



**Danmarks
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42123 Optimization In Finance

Midterm Project Report

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1. Data Collection

Relevant data were collected from the Nasdaq OMX Nordic homepage, including 8 government bonds and their corresponding coupons, prices, maturities and yield rates respectively.

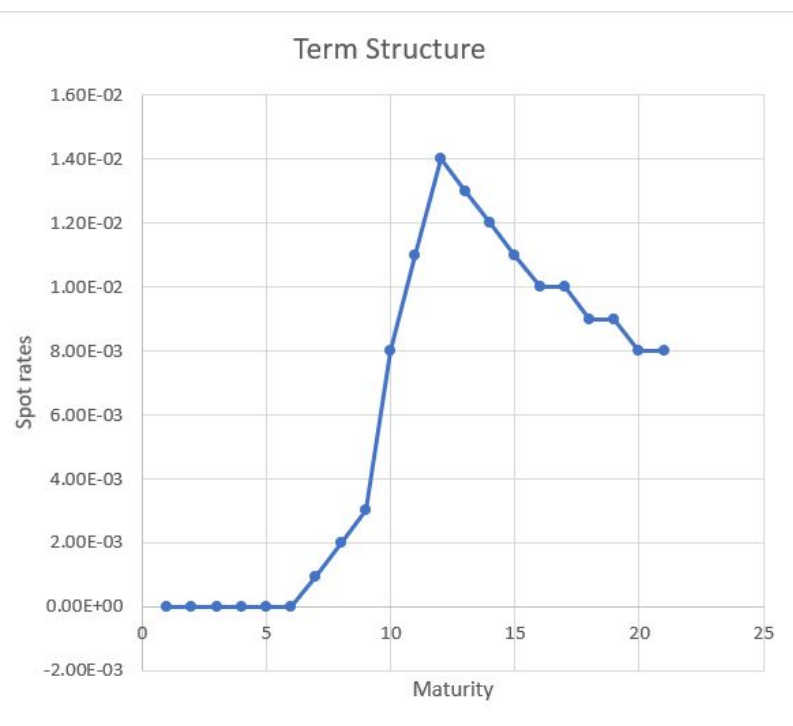
Bond	Bond Price (/100)	Coupons (/100)	Maturities (-2018)	Yield rate (15/11/18)
1,5St.l.23 GB	1.08485	0.0150	5	-0.163
3St.l. 21 GB	1.10626	0.0300	3	-0.446
4,5 St.l 39 GB	1.7114	0.0450	21	0.802
4St.l.19 GB	1.04659	0.0400	1	-0.679
7 St.l 24 GB	1.424	0.0700	6	-0.067
Danske Stat 2020	1.0176	0.0025	2	-0.627
Danske Stat 2025	1.11595	0.0175	7	0.107
Danske Stat 2027	1.0173	0.0050	9	0.308

2. Bootstrap Method

2.1 Bootstrap Without Smoothing

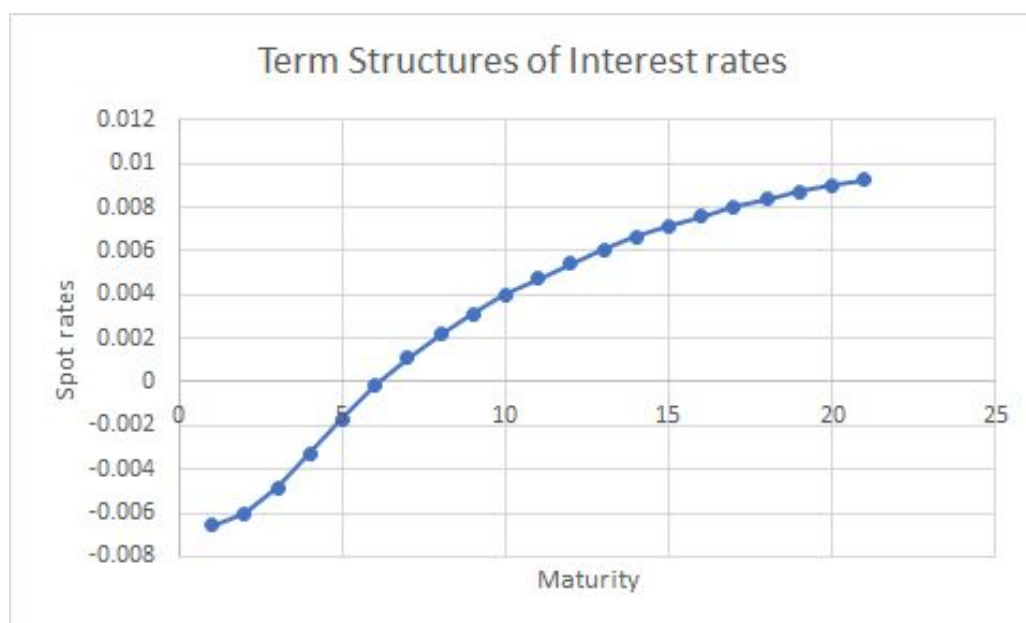
The bootstrap method was used to obtain the following term structure diagram.

