Q1

```
clear all
syms k integer
assumeAlso(k>=0)
syms a b n
assume(a ~= b)
x(k) = a^k;
h(k) = b^k;
```

直接法

```
y1(k) = symsum(h(n)*x(k-n), n, [0, k])
```

y1(k) =

$$\frac{a\,a^k - b\,b^k}{a-b}$$

Z变换法

```
syms z
assumeAlso([a>0, b>0, k>0])
y2(k) = iztrans(ztrans(h, k, z)*ztrans(x, k, z), z, k)
```

y2(k) =

$$\frac{a\,a^k}{a-b} - \frac{b\,b^k}{a-b}$$

Q2

```
clear all
syms f(t) s
Df = diff(f, t);
LDf = laplace(Df, t, s)
```

LDf = $s\,\mathrm{laplace}(f(t), t, s) - f(0)$

Q3

```
clear all
syms x y
eq = x^2+y^2 == 1;
cond = x*y == 2;
s = solve(eq, cond);
```

```
disp([s.x, s.y])
```

$$\begin{pmatrix} -\frac{\sigma_1}{2}+\sigma_3 & -\sigma_1 \\ -\frac{\sigma_2}{2}+\sigma_4 & -\sigma_2 \\ \frac{\sigma_1}{2}-\sigma_3 & \sigma_1 \\ \frac{\sigma_2}{2}-\sigma_4 & \sigma_2 \end{pmatrix}$$

where

$$\sigma_1 = \sqrt{\frac{1}{2}-\frac{\sqrt{15}\,\mathrm{i}}{2}}$$

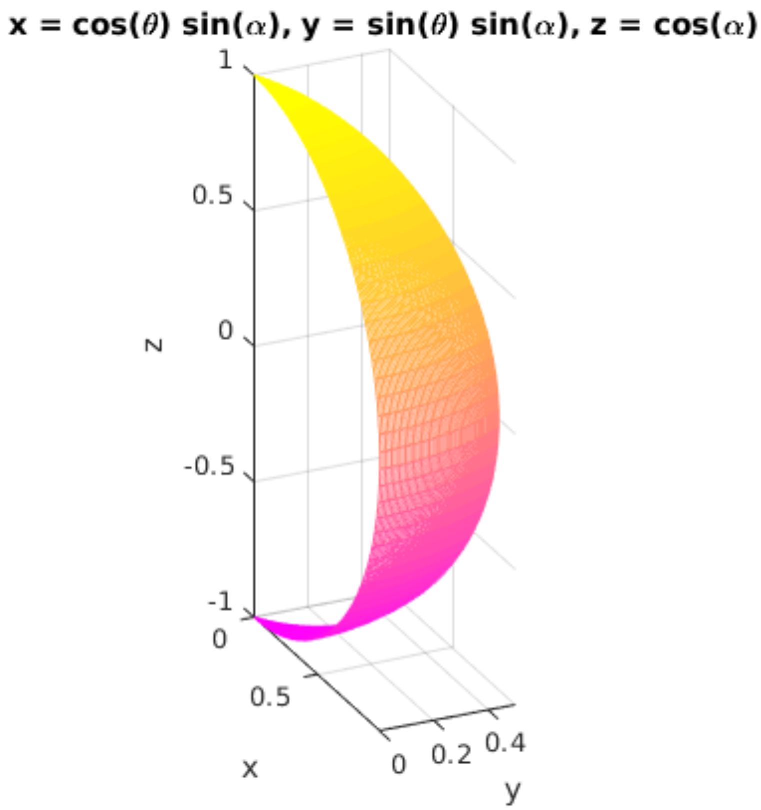
$$\sigma_2 = \sqrt{\frac{1}{2}+\frac{\sqrt{15}\,\mathrm{i}}{2}}$$

$$\sigma_3 = \frac{\left(\frac{1}{2}-\frac{\sqrt{15}\,\mathrm{i}}{2}\right)^{3/2}}{2}$$

