Sequences and Series

Baboo J. Cui

June 10, 2019

Contents

1 Sequence

A sequence is a list of numbers written in a definite order:

$$a_1, a_2, \cdots, a_n$$

- n could be either **finite** or **infinite**
- a_i is the i-th **term** of the sequence
- can be defined as a function whose domain is $\mathbb{Z}^+$
- can be written as $\{a_n\}$ or $\{a\}_{n=1}^{+\infty}$

1.1 Limit and Convergence

A sequence $\{a_n\}$ has the **limit** L, and it is denoted as

$$\lim_{n\to\infty} a_n = L$$

if we can make a_n as close to L as taking n sufficiently large. If L exists, the sequence is said to be **convergent**, otherwise **divergent**.

1.2 Precise Definition of Limit of Sequence

A sequence $\{a_n\}$ has the **limit** L, and it is denoted as

$$\lim_{n\to\infty} a_n = L$$

if for every $\epsilon > 0$, there is a corresponding integer N such that if $n > N$

$$|a_n - L| < \epsilon$$

And

$$\lim_{n\to\infty} a_n = \infty$$

indicates that for every positive number M, there is an integer N such that if $n > N$, then $a_n > M$

1.3 Limit of a Sequence by Function

If

$$\lim_{x\to\infty} f(x) = L$$

and

$$f(n) = a_n, \forall n \in \mathbb{Z}^+$$

then

$$\lim_{n\to\infty} a_n = L$$

1.4 Limit Laws for Sequences

If $\{a_n\}$ and $\{b_n\}$ are convergent and c is a constant:

- $\lim c = c$
- $\lim(a \pm b) = \lim a \pm \lim b$
- $\lim ca = c \lim a$
- $\lim(ab) = \lim a \lim b$
- $\lim(a/b) = \lim a / \lim b$
- $\lim a^p = (\lim a)^p$

1.5 Squeeze Theorem

If $a_n \leq b_n \leq c_n$, for $n > n_0$ and

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$$

then

$$\lim_{n\to\infty} b_n = L$$

1.6 Absolute Convergence

If

$$\lim_{n\to\infty} |a_n| = 0$$

then

$$\lim_{n\to\infty} a_n = 0$$

1.7 Convergence of a Sequence Applied to Function

If

$$\lim_{n\to\infty} a_n = L$$

and a function f is continuous at L, then

$$\lim_{n\to\infty} f(a_n) = f(L)$$

1.8 title

1.9 title

1.10 title

2 Extra

2.1 Fibonacci Sequence

It is defined recursively by the conditions:

$$a_1 = 1 \qquad a_2 = 1 \qquad a_n = a_{n-1} + a_{n-2}, n \geq 3$$