# Distributed Control of Multi-zone HVAC Systems Considering Indoor Air Quality

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*Abstract*—This investigation studies a scalable control method for multi-zone heating, ventilation and air-conditioning (HVAC) systems with the objective to reduce energy cost while satisfying thermal comfort and indoor air quality (IAQ) (represented by CO2) simultaneously. This problem is challenging as we need to cope with the complex system behaviours, various couplings and the conflicting nature of objectives and constraints. To address the computational challenges, we propose a two-level distributed method (TLDM) by exploring the problem structures. Specifically, the *upper* level control (ULC) computes the optimal zone mass flow rates for maintaining zone thermal comfort, and the *lower* level control (LLC) regulates zone mass flow rates calculated from the *upper* level and the ventilation rate to achieve IAQ. As both the *upper* and *lower* level subproblems can be solved in a distributed manner w.r.t the zones, the proposed method is scalable and computationally efficient. The sub-optimality of the method is demonstrated through comparison with central-ized method in a *benchmark*. Through comparison with the distributed-based scheduling strategy (DTBSS) for HVAC control [1] which hasn't been able to involve IAQ, we find that the proposed method can achieve both thermal comfort and IAQ with a slight increase of energy cost. Finally, we compare the proposed method with the commonly-used demand controlled ventilation strategies (DCVs) for IAQ management [2, 3]. The numeric results imply an 8-10% energy cost saving for the proposed method.

*Note to Practitioners*—Designing energy-efficient controllers for HVAC systems has stimulated extensive discussions. However, the status quo has mostly focused on thermal comfort requirements only. The indoor air quality (IAQ) (usually represented by CO2 level), which closely relates to human's health and working productivity, has been less aware of.

This work is mainly motivated by our previous work [4] where we observed that the CO2 level may arise insufferably during the periods with high occupancy if only the temperature is cared while designing energy-efficient controllers for HVAC systems. Therefore, this paper aims to involve the requirements on IAQ into HVAC controller design. This task is usually computationally challenging as *i)* the necessities to cope with the complex thermal and CO2 dynamics simultaneously, which usually makes it intractable even to find a viable control both to maintain the comfortable temperature and CO2 bounds, and *ii)* the cooperative control of different parts for the HVAC systems, i.e., Variable Air Volume (VAV) boxes and fresh air damper in AHU, the latter of which are usually assumed fixed for thermal comfort control.

To cope with the challenges, this work developed a two-level (i.e., *upper* and *lower* level) distributed paradigm for HVAC control based on the independent feature of temperature and CO2 dynamics. Specifically, the *upper* level controllers calculate the estimated zone mass flow rates by minimizing HVAC energy cost while respecting the comfortable temperature bounds, and the *lower* level regulates the zone mass flow rates from the upper level and the fresh air damper to satisfy IAQ. As both the *upper* and *lower* level calculation can be implemented in a distributed and parallel mode by deploying zone controllers on single-board computers such as Raspberry Pi, the proposed control method is scalable for large multi-zone buildings. The method's performance is demonstrated against the distributed Token-based scheduling strategy (DTBSS) [1] and the widely-used demand controlled ventilation strategies (DCVs) [2, 3] through simulation. Our results show a perceived reduction in energy cost while the prescribed comfortable temperature and CO2 bounds are satisfied.

*Index Terms*—multi-zone HVAC system, two-level, distributed approach, IAQ, CO2.

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## I. INTRODUCTION

The heating, ventilation, and air-conditioning (HVAC) sys-tems account for a major and incremental proportion of buildings' energy consumption to maintain comfortable indoor environment [5]. This has stimulated widespread attention both from the industries and research stand-point towards HVAC energy (cost) saving while not compromising human comfort. While thermal comfort (e.g., temperature, humidity, etc.) has been widely perceived for HVAC control, the indoor air quality (IAQ) (usually represented by CO2) has been seldom concerned yet. In such situations, the energy (cost) saving target may be achieved at the sacrifice of IAQ, which is closely related to human health and working productivity [6, 7]. Along with the increasing awareness of human health and working productivity, it becomes imperative to jointly consider thermal comfort and IAQ while investigating HVAC control.