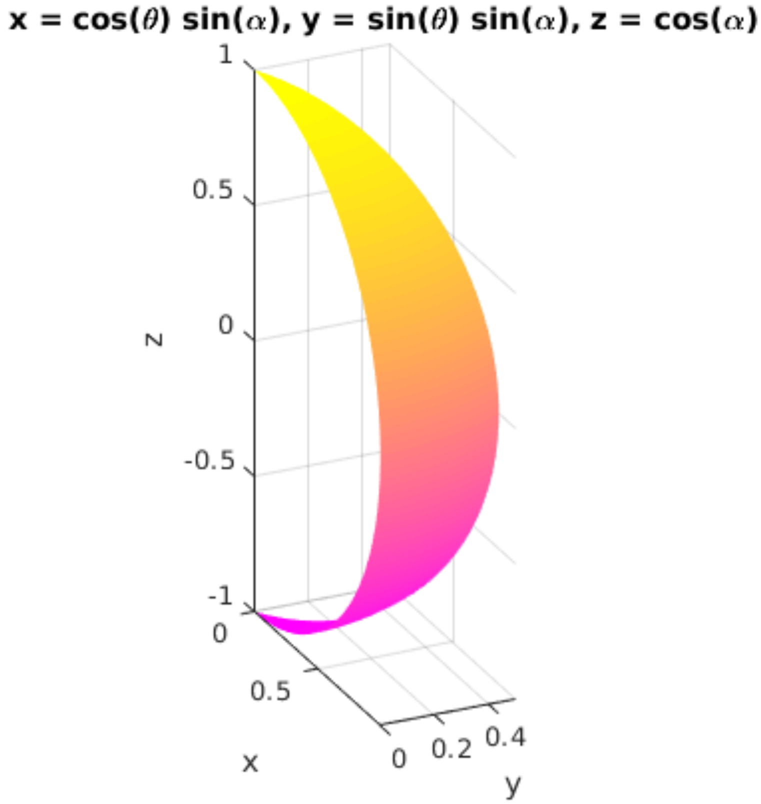
$$\sigma_4 = \frac{\left(\frac{1}{2}+\frac{\sqrt{15}\,\mathrm{i}}{2}\right)^{3/2}}{2}$$

Q4

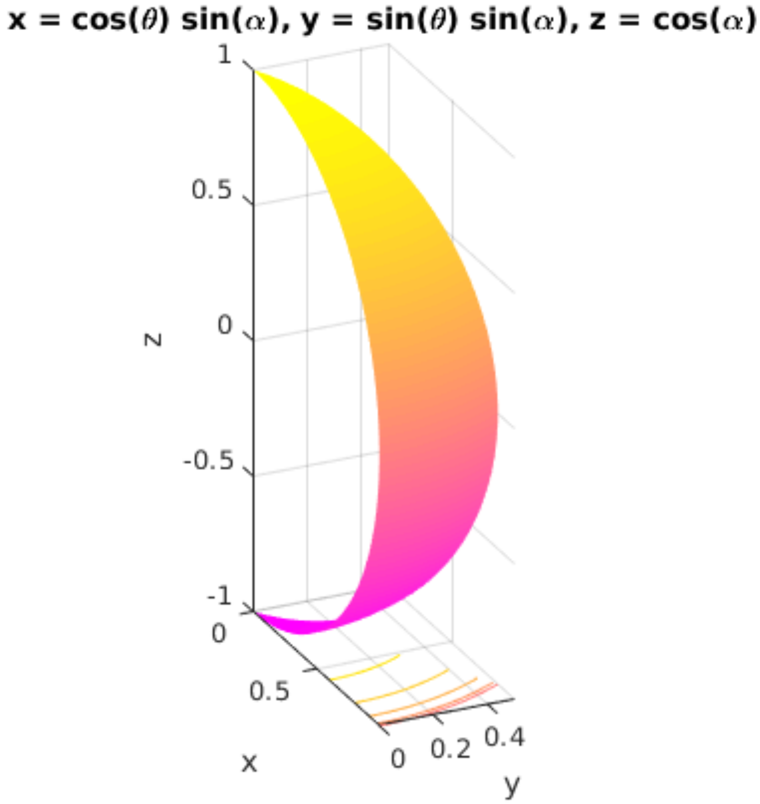
```
clear all
syms alpha theta
x = 'cos(theta)*sin(alpha)';
y = 'sin(theta)*sin(alpha)';
z = 'cos(alpha)';
ezmesh(x, y, z, [0, pi, 0, pi/6])
xlabel('x'), ylabel('y'), zlabel('z')
colormap('spring')
axis equal
view([65, 25])
```




```
ezsurf(x, y, z, [0, pi, 0, pi/6])
shading interp
axis equal
view([65, 25])
```




```
ezsurfc(x, y, z, [0, pi, 0, pi/6])
shading interp
axis equal
view([65, 25])
```



