [R_1 \mid t_1]$, $\hat{P}' = [R_2 \mid t_1]$

$\hat{P}' = [R_1 \mid t_2]$, $\hat{P}' = [R_2 \mid t_2]$

DECOMPOSE FUNDAMENTAL MATRIX

$$P = [I \mid 0] \text{ AND } P' = [[e']_\times F + e' v^T \mid \lambda e']$$

WHERE v IS ANY 3-VECTOR AND λ IS A NON ZERO SCALAR

IF $v = 0$ AND $\lambda = 1$, THEN

$$P' = [\underbrace{[e']_\times F}_{\text{RANK 2}} \mid e']$$

F IS RANK 2

$[e']_\times$ IS RANK 2