1	Year	Maturity	Spot Rate
2	2019	1	7.59E-17
3	2020	2	3.28E-17
4	2021	3	1.86E-17
5	2022	4	-5.11E-17
6	2023	5	1.63E-17
7	2024	6	2.67E-17
8	2025	7	9.23E-04
9	2026	8	0.002
10	2027	9	0.003
11	2028	10	0.008
12	2029	11	0.011
13	2030	12	0.014
14	2031	13	0.013
15	2032	14	0.012
16	2033	15	0.011
17	2034	16	0.01
18	2035	17	0.01
19	2036	18	0.009
20	2037	19	0.009
21	2038	20	0.008
22	2039	21	0.008



2.2 Bootstrap With Smoothing

The bootstrap method was then modified to obtain a smoother output. Our group chose $\lambda = 0.7$ where the graph is the smoothest and does not have kinks.

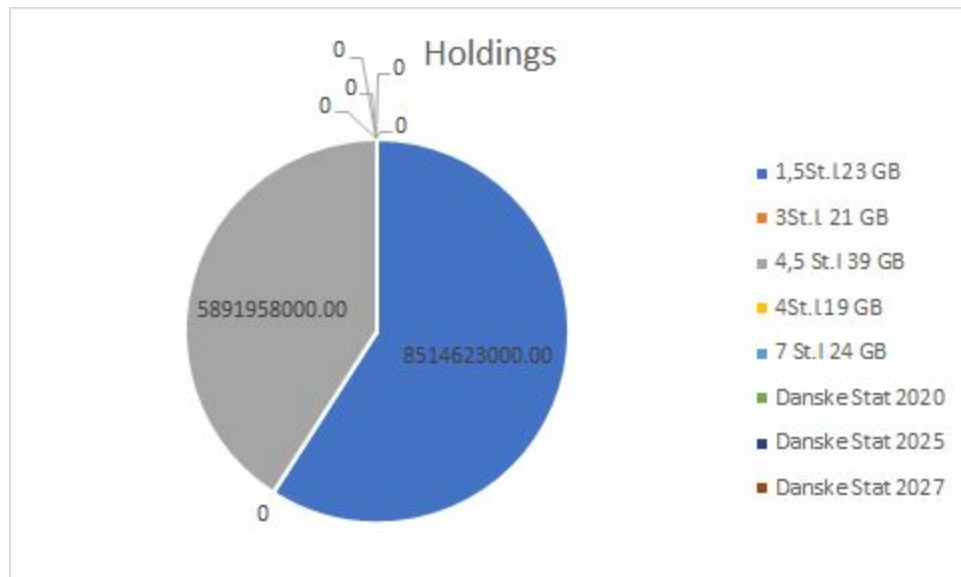


3. Portfolio Immunization Model

3.1 Match Duration

If only the convexity matching constraint matches the duration, the result is shown below with the duration match being $2.01840E+11$.

Bond	Holdings
1,5St.I.23 GB	8514623000
4,5 St.I 39 GB	5891958000



3.2 Match Duration and Convexity

If both the duration and the convexity are matched, bond holdings will not change. However, the duration match will be 10.492 and convexity will be 173.017.

Matching both duration and convexity produces better results. This is because the duration decreases significantly. Duration measures the sensitivity of a portfolio's price to changes in interest rates. A smaller duration means the price will not drop significantly if interest rates rise. This is preferable for pension funds because it involves

smaller risks. Additionally, convexity reflects the change in duration as the interest rate changes. The larger the convexity, in general, the better.

4. Portfolio Dedication Model

There are two types of portfolio dedication models. One is minimising the initial amount of investment needed, while the other is to maximise horizon returns (given a fixed budget).