Q5

(1)

(a)

$xy - z = 0$ 的二次项部分的矩阵为 $\begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$，

正交特征向量组为 $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$，

作对应的坐标变换之后得

$\frac{x'^2}{2} - \frac{y'^2}{2} - z' = 0$，

是双曲抛物面方程

(b)

$x^2-2xy+2y+z^2-4=0$的二次项部分的矩阵为$\begin{bmatrix}1 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$，

特征向量组为$\begin{bmatrix}1 & \frac{1-\sqrt{5}}{2} & 0\\ \frac{\sqrt{5}-1}{2} & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$，

作对应的坐标变换之后得

$$\sqrt{5}\,x'^2+\frac{5-3\sqrt{5}}{2}y'^2+(1-\sqrt{5})x'+2y'+z'^2-4=0$$

再做平移 $x''=x'+\frac{1-\sqrt{5}}{2\sqrt{5}}$， $y''=y'+\frac{2}{5-3\sqrt{5}}$ 后得

$$\sqrt{5}\,x''^2+\frac{5-3\sqrt{5}}{2}y''^2+z''^2-3=0$$

是单叶双曲面方程

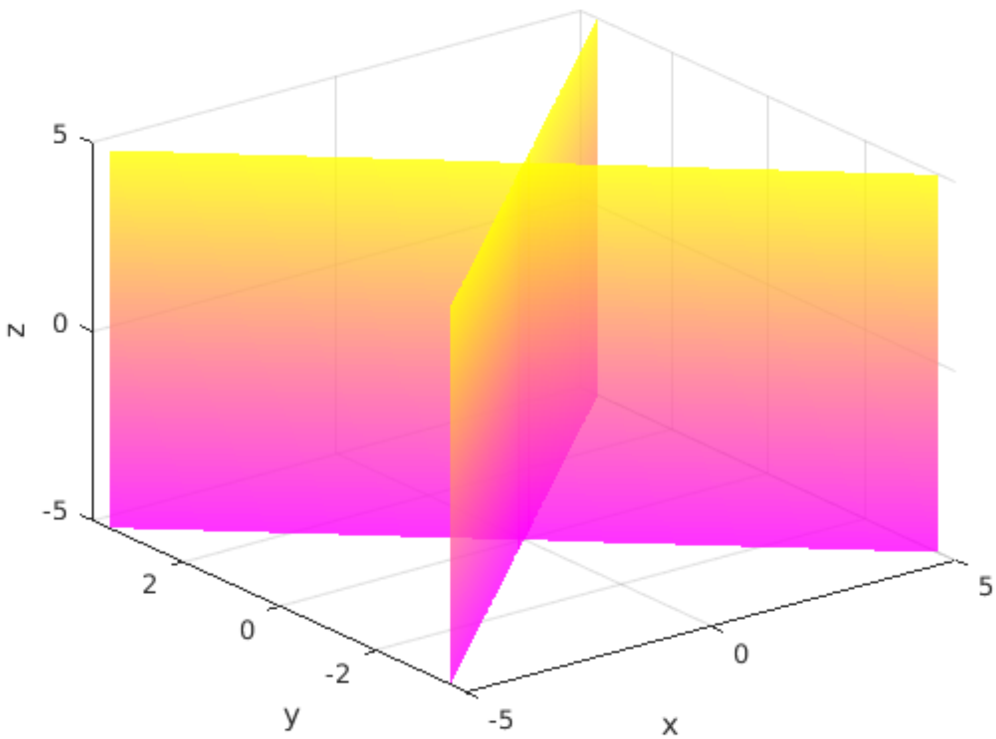
(2)(3)

```
syms x y z
```

1) 平面

$x^2-2y^2=0,\quad z\in R$

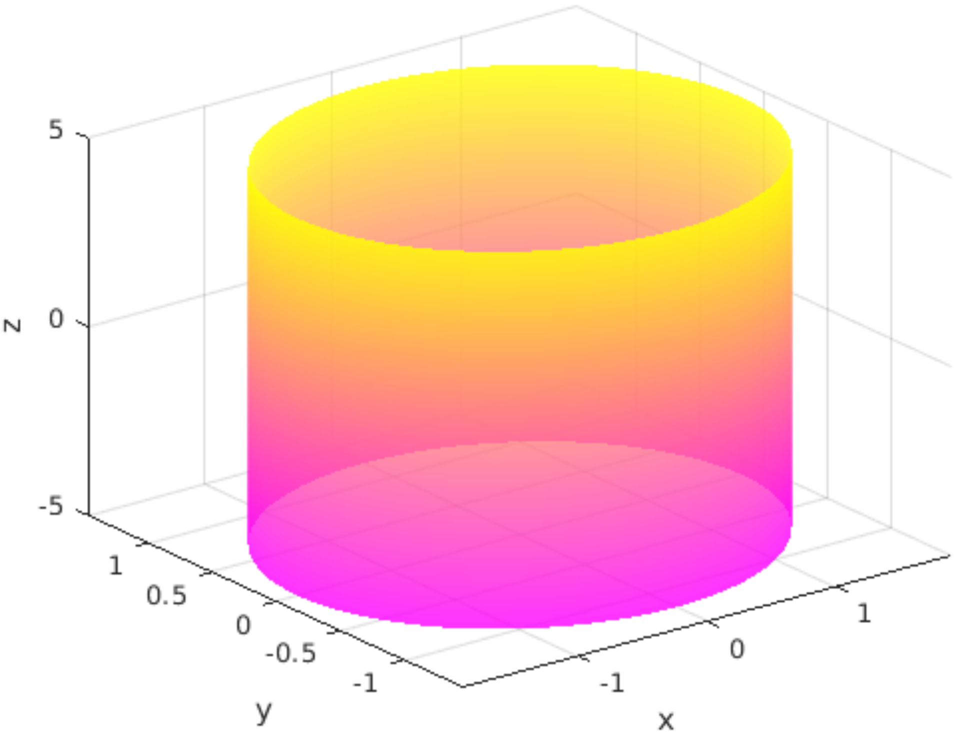
```
f1 = x^2 - 2*y^2;
fimplicit3(f1,'EdgeColor','none','FaceAlpha',.8)
xlabel('x'), ylabel('y'), zlabel('z')
```



