Part 01 - Introduction

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Contents

1 Basics

1.1 Signals, Systems, and Signal Processing

- Signal: Physical quantity that varies independent variable or variables
- System: Device that performs an operation on a signal
- Algrithm: Method or set of rules for implementing the system
- Signal Processing: Passing a signal through a system
- A/D Converter: Converting the signal from analog to digital
- D/A Converter: Converting the signal from digital to analog

1.2 Classification of Signals

- Multichannel and Multidimensional Signals
  - Multichannel Signals: Signal with one independent variable
  - Multidimensional Signals: Signal with multiple independent variables
- Continuous-Time Versus Discrete-Time Signals
  - Continuous-time Signals: Signal with every value of time
  - Discrete-time Signals: Signal with specific values of time
- Continuous-Valued Versus Discrete -Valued Signals
  - Continuous-valued Signals: Signal with continuous values
  - Discrete-valued Signals: Signal with discrete values
- Deterministic Versus Random Signals
  - Deterministic Signals: Signals that can be uniquely described
  - Random Signals: Signals evolve in an unpredictable manner

2 The Concept of Frequency in Continuous-Time and Discrete-Time Signals

We expect that the nature of time (continuous or discrete) would affect the nature of the frequency accordingly.

2.1 Continuous-Time Sinusoidal Signals

A simple harmonic oscillation:

$$x_a(t) = A\cos(\Omega t + \theta), -\infty < t < \infty$$

A is the amplitude of sinuoid, Ω is the frequency, and θ is the phase in radians. Instead of Ω, we often use the frequency F in cycles per second or hertz(Hz), where

$$\Omega = 2\pi F$$

The analog sinusoidal signal is characterized by the following properties:

- $x_a(t)$ is periodic if $x_a(t+T_p) = x_a(t)$, where $T_p = 1/F$ is the fundamental period of the sinusoidal signal.
- Continuous-time sinusoidal signals with distinct frequencies are themselves distinct.
- Increasing the frequency F results in an increase in the rate of oscillation of the signal.

Corresponding complex exponential form

$$x_a(t) = Ae^{j(\Omega t+\theta)}$$

This can easily be seen by expressing these signals in terms of sinusoids using the Euler indentity

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi$$

2.2 Discrete-Time Sinusoidal Signals

A discrete-time sinusoidal signal:

$$x(n) = A\cos(\omega n + \theta), -\infty < n < \infty$$

where n is an integer variable, called the sample number, A is the amplitude of the sinusoid, ω is the frequency in radians per sample, and θ is the phase in radians.

The discrete-time sinusoids are characterized by following properties:

- A discrete-time sinusoid is periodic only if its frequency f is a rational number.
- Discrete-time sinusoids whose frequencies are separated by an integer multiple of 2π are identical.
- The highest rate of oscillation in a discrete-time sinusoid is attained when $\omega = \pi$(or $\omega = -\pi$) or, equivalently, $f = \frac{1}{2}$ (or $f = -\frac{1}{2}$).

Usually, we choose the range $0 \leqslant \omega \leqslant 2\pi$ or $-\pi \leqslant \omega \leqslant \pi$ $(0 \leqslant f \leqslant 1, -\frac{1}{2} \leqslant f \leqslant \frac{1}{2})$, which we call the *fundamental range*.

2.3 Harmonically Related Complex Exponentials

These are sets of periodic complex exponentials with fundamental frequencies that are multiples of a single positive frequency. We can construct a linear combination of harmonically related complex exponentials by adding continuous-time, harmonically related exponentials, which is called *Fourier series expansion*. For discrete-time, $f_0 = 1/N$, they are only N distinct periodic complex exponentials, this is called *Fourier series*.

3 Analog-to-Digital and Digital-to-Analog Conversion

We view A/D conversion as a three-step process: sampling, quantization and coding. The accuracy of D/A conversion depends on the quality of D/A conversion, a simple form of D/A conversion is called zero-order hold or staircase approximation. The sampling rate is sufficiently high to avoid the problem commonly called *aliasing*.

3.1 Sampling of Analog Signals

We limit our discussion to *periodic* or $uniform$ *sampling*, $F_s = 1/T$ is called the *sampling rate*(hertz). F is frequency of analog signal, the frequency variables F and f are linearly related as

$$f = \frac{F}{F_s}$$

and f is also called *relative* or *normlized* $frequency$.

We observe that the fundamental difference between continuous-time and discrete-time signals is in their range of values of the frequency variable F and f, or Ω and ω.

$$F_{max} = \frac{F_s}{2} = \frac{1}{2T}$$
$$\Omega_{max} = \pi F_s = \frac{\pi}{T}$$

For example, $F_2 = 10Hz$ is an alias of the frequency $F_1 = 50Hz$ when $F_s = 40Hz$. The relationship is

$$F_k = F_0 + kF_s,\ k = \pm 1, \pm 2, ...$$

$F_s/2$ is called $folding\ frequency$.

3.2 The Sampling Theorem

F_s is selected that

$$F_s > 2F_{max}$$

where F_{max} is the largest frequency component in the analog signal. The sampling rate $F_N = 2B = 2F_{max}$ is called the $Nyquist$ *rate*.

3.3 Quantization of Continuous-Amplitude Signals

- Quantization: The process of converting a discrete-time continuous-amplitude signal into a digital signal by expressing each sample value as a finite number of digits
- Quantization Error: Difference between the quantized value and the actual sample value
- Quantization Level: Values allowed in the digital signal
- Quantization step size or resolution: Distance δ between two successive quantization levels
- Two ways of quantization: Truncation and rounding

3.4 Quantization of Sinusoidal Signals

Sinusoids are used as test signals in A/D converters. If the sampling rate F_s satisfies the sampling theorem, quantization is the only error in the A/D conversion process. The quality of the output of the A/D converter is usually measured by the $signal - to - quantization\ noise\ ratio (SQNR)$.

$$\text{SQNR} = \frac{P_x}{P_q} = \frac{3}{2} \cdot 2^{2b}$$

Expressed in decibels(dB), the SQNR is

$$\text{SQNR(dB)} = 10 \log_{10} \text{SQNR} = 1.76 + 6.02b$$

3.5 Coding of Quantized Samples

The number of bits required in the coder is the smallest integer greater than or equal to $\log_2 L$.