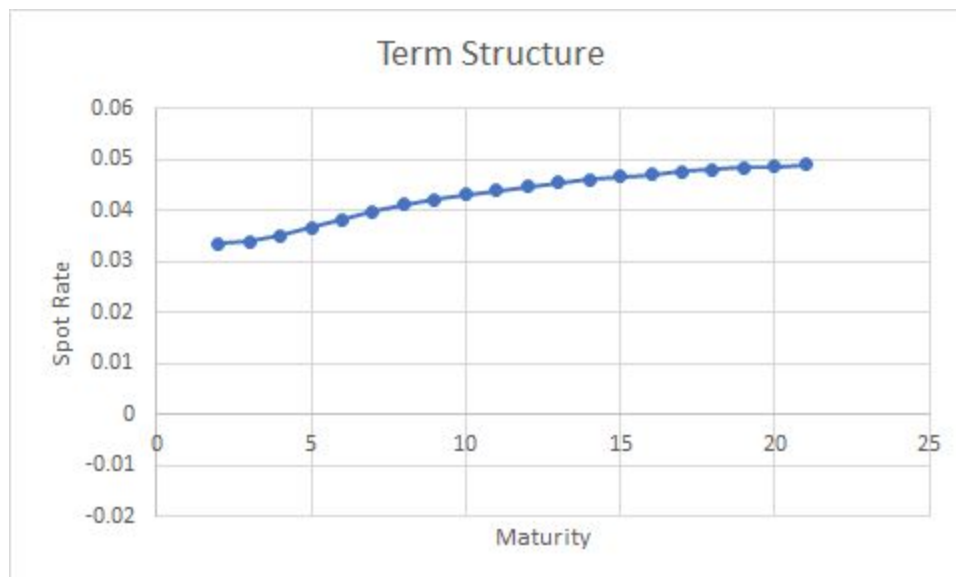
When running the portfolio dedication model in GAMS, the upfront investment V_0 for the first model is 19930700000. It is a very large value which contradicts the objective function of minimising the initial amount of investment needed.

By running the second model, the final value of V_{plus} is 0.00001049042 which is very small. However, we want to maximise the returns instead.

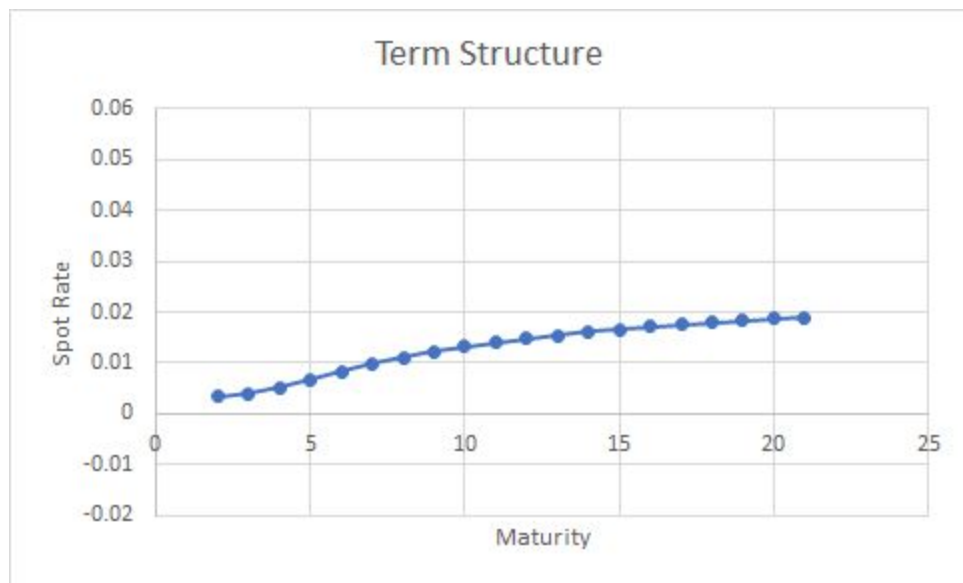
In conclusion, since the objectives are met for neither models, re-investment rate = -0.02 and spread = 0.04 would not be reasonable.