2) 柱面

$x^2 + 2y^2 - 3 = 0, \quad z \in R$

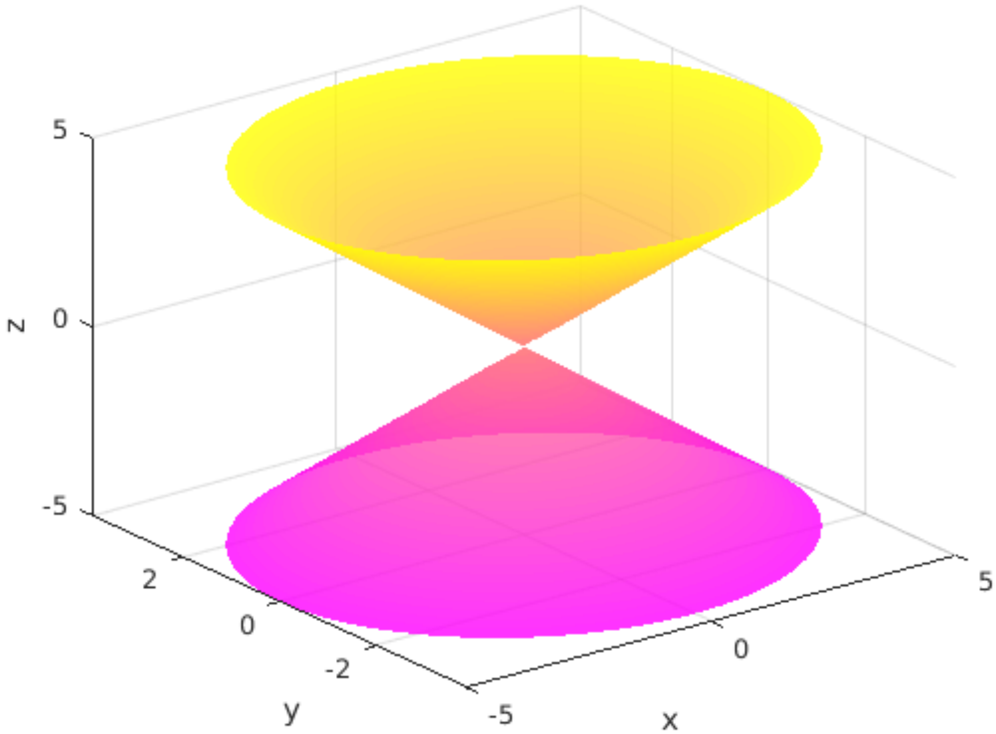
```
f2 = x^2 + 2*y^2 - 3;
fimplicit3(f2,'EdgeColor','none','FaceAlpha',.8)
xlabel('x'), ylabel('y'), zlabel('z')
```



3) 锥面

$x^2 + 2y^2 - z^2 = 0$

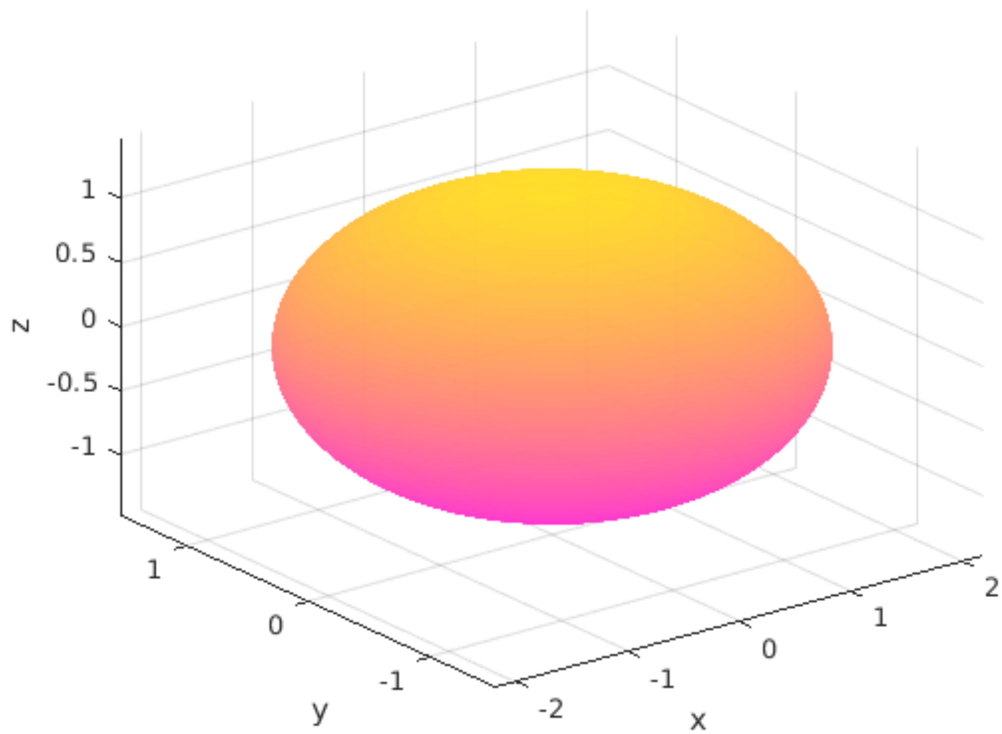
```
f3 = x^2 +2*y^2 - z^2;
fimplicit3(f3,'EdgeColor','none','FaceAlpha',.8)
xlabel('x'), ylabel('y'), zlabel('z')
```



