Wah-Wah Filter Design

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The parametric filters discussed in the previous documents allow the time-varying control of the filter parameters gain, cut-off frequency, and bandwidth or Q factor. Special applications of time-varying audio filters play important roles in music signal processing, one of them is wah-wah filter.

The wah-wah effect is produced mostly by foot-controlled signal processors containing a bandpass filter with variable center frequency and a small bandwidth. Moving the pedal back and forth changes the bandpass center frequency. The "wah-wah" effect is then mixed with the direct signal. This effect leads to a spectrum shaping similar to speech and produces a speech-like "wah-wah" sound.

Instead of manually changing the center frequency, it is also possible to let a low-frequency oscillator control the center frequency, which in turn is controlled based on parameters derived from the input signal. Such an effect is called an auto-wah filter. If the effect is combined with a low-frequency amplitude variation, which produces a tremolo, the effect is denoted a tremolo-wah filter. Replacing the unit delay in the bandpass filter by an M tap delay leads to the M-fold wah-wah filter. M bandpass filters are spread over the entire spectrum and simultaneously change their center frequency. Effects with M-fold wah-wah filter are shown in following table.

	M	Q^{-1}/f_m	Δf
Wah-Wah	1	$-$/3kHz	200Hz
M-fold Wah-Wah	5-20	0.5/$-$	200-500Hz
Bell effect	100	0.5/$-$	100Hz

Based on the measured shape of the amplitude response (a bandpass-resonator characteristic), and knowledge (from circuit schematics), the transfer function of the second-order bandpass can be presumed to be of the form

$$H(s) = g\frac{s-\xi}{\left(\frac{s}{\omega_r}\right)^2 + \frac{2}{Q}\left(\frac{s}{\omega_r}\right) + 1},$$

where g is an overall gain factor, ξ is a real zero at or near dc (the other being at infinity), ω_r is the pole resonance frequency, and Q is the so-called "quality factor" of the resonator. The measurements reveal that ω_r, Q, and g all vary significantly with pedal angle θ.Good choices for these functions are as shown in following code block, where the controlling wah variable is the pedal-angle normalized to a [0, 1] range.