5. Term Structure Of Interest Rates

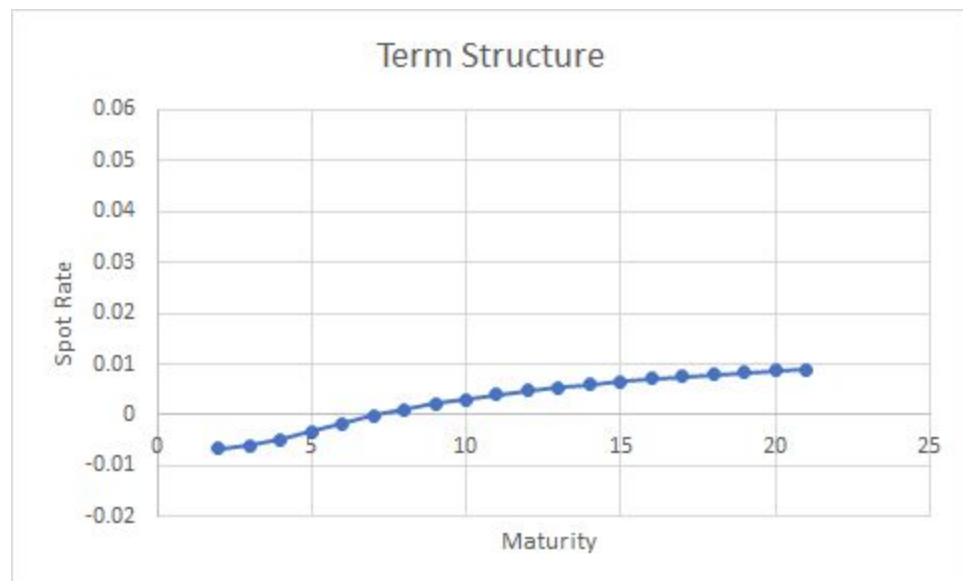
5.1 Parallel shift of +4%



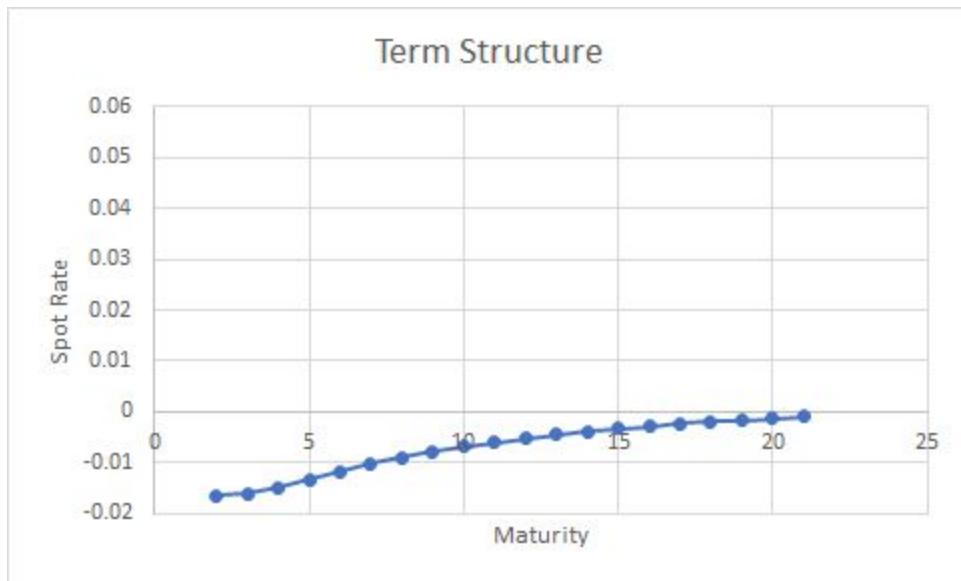
5.2 Parallel shift of +1%



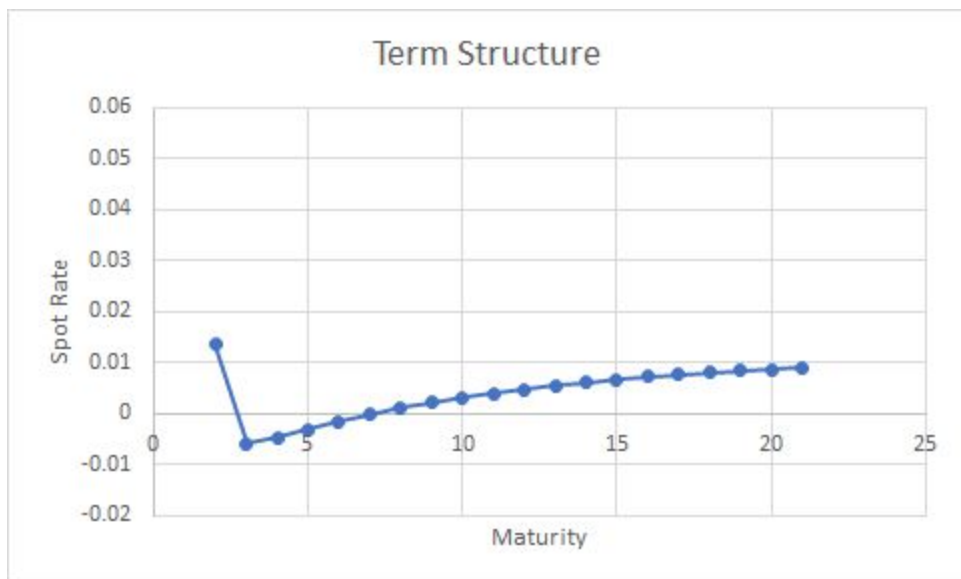
5.3 Term Structure unchanged



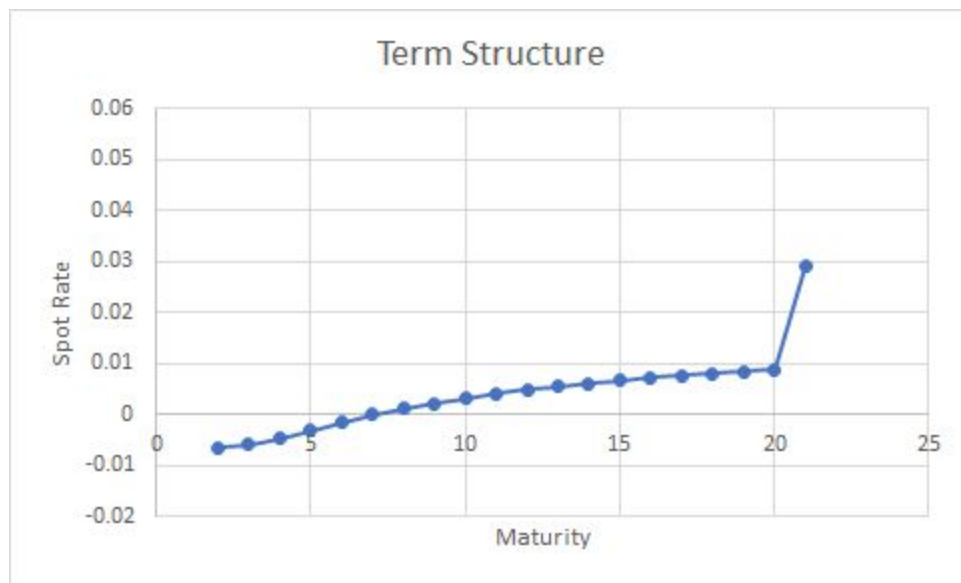
5.4 Parallel shift of -1%



5.5 Short end goes up by 2% and the long end stays unchanged



5.6 Short end stays unchanged and the long end goes up by 2%



6. Asset Liability Gap

Our group chose scenarios a, c and e and obtained the following results.

6.1 Parallel shift of +4%

Bonds	Holdings	Liability (Initial Purchase + 2019 Liability Stream)	Asset (2019 coupons)	Gap	Percentage (Liability over Asset)
1,5St.I. 23 GB	7530455000	-15416640700	422835181.5	-14993805520	3646.02%
4,5 St.I 39 GB	6886185700				

6.2 Term structure unchanged

Bonds	Holdings	Liability	Asset	Gap	Percentage (Liability
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		(Initial Purchase + 2019 Liability Stream)	(2019 coupons)		over Asset)
1,5St.I. 23 GB	8449463900	-15611837900	404048788.5	-15207789 110	3863.85%
4,5 St.I 39 GB	6162374000				

6.3 Short end goes up by 2% and the long end stays unchanged

Bonds	Holdings	Liability (Initial Purchase + 2019 Liability Stream)	Asset (2019 coupons)	Gap	Percentage (Liability over Asset)
1,5St.I. 23 GB	8424012500	-15588825200	403776759	-15185048 440	3860.75%
4,5 St.I 39 GB	6164812700				

7.Second Immunization Model Expansion

Our group understands that the objective is to minimize the biggest mismatch in the year 2019, under different scenarios given in question 5. In other words, it is to minimize $\text{abs}(\text{Present Value Of Assets} - \text{Present Value Of Liabilities})$. However, we could not come up with an implementation in GAMS because there are too many spot rates variations. Additionally, maximising the return and minimizing the biggest mismatch at the same time seems to be a dual-objective model which is something beyond our abilities despite our best efforts in sourcing for information online.