4) 楕球面

$x^2 + 2y^2 + 3z^2 - 4 = 0$

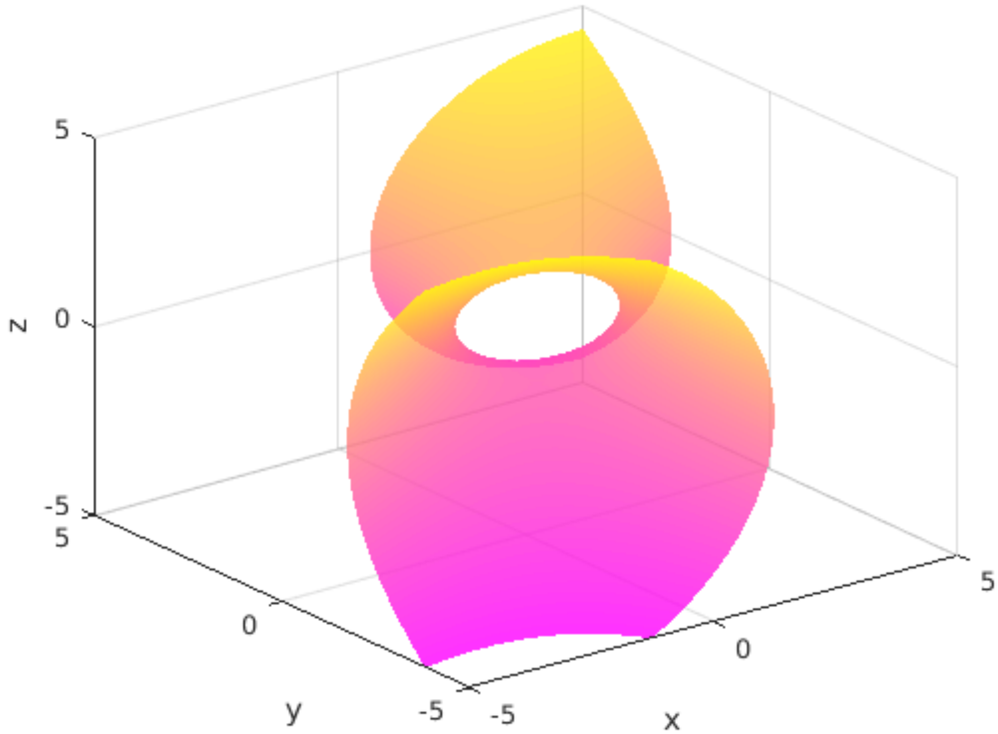
```
f4 = x^2 + 2*y^2 + 3*z^2 - 4;
fimplicit3(f4,'EdgeColor','none','FaceAlpha',.8)
xlabel('x'), ylabel('y'), zlabel('z')
```



