State and Output Equations of Filters

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1 First-Order Allpass Filter

The difference equations of first-order allpass filter are:

$$x(n) = u(n) - cx(n-1)$$

$$y(n) = cx(n) + x(n-1)$$

and corresponding state and output equations are:

$$x(n) = -cx(n-1) + u(n)$$

$$y(n) = (1-c^2)x(n-1) + cu(n)$$

2 First-Order Lowpass Filter

The difference equations of first-order lowpass filter are:

$$x(n) = u(n) - cx(n-1)$$

$$y(n) = \frac{1+c}{2}x(n) + \frac{1+c}{2}x(n-1)$$

and corresponding state and output equations are:

$$x(n) = -cx(n-1) + u(n)$$

$$y(n) = \frac{1-c^2}{2}x(n-1) + \frac{1+c}{2}u(n)$$

3 First-Order Highpass Filter

The difference equations of first-order highpass filter are:

$$x(n) = u(n) - cx(n-1)$$

$$y(n) = \frac{1-c}{2}x(n) + \frac{c-1}{2}x(n-1)$$

and corresponding state and output equations are:

$$x(n) = -cx(n-1) + u(n)$$

$$y(n) = \frac{c^2-1}{2}x(n-1) + \frac{1-c}{2}u(n)$$

4 First-Order Low Frenquency Shelving Filter

The difference equations of first-order low frequency shelving filter are:

$$x(n) = u(n) - c_{B/C}x(n-1)$$

$$y_1(n) = c_{B/C}x(n) + x(n-1)$$

$$y(n) = \frac{H_0}{2}[u(n) + y_1(n)] + u(n).$$

and corresponding state and output equations are:

$$x(n) = -c_{B/C}x(n-1) + u(n)$$

$$y(n) = \frac{H_0}{2}(1-c^2)x(n-1) + [\frac{H_0}{2}(1+c) + 1]u(n)$$

5 First-Order High Frenquency Shelving Filter

The difference equations of first-order high frequency shelving filter are:

$$x(n) = u(n) - c_{B/C}x(n-1)$$

$$y_1(n) = c_{B/C}x(n) + x(n-1)$$

$$y(n) = \frac{H_0}{2}[u(n) - y_1(n)] + u(n).$$

and corresponding state and output equations are:

$$x(n) = -c_{B/C}x(n-1) + u(n)$$

$$y(n) = \frac{H_0}{2}(c^2-1)x(n-1) + [\frac{H_0}{2}(1-c) + 1]u(n)$$

6 Second-Order Allpass Filter

The difference equations of second-order allpass filter are:

$$x(n) = u(n) - d(1-c)x(n-1) + cx(n-2)$$

$$y(n) = -cx(n) + d(1-c)x(n-1) + x(n-2),$$

and corresponding state and output equations are:

$$\begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} = \begin{bmatrix} -d(1-c) & c \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(n)$$

$$y(n) = \begin{bmatrix} (1-c^2)d & 1-c^2 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + (-c)u(n)$$

7 Second-Order Bandpass Filter

The difference equations of second-order bandpass filter are:

$$x(n) = u(n) - d(1-c)x(n-1) + cx(n-2)$$

$$y(n) = \frac{1+c}{2}x(n) - \frac{1+c}{2}x(n-2),$$

and corresponding state and output equations are:

$$\begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} = \begin{bmatrix} -d(1-c) & c \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(n)$$

$$y(n) = \begin{bmatrix} \frac{d(c^2-1)}{2} & \frac{c^2-1}{2} \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \frac{1+c}{2}u(n)$$

8 Second-Order Bandreject Filter

The difference equations of second-order bandreject filter are:

$$x(n) = u(n) - d(1-c)x(n-1) + cx(n-2)$$

$$y(n) = \frac{1-c}{2}x(n) + d(1-c)x(n-1) + \frac{1-c}{2}x(n-2),$$

and corresponding state and output equations are:

$$\begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} = \begin{bmatrix} -d(1-c) & c \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(n)$$

$$y(n) = \begin{bmatrix} \frac{d(1-c^2)}{2} & \frac{1-c^2}{2} \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \frac{1-c}{2}u(n)$$

9 Second-Order Peak Filter

The difference equations of second-order peak filter are:

$$x(n) = u(n) - d(1-c_{B/C})x(n-1) + c_{B/C}x(n-2)$$

$$y_1(n) = -c_{B/C}x(n) + d(1-c_{B/C})x(n-1) + x(n-2)$$

$$y(n) = \frac{H_0}{2}[u(n) - y_1(n)] + u(n).$$

and corresponding state and output equations are:

$$\begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} = \begin{bmatrix} -d(1-c_{B/C}) & c_{B/C} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(n)$$

$$y(n) = \begin{bmatrix} \frac{H_0}{2}(c_{B/C}^2 - 1)d & \frac{H_0}{2}(c_{B/C}^2 - 1) \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + [\frac{H_0}{2}(1 + c_{B/C}) + 1]u(n)$$