# Introduction

## Yangtao Ge

June 18, 2019

## 1 Preface

## 1.1 Purpose

 $\frac{\text{How does the books go:}}{Specific\ problems = Coding + Math\ Analysing}$ 

### Knowlegde preferred:

- intermediate programming(OOP & recursion)
- discrete Math Ref: COMP0147 & "Discrete Mathematics and Its Application"

### 1.2 Overview

#### $Part1: Basic\ Knowlegde$

- Chapter 1: Reviewing material on discrete math & recursion + Java related(out of date, not focus on)
- Chapter 2: Algorithm analysis (important and doing exercise)
- Chapter 3: List, Stack and Queues
- Chapter 4: Tress (Basic, AVL & game trees refer to advanced part)

- Chapter 5: Hash tables
- Chapter 6: Priority Queues
- Chapter 7: Sorting
- Chapter 8: Disjoint set
- Chapter 9: Graph Algorithm

## $Part 2: Advanced\ Knowleg de$

- Chapter 10: Algorithm on problem-solving techniques (Lots of Examples)
- Chapter 11: amortized analysis(Three data structure from C4 & C6 + Fibonacci heap)
- Chapter 12: Search tree Algorithms (advanced trees)

#### 1.3 Exercise

From easy to hard(marked with \*), Last question demo the whole Chapter Ref: www.pearsonhighered.com/cssupport

# 2 Chapter 1: Introduction

#### 2.1 What is the Book About?

Running code fast and analysis them

N.B. detail contents for every chapter are in the previous section

#### 2.2 Mathematics Review

Ref: pp.3-8

#### 2.2.1 Exponents

$$X^{A}X^{B} = X^{A+B}$$

$$\frac{X^{A}}{X^{B}} = X^{A-B}$$

$$(X^{A})^{B} = X^{AB}$$

$$X^{N} + X^{N} = 2X^{N} \neq X^{2N}$$

$$2^{N} + 2^{N} = 2^{N+1}$$

#### 2.2.2 Logarithms

In computer science, all Logarithms are to the 'base 2' unless specified otherwise.

**Definition 2.1.**  $X^A = B$  iff  $\log_X B = A$ 

Theorem 2.1.

$$\log_A B = \frac{\log_C B}{\log_C A}; \ A, B, C > 0, A \neq 1$$

Theorem 2.2.

$$\log AB = \log A + \log B$$
;  $A, B > 0$ 

#### **2.2.3** Series

**Theorem 2.3** (Sum of Geometric Progression).

$$\sum_{i=0}^{N} A^{i} = \frac{A^{N+1} - 1}{A - 1}$$

Theorem 2.4 (Infinite Geometric Series).

$$\sum_{i=0}^{N} A^i \le \frac{1}{1-A}$$

Hence, when  $N \to \infty$ , we have:

$$\sum_{i=0}^{\infty} ar^i = \frac{a}{1-i}$$

Theorem 2.5 (Sum of Arithmetic Series).

$$\sum_{i=1}^{N} i = \frac{N(N+1)}{2} \approx \frac{N^2}{2}$$

Theorem 2.6 (Inference of Arithmetic Progression).

$$\sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{2} \approx \frac{N^3}{3}$$

$$\sum_{i=1}^{N} i^{k} \approx \frac{N^{k+1}}{|k+1|}; \ k \neq -1$$

Theorem 2.7 (Harmonic Numbers).

$$H_N = \sum_{i=1}^{N} \frac{1}{i} \approx \log_e N$$

The error is called **Euler's constant**, which value is  $\gamma \approx 0.57721566$ 

#### 2.2.4 Modular Arithmetic

**Definition 2.2.** We say A os *congruent* to B modulo N, written in  $A \equiv B \pmod{N}$ 

**Theorem 2.8.** If a prime number N divides a product of two numbers, it divides at least one of the two numbers

$$ab \equiv 0 \pmod{N}$$
 iff  $a \equiv 0 \pmod{N}$  or  $b \equiv 0 \pmod{N}$ 

**Theorem 2.9.** If N is prime, then the equation  $ax \equiv 1 \pmod{N}$  has a unique solution (mod N) for all 0 < a < N

This solution 0 < x < N is the *multiplicative inverse* 

#### 2.2.5 The P Word

Three common ways of proving statements in data structure analysis:

- Proof by induction
- Proof by contradiction
- Proof by counterexample

Ref: pp.6 - 8 & COMP 0147 & COMP 0003(1)

#### 2.3 A Brief Introduction to Recursion

Ref: pp. 8-12 & COMP 0002 Haskell & COMP0005 Algorithm Basic Rules of recursion:

- Base cases: you must have some base cases, which can be solved without recursion
- Making progress: For the case that are to be solved recursively, the recursive call must always be to a case that makes progress toward a base case (from base case)
- Design rule: Assume that all the recursive calls work
- Compound interest rule: Never duplicate work by solving the same instance of a problem in separate recursive calls

## 2.4 Implementing Generic Components

Review after finish Core Java 'Generic Programming'

## 2.5 Function Objects

Review after finish Core Java 'Generic Programming'