*Note:* In practice, there are many variables such as particulate matter, total volatile components, or chemical concentration related to IAQ. However, in HVAC control, CO2 concentration is a commonly used IAQ indicator and is therefore used in this paper as well to simplify our analysis.

## A. Related Works

In the literature, model predictive control (MPC) has emerged as a promising choice for HVAC systems as they allow to incorporate dynamic information (e.g., weather, occupancy, etc.) (see, [8, 9] and the references therein). There already exist various MPC-based control methods for HVAC systems. We refer the reader to [8, 9] for interest. They can generally be categorized based on the computation paradigms in implementation: centralized [10] and decentralized [1]. Wherein centralized approaches are usually developed for single-room/zone cases, in which relatively accurate models are usually used to capture the system dynamics (see, [11, 12]). However, they are usually not scalable or viable for large commercial buildings due to the high computation burden. Motivated by such applications, various decentralized or distributed approaches have been proposed and comprehensively studied (see [1, 4]).

However, most of the existing methods have been mainly focused on thermal comfort (e.g, temperature, humidity, etc) (see, [13, 14] and the references therein) and haven't been able to incorporate IAQ while investing HVAC energy (cost) savings. In this backdrop, the IAQ could be violated at time periods with high occupancy due to insufficient fresh air infusion. This can be briefly understood that the HVAC controller prefers more inside recirculated air (with lower temperature) than outside fresh air (higher temperature) to minimize cooling loads. Probably because of that, there appeared some standards for determining fresh air infusion for building HVAC control to manage IAQ. Wherein the demand controlled ventilation strategies (DCVs) have been the currently widely-used ones. Such methods could be CO2-based [15–17] or occupancy-based [18–20]. Their main ideas are either adjusting the fresh air infusion based on the detected instantaneous CO2 concentration or occupancy. Due to the zone CO2 or occupancy variations, it has been widely recognized that the DCVs tend to cause over-ventilation or under-ventilation for multi-zone commercial buildings [3]. Moreover, another drawback is that they are only developed for IAQ management regardless of the thermal comfort and energy savings target.

Therefore, it's imperative to jointly consider both thermal comfort and IAQ simultaneously for HVAC control while achieving energy cost savings. Such awareness has motivated some works on the joint management of temperature and CO2 in single-zone HVAC control using some simplified linear models [21–23]. In principle, these methods are not amenable to multi-zone commercial buildings as the used models can not capture the real system dynamics in such situations. As a scarce exception, [24] investigated commercial HVAC control by using Lyapunov optimization technique to achieve both thermal comfort and IAQ while reducing HVAC energy cost. Mainly due to the computational challenges to tackle the various temporally and spatially coupled non-linear constraints, the original multi-step problem was tackled by using successive single-step optimization and the performance of the method was reported on a 4-zone case study.

To our best knowledge, though both industries and research communities have realized the importance of IAQ in buildings and the necessity to incorporate it into buildings' HVAC control, such problem hasn't been well studied yet as: *i)* the existing related works are fairly limited (see the references therein); *i)* most of them were developed for single-zone case and are not amenable or scalable to multi-zone commercial buildings. With humans' increasing standards for indoor environment, there is an urgent need for scalable HVAC control methods for commercial buildings, which are capable of handling both thermal comfort and IAQ while still energy-efficient. However, this is a challenging task as:

*(C1)* Conflicting objectives and constraints imposed by the energy saving targets and the requirements on IAQ and thermal comfort, i.e., a good IAQ generally requires sufficient outdoor fresh air infusion and zone mass flow rates, which may result in the violations of lower zone temperature bounds and the high energy cost.

*(C2)* Intrinsic non-linearity and non-convexity caused by the complex HVAC system behaviors, i.e., both the energy cost and the system (i.e., temperature and CO2) dynamics are non-linear w.r.t the control inputs of HVAC system.

*(C3)* Various couplings both arising from inter-zone heat transfer and zone recirculated air, i.e., both the zone temperature and CO2 dynamics are coupled with each other.

The above challenges make the problem NP-hard. Moreover, due to the two bundles of complex non-linear constraints that related to temperature and CO2, it's even computationally intensive to search for a feasible operation point to maintain the two indexes (i.e., thermal comfort and IAQ).