5) 单叶双曲面

Q5(1)(b): $x^2 - 2xy + 2y + z^2 - 4 = 0$

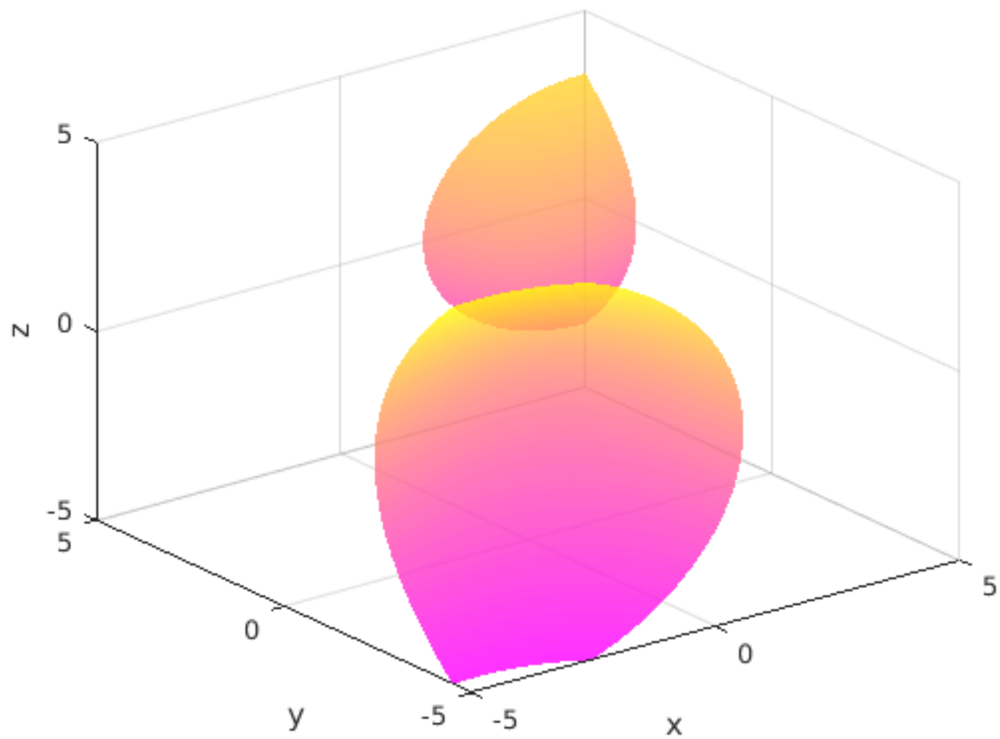
```
f5 = x^2 - 2*x*y + 2*y + z^2 - 4;
fimplicit3(f5,'EdgeColor','none','FaceAlpha',.8)
xlabel('x'), ylabel('y'), zlabel('z')
```



6) 双叶双曲面

$x^2 - 2xy + 2y + z^2 + 4 = 0$

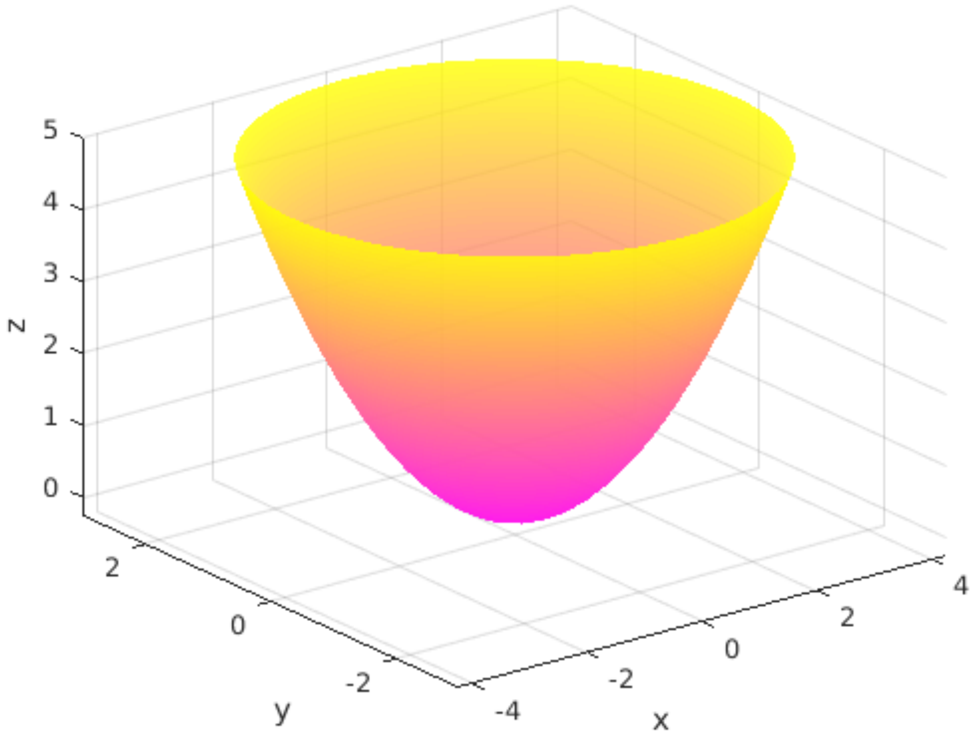
```
f6 = x^2 - 2*x*y + 2*y + z^2 + 4;
fimplicit3(f6,'EdgeColor','none','FaceAlpha',.8)
xlabel('x'), ylabel('y'), zlabel('z')
```



