Second-Order Bandpass/Bandreject Filter Design

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The signal can be seen as a set of partials having different frenquencies and amplitudes. The filter can modify the amplitude of partials according to their frenquency. The two types of filters can be defined according to the following classification:

- **Bandpass (BP)** filters select frenquencies between a lower cut-off frenquency f_{cl} and a higher cut-off frenquency f_{ch}. Frenquencies below f_{cl} and frenquencies higher than f_{ch} are attenuated.
- **Bandreject (BR)** filters attenuate frenquencies between a lower cut-off frenquency f_{cl} and a higher cut-off frenquency f_{ch}. Frenquencies below f_{cl} and frenquencies higher than f_{ch} are passed.

The bandpass can produce effects such as the imitation of a telephone line or of a mute on an acoustical instrument; the bandreject can divide the audible spectrum into two bands that seem to be uncorrelated.

Second-order bandpass and bandreject filters can be described by the following transfer function

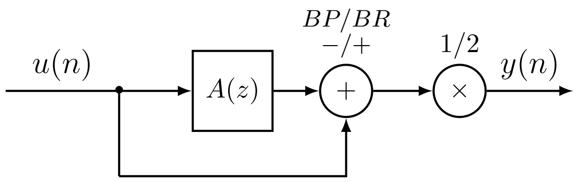
$$H(z) = \frac{1}{2}[1 \mp A(z)] \quad (BP/BR - /+)$$

$$A(z) = \frac{-c + d(1-c)z^{-1} + z^{-2}}{1 + d(1-c)z^{-1} - cz^{-2}}$$

$$c = \frac{\tan(\pi f_b/f_S) - 1}{\tan(\pi f_b/f_S) + 1}$$

$$d = -\cos(2\pi f_c/f_S),$$

where a tunable second-order allpass $A(z)$ with tuning parameters c and d is used. The minus sign (-) denotes the bandpass operation and the plus sign (+) the bandreject operation. The block diagram in following figure represents the operations involved in performing the bandpass/bandreject filtering.



The difference equations of second-order bandpass filter are

$$x(n) = u(n) - d(1-c)x(n-1) + cx(n-2)$$

$$y(n) = \frac{1+c}{2}x(n) - \frac{1+c}{2}x(n-2),$$

and corresponding state and output equations are

$$\begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} = \begin{bmatrix} -d(1-c) & c \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(n)$$

$$y(n) = \begin{bmatrix} \frac{d(c^2-1)}{2} & \frac{c^2-1}{2} \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \frac{1+c}{2}u(n).$$

A second-order bandpass filter implementation can be obtained by the following **Matlab** code.

```
function y = apbandpassunit(audio, para)
% Applies a bandpass filter to the input signal.
% para(1) is the normalized center frequency in (0,1), i.e. 2*fc/fs.
% para(2) is the normalized bandwidth in (0,1) i.e. 2*fb/fs.
c = (tan(pi*para(2)/2)-1) / (tan(pi*para(2)/2)+1);
d = -cos(pi*para(1));
x = [0; 0];
x_1 = 0;
A = [-d*(1-c), c; 1, 0];
B = [1; 0];
C = [d*(c^2-1)/2, (c^2-1)/2];
D = (1+c)/2;
for n=1:length(audio)
    x_1 = A * x + B * audio(n);
    y(n) = C * x + D * audio(n);
    x = x_1;
end
```

The difference equations of second-order bandreject filter are

$$x(n) = u(n) - d(1-c)x(n-1) + cx(n-2)$$

$$y(n) = \frac{1-c}{2}x(n) + d(1-c)x(n-1) + \frac{1-c}{2}x(n-2),$$

and corresponding state and output equations are

$$\begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} = \begin{bmatrix} -d(1-c) & c \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(n)$$

$$y(n) = \begin{bmatrix} \frac{d(1-c^2)}{2} & \frac{1-c^2}{2} \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \frac{1-c}{2}u(n).$$

A second-order bandreject filter implementation can be obtained by the following **Matlab** code.

```
function y = apbandrejectunit(audio, para)
% Applies a bandreject filter to the input signal.
% para(1) is the normalized center frequency in (0,1), i.e. 2*fc/fs.
% para(2) is the normalized bandwidth in (0,1) i.e. 2*fb/fs.
```

```
c = (tan(pi*para(2)/2)-1) / (tan(pi*para(2)/2)+1);
d = -cos(pi*para(1));
x = [0; 0];
x_1 = 0;
A = [-d*(1-c), c; 1, 0];
B = [1; 0];
C = [d*(1-c^2)/2, (1-c^2)/2];
D = (1-c)/2;
for n=1:length(audio)
    x_1 = A * x + B * audio(n);
    y(n) = C * x + D * audio(n);
    x = x_1;
end
```