## B. Contributions

Motivated by the literature, this paper aims to study energy-efficient control methods for commercial HVAC systems while jointly considering both thermal comfort and IAQ. Our main contributions to overcome such challenges are outlined.

*(i)* We propose a two-level control framework (i.e., ULC and LLC) to manage thermal comfort and IAQ separately while reducing HVAC energy cost.

*(ii)* While the ULC can adopt some existing distributed methods, we develop a distributed method for the LLC to achieve scalable computation.

*(iii)* Both the performance in energy cost saving and computation efficiency of the method are studied by comparisons with the existing methods.

Our two-level control framework is mainly motivated by *i)* the computational challenges imposed by the two bundles of complex non-linear constraints (i.e., temperature and CO2) simultaneously, and *ii)* the independent zone temperature and CO2 dynamics. The latter makes it possible to decompose the problem into two levels (i.e., *upper* and *lower* level) and tackle the two bundles of non-linear constraints sequentially. To be specific, the ULC first computes the optimal zone mass flow rate to satisfy zone thermal comfort while minimizing the HVAC energy cost. Successively, the LLC starts zone CO2 controllers to optimally regulate the computed zone mass flow rates from the *upper* level as well as the fresh air infusion to achieve the desirable IAQ metric. Such two-level paradigm makes it computationally tractable to achieve the

two comfort indexes without much compromise in energy cost savings. Moreover, such two-level structure can help reduce computation as the LLC only needs to be activated when the CO2 concentration bounds are violated. Particularly, as the ULC on thermal comfort has been comprehensively studied in our previous work [4], this paper adopts such existing method and place our main focus on achieving scalable (distributed) computation in the LLC.

The remainder of this paper is outlined. In Section II, we present the problem formulation. In Section III, we discuss the two-level distributed method. In Section IV, the performance of the method is evaluated through simulations. In Section V, we briefly conclude this paper.

## II. PROBLEM FORMULATION

### A. HVAC Systems for Commercial Buildings

A typical schematic for a commercial HVAC system is shown in Fig. 1. The main parts contain the Air Handling Unit (AHU), the Variable Air Volume (VAV) box, and the chiller water system (not shown in the figure). The central AHU is usually equipped with a damper, a cooling/heating coil and a supply fan. The heating/cooling coil will cool down/heat up the mixed air (the outside fresh air and the inside recirculated air) to the set-point temperature before delivered to the zones. Without loss of generality, this paper considers the cooling mode with a set-point temperature $15°$ (the usual scope 12-16°C). The damper within AHU can regulate the fraction of return air $d_r$ (ventilation rate). Generally, a smaller $d_r$ (more fresh air infusion) tends to yield better IAQ by diluting inside CO2 concentration but usually with higher energy consumption cost. This is mainly caused by the increased cooling demand caused by the increased outside fresh air. There usually exists a local VAV box attached to each zone, which consists of a damper and an heating coil. The damper regulates the zone mass flow rate and the heating coil can reheat the supply air if necessary (this is not discussed in this paper). Besides, the operation of HVAC system depends on a chiller water system (i.e., a chiller pump, water tank and the chiller) providing continuous chilled water to the cooling coils in the AHU. Except for the chiller, the chiller pump is also partially responsible for the HVAC's energy consumption to circulate the water between the water tank and the chiller. This paper mainly studies HVAC systems with the standard constant water flow system [25], in which the energy consumption of chiller pump can be regarded as fixed and therefore not explicitly discussed. More details for commercial HVAC systems can refer to [10, 26].

This paper studies HVAC control to maintain both zone thermal comfort and IAQ. To achieve it, both the zone air flow rates and the ventilation rate ($d_r$) need to be jointly coordinated. The problem is studied in a discrete-time framework with $\Delta_k = 30$ min's sampling and calculation interval. The results within a daily optimization horizon is inspected (48 stages). As there exist uncertainties (e.g., weather, occupancy, etc.), the problem is studied under MPC framework, i.e., at each decision epoch, the control inputs are computed based on the predicted information over the look-ahead planning
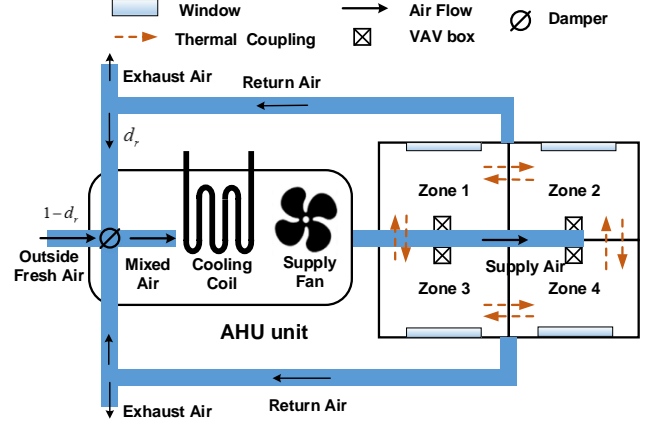


