Second-Order Peak Filter Design

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In contrast to low/highpass and bandpass/reject filters, which attenuate the audio spectrum above or below a cut-off frequency, equalizers shape the audio spectrum by enhancing certain frequency bands while others remain unaffected. They are typically built by a series connection of first-and second-order shelving and peak filters, which are controlled independently.

Peak filters boost or cut mid-frequency bands with parameters center frequency fc, bandwidth fb and gain G. One often-used filter type is the constant Q peak filter. The Q factor is defined by the ratio of the bandwidth to center frequency $Q = \frac{fc}{fb}$. The center frequency of peak filters is then tuned while keeping the Q factor constant. This means that the bandwidth is increased when the center frequency is increased and vice versa.

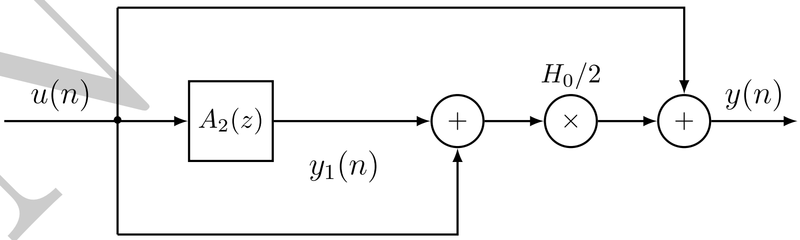
Similarly to first-order shelving filters, a second-order peak filter based on second-order allpass filter is given by the transfer function

$$H(z) = 1 + \frac{H_0}{2}[1 - A_2(z)],$$

where

$$A_2(z) = \frac{-c_{B/C} + d(1 - c_{B/C})z^{-1} + z^{-2}}{1 + d(1 - c_{B/C})z^{-1} - c_{B/C}z^{-2}}$$

is a second-order allpass filter. The block diagram in following figure shows the second-order peak filter, the part of $A_2(z)$ can be reviewed in previous document,



which leads to the difference equations

$$x(n) = u(n) - d(1 - c_{B/C})x(n-1) + c_{B/C}x(n-2)$$

$$y_1(n) = -c_{B/C}x(n) + d(1 - c_{B/C})x(n-1) + x(n-2)$$

$$y(n) = \frac{H_0}{2}[u(n) - y_1(n)] + u(n),$$

and corresponding state and output equations are

$$\begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} = \begin{bmatrix} -d(1-c_{B/C}) & c_{B/C} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(n)$$

$$y(n) = \begin{bmatrix} \frac{H_0}{2}(c_{B/C}^2 - 1)d & \frac{H_0}{2}(c_{B/C}^2 - 1) \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + [\frac{H_0}{2}(1 + c_{B/C}) + 1]u(n).$$

The center frenquency parameter d and the coefficient H_0 are given by

$$d = -\cos(2\pi f_c/f_S)$$

$$V_0 = H(e^{j2\pi f_c/f_S}) = 10^{G/20}$$

$$H_0 = V_0 - 1.$$

The bandwidth f_b is adjusted through the parameters c_B and c_C for boost and cut given by

$$c_B = \frac{\tan(\pi f_b/f_S) - 1}{\tan(\pi f_b/f_S) + 1}$$

$$c_C = \frac{\tan(\pi f_b/f_S) - V_0}{\tan(\pi f_b/f_S) + V_0}.$$

A possible peak filter implementation using this approach is given in the following **Matlab** code.

```
function y = peakfiltunit(audio, para)
% Applies a peak filter to the input signal.
% para(1) is the normalized center frequency in (0,1), i.e. 2*fc/fs.
% para(2) is the normalized bandwidth in (0,1), i.e. 2*fb/fs.
% prar(3) is the gain in dB.
V0 = 10^(para(3)/20); H0 = V0 - 1;
if para(3) ≥ 0
     c = (tan(pi* para(2)/2)-1) / (tan(pi* para(2)/2)+1);     % boost
else
     c = (tan(pi* para(2)/2)-V0) / (tan(pi* para(2)/2)+V0);   % cut
end;
d = -cos(pi*para(1));
x = [0; 0];
x_1 = 0;
A = [-d*(1-c), c; 1, 0];
B = [1; 0];
C = [H0/2*(c^2-1)*d, H0/2*(c^2-1)];
D = [H0/2*(1+c) + 1];
for n=1:length(audio)
     x_1 = A * x + B * audio(n);
     y(n) = C * x + D * audio(n);
     x = x_1;
end
```