6) 椭圆抛物面

$$x^2 + 2y^2 - 3z = 0$$

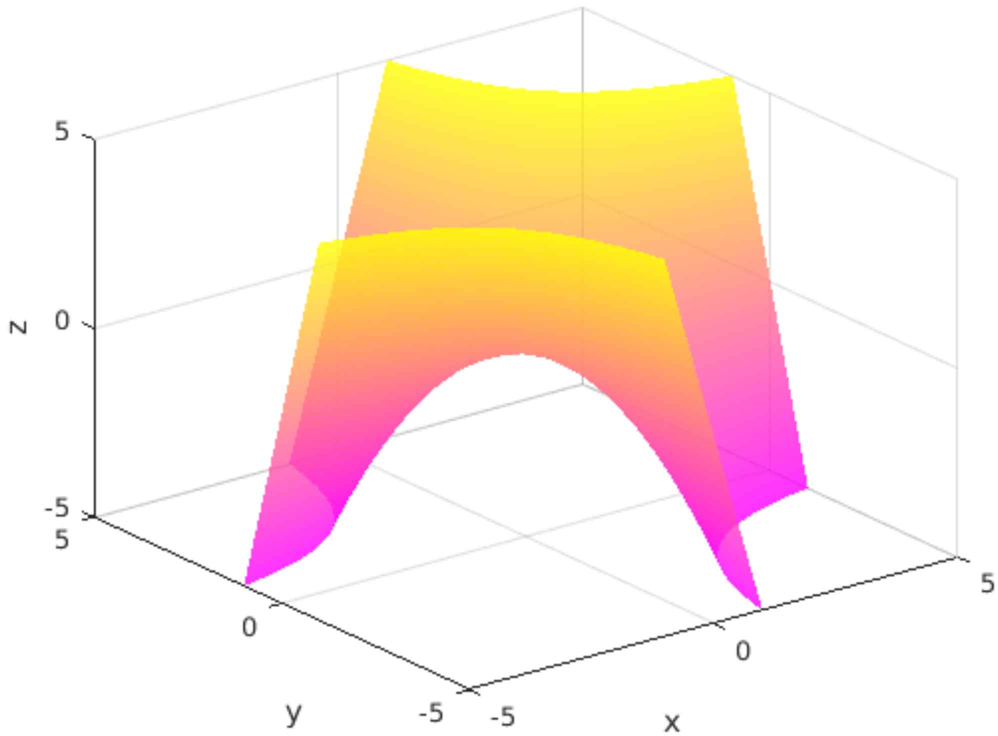
```
f7 = x^2 + 2*y^2 - 3*z;
fimplicit3(f7,'EdgeColor','none','FaceAlpha',.8)
xlabel('x'), ylabel('y'), zlabel('z')
```



7) 双曲抛物面

Q5(1)(a): $xy - z = 0$

```
f8 = x*y - z;
fimplicit3(f8,'EdgeColor','none','FaceAlpha',.8)
xlabel('x'), ylabel('y'), zlabel('z')
```


5
0
-5
z
5
0
y
-5
-5
0
x
5