Note Template

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October 7, 2021

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Conventions

$\mathbb{F}$ denotes either $\mathbb{R}$ or $\mathbb{C}$.
$\mathbb{N}$ denotes the set $\{1, 2, 3, \ldots\}$ of natural numbers (excluding 0).

1 Sample Chapter

Let's dive right in!

1.1 Some Definitions

Definition 1.1. The **derivative** of a function $f : I \to \mathbb{R}$ at $a \in I$ is given by:

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

You know those awesome commutative diagrams?

$$\begin{tikzcd} A \arrow[r, "p"] \arrow[d, "q"'] & B \arrow[d, "r"] \\ C \arrow[r, "s"'] & D \end{tikzcd}$$

The derivative has *nothing* to do with them!

Proposition 1.2. If f is differentiable at a, then f is continuous at a.

Proof. Exercise (but only because this is a template). 😎

The converse of Proposition 1.2 is not true in general.

Examples.

1. $f(x) = |x|$
2. $f(x) = \begin{cases} \sin(x) & x \geq 0 \\ 0 & x < 0 \end{cases}$

Theorem 1.3. The following statements are true:

1. First statement
2. Second statement

Proof.

1. Trivial.
2. Trivial. 😎

Corollary 1.4. We are both very lucky to have each other as a collaborator.

Proof. We simply note that:

$$\frac{1}{1} + \frac{1}{1} \gg \frac{1}{1}$$

😎

Remark. This corollary is also obvious from empirical evidence.

Lemma 1.5. $(a + b)^2 = a^2 + 2ab + b^2$

Proof. Expand the left side. 😎

Remarks.

1. It's also kind of obvious.
2. No extra points for guessing what $(a - b)^2$ is.

Example. $(2 + 4)^2 = 2^2 + 2 \cdot 2 \cdot 4 + 4^2 = 36$

Theorem 1.6 (Pythagoras' Theorem)**.** If c is the hypotenuse of a right triangle and a and b are the other two sides, then $a^2 + b^2 = c^2$.

Proof. Draw a picture and convince yourself. 😎

Pythagoras' theorem helps motivate the study of metric spaces, which you can learn about in [**sekhon**].

A lot of nice integrals can be computed using the residue theorem, see [**taylor**].

A Bonus Material

The `talign` and `talign*` environments work like the `align` and `align*` environments, except they render equations in inline size. For example, `\begin{align*}...\end{align*}` yields:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

While `\begin{talign*}...\end{talign*}` yields:

$$\textstyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

As usual, the purpose of `*` is to prevent numbering of the equation.

Some commands, like `\sumn`, can be used with or without a starting value (the default starting value is 1). For example, `$\sumn\frac{1}{n^2}$` yields $\sum_{n=1}^{\infty} \frac{1}{n^2}$, while `$\sumn[69]\frac{1}{n^2}$` yields $\sum_{n=69}^{\infty} \frac{1}{n^2}$. This can be used in inline mode as well as display mode.