FFT

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1 Fourier Series (FS)

$x(t)$ is a continuous-time periodic signal, we suppose that

$$x(t) = \sum_{k=-\infty}^{\infty} c_k \mathrm{e}^{j2\pi kF_0 t}$$

$$c_k = \frac{1}{T_p}\int_{T_p} x(t)\mathrm{e}^{j2\pi kF_0 t}\mathrm{dt}$$

$$P_x = \frac{1}{T_p}\int_{T_p} |x(t)|^2\mathrm{dt} = \sum_{k=-\infty}^{\infty} |c_k|^2$$

2 Fourier Transform (FT)

$x(t)$ is a continuous-time aperiodic signal

$$c_k = \frac{1}{T_p}\int_{-\infty}^{\infty} x(t)\mathrm{e}^{j2\pi kF_0 t}\mathrm{dt}$$

we define

$$X(F) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-j2\pi Ft}\mathrm{dt}$$

we have

$$c_k = \frac{1}{T_p} X(kF_0)$$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\mathrm{dt} = \int_{-\infty}^{\infty} |X(F)|^2\mathrm{dF}$$

$$S_{xx}(F) = |X(F)|^2$$

3 Discrete Time Fourier Series (DTFS)

$x(n)$ is a discrete-time periodic signal

$$x(n+N) = x(n)$$

$$x(n) = \sum_{k=0}^{N-1} c_k \mathrm{e}^{j2\pi kn/N}$$

$$c_k = c_{k+N} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\mathrm{e}^{-j2\pi kn/N}$$

$$P_x = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{k=0}^{N-1} |c_k|^2$$

4 Discrete Time Fourier Transform (DTFT)

$x(n)$ is a discrete-time aperiodic signal

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\mathrm{e}^{-j\omega n}$$

$$X(\omega + 2\pi k) = X(\omega)$$

$$E_x = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega m)|^2$$

5 Discrete Fourier Transform (DFT)

$$X(k) = \sum_{n=0}^{N-1} x(n)\mathrm{e}^{-j2\pi kn/N} \quad k = 0, 1, \ldots, N-1$$

$$X(k+N) = X(k)$$

we suppose $x(n)$ is real series

$$X(N-k) = X^*(k) = X(-k)$$

$$|X(N-k)| = |X(k)|$$

$$\angle X(N-k) = -\angle X(k)$$