First-Order Low/High-Frequency Shelving Filter Design

Yangang Cao

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In contrast to low/highpass and bandpass/reject filters, which attenuate the audio spectrum above or below a cut-off frequency, equalizers shape the audio spectrum by enhancing certain frequency bands while others remain unaffected. They are typically built by a series connection of first-and second-order shelving and peak filters, which are controlled independently.

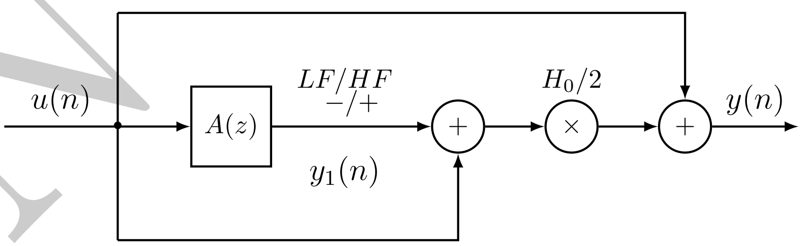
Shelving filters boost or cut the low- or high-frequency bands with the parameters cut-off frequency f_c and gain G. Similar to the first-order lowpass/highpass filters design, first-order low/high frequency shelving filters can be constructed based on a first-order allpass, yielding the transfer function

$$H(z) = 1 + \frac{H_0}{2}[1 \pm A(z)] \quad (LF/HF + /-)$$

with the first-order allpass

$$A(z) = \frac{z^{-1} + c_{B/C}}{1 + c_{B/C} z^{-1}}.$$

The block diagram in following figure shows a first-order low/high-frequency shelving filter.



The difference equations of first-order low shelving filter are

$$x(n) = u(n) - c_{B/C} x(n-1)$$

$$y_1(n) = c_{B/C} x(n) + x(n-1)$$

$$y(n) = \frac{H_0}{2}[u(n) - y_1(n)] + u(n),$$

and corresponding state and output equations are

$$x(n) = -c_{B/C}x(n-1) + u(n)$$

$$y(n) = \frac{H_0}{2}(1-c^2)x(n-1) + [\frac{H_0}{2}(1+c)+1]u(n).$$

The gain G in dB for low/high frequencies can be adjusted by the parameter

$$H_0 = V_0 - 1 \quad \text{with} \quad V_0 = 10^{G/20}.$$

The cut-off frenquency parameters, c_B for boost and c_C for cut, for a first-order low-frequency shelving filter can be calculated as

$$c_B = \frac{\tan(\pi f_c/f_S) - 1}{\tan(\pi f_c/f_S) + 1}$$

$$c_C = \frac{\tan(\pi f_c/f_S) - V_0}{\tan(\pi f_c/f_S) + V_0}.$$

An implementation of first-order low-frenquency shelving filter is given in following **Matlab** code.

```
function y = lowshelvingunit(audio, para)
% Applies a low-frequency shelving filter to the input signal.
% para(1) is the normalized cut-off frequency in (0,1), i.e. 2*fc/fs
% para(2) is the gain in dB
V0 = 10^(para(2)/20); H0 = V0 - 1;
if para(2) ≥ 0
    c = (tan(pi*para(1)/2)-1) / (tan(pi*para(1)/2)+1);     % boost
else
    c = (tan(pi*para(1)/2)-V0) / (tan(pi*para(1)/2)+V0);   % cut
end
x = 0;
x_1 = 0;
for n=1:length(audio)
    x_1 = -c * x + audio(n);
    y(n) = H0 / 2 * (1-c^2) * x + [H0 / 2 * (1+c) + 1] * audio(n);
    x = x_1;
end
```

The difference equations of first-order high-shelving filter are

$$x(n) = u(n) - c_{B/C}x(n-1)$$

$$y_1(n) = c_{B/C}x(n) + x(n-1)$$

$$y(n) = \frac{H_0}{2}[u(n) + y_1(n)] + u(n),$$

and corresponding state and output equations are

$$x(n) = -c_{B/C}x(n-1) + u(n)$$

$$y(n) = \frac{H_0}{2}(c^2-1)x(n-1) + [\frac{H_0}{2}(1-c)+1]u(n).$$

The gain G in dB for low/high frequencies can be adjusted by the parameter

$$H_0 = V_0 - 1 \quad \text{with} \quad V_0 = 10^{G/20}.$$

The cut-off frenquency parameters, c_B for boost and c_C for cut, for a first-order high-frequency shelving filter can be calculated as

$$c_B = \frac{\tan(\pi f_c/f_S) - 1}{\tan(\pi f_c/f_S) + 1}$$

$$c_C = \frac{V_0 \tan(\pi f_c/f_S) - 1}{V_0 \tan(\pi f_c/f_S) + 1}.$$

An implementation of first-order high-frenquency shelving filter is given in following **Matlab** code.

```
function y = highshelvingunit(audio, para)
% Applies a high-frequency shelving filter to the input signal.
% para(1) is the normalized cut-off frequency in (0,1), i.e. 2*fc/fs
% para(2) is the gain in dB
V0 = 10^(para(2)/20); H0 = V0 - 1;
if para(2) ≥ 0
c = (tan(pi*para(1)/2)-1) / (tan(pi*para(1)/2)+1);      % boost
else
c = (tan(pi*para(1)/2)-V0) / (tan(pi*para(1)/2)+V0);    % cut
end
x = 0;
x_1 = 0;
for n=1:length(audio)
    x_1 = -c * x + audio(n);
    y(n) = H0/2 * (c^2-1) * x + (H0/2 * (1-c) + 1) * audio(n);
    x = x_1;
end
```