Fig. 1. The schematic of the HVAC system for multi-zone buildings.

horizon $H = 10$ (5h) but the results for the current stage is executed. This process is repeated with the evolving of time.

### B. Zone Thermal Dynamics

We consider a commercial building with $I$ thermal zones indexed by $\mathcal{I} = \{1, 2, \cdots, I\}$. At each decision epoch, the zone thermal dynamics over the planning horizon $\mathcal{H} = \{0, 1, \cdots, H-1\}$ can be captured using the Resistance-Capacitance (RC) network [27, 28], i.e.,

$$C_i(T_i(k+1) - T_i(k)) = \sum_{j \in \mathcal{N}_i} \frac{T_j(k) - T_i(k)}{R_{ij}} \Delta_k + \frac{T_o(k) - T_i(k)}{R_{oi}} \Delta_k$$
$$+ c_p m_i^z(k)(T_c - T_i(k))\Delta_k + Q_i(k)\Delta_k, \ \forall i \in \mathcal{I}, k \in \mathcal{H}. \tag{1}$$

where $k \in \mathcal{H}$ and $i, j \in \mathcal{N}$ denote the time and zone index. $C_i$ is the zone air heat capacity. $T_i(k)$, $T_o(k)$ and $T_c$ denote the zone temperature, outside air temperature and the set-point temperature of supply air, respectively. $R_{oi}$ denotes the thermal resistance between zone $i$ and the outside, and $R_{ij}$ ($R_{ji}$) denotes the thermal resistance between the neighboring zones $i, j$. We use $\mathcal{N}_i$ to indicate the collection of adjacent zones to zone $i$. $c_p$ is the specific heat of the air. $m_i^z(k)$ indicates zone mass flow rates. $Q_i(k)$ quantifies zone internal heat gains mainly from the occupants and equipments [29].

We organize (1) in a standard form as

$$T_i(k+1) = A_{ii}T_i(k) + \sum_{j \in \mathcal{N}_i} A_{ij}T_j(k)$$
$$+ C_{ii}m_i^z(k)(T_i(k) - T_c) + D_i(k), \ \forall i \in \mathcal{I}, k \in \mathcal{H}. \tag{2}$$

where $A_{ii} = 1 - (\sum_{j \in \mathcal{N}_i} \frac{\Delta_k}{R_{ij}C_i} + \frac{\Delta_k}{C_iR_{oi}})$, $A_{ij} = \frac{\Delta_k}{C_iR_{ij}}$, $C_{ii} = -\frac{\Delta_k \cdot c_p}{C_i}$, and $D_i(k) = \frac{\Delta_k T_o(k)}{C_iR_{oi}} + \frac{\Delta_k \cdot Q_i(k)}{C_i}$.

### C. Zone IAQ Dynamics

Similar to [21, 24], this paper uses CO2 concentration as an IAQ indicator. The zone CO2 dynamics for multi-zone commercial buildings can be described by [24]

$$m_i(C_i(k+1) - C_i(k)) = N_i(k)C_g\Delta_k$$
$$+ m_i^z(k)(C_z(k) - C_i(k))\Delta_k \ \forall i \in \mathcal{I}, k \in \mathcal{H} \tag{3}$$

where $m_i$ denotes zone air mass. $C_i(k)$ (ppm) denotes zone CO2 concentration. As the occupants are the main source

of CO2 generation, we dynamically estimate the zone CO2 accumulation based on the average CO2 generation rate per person $C_g$ (g h$^{-1}$) and the occupancy $N_i(k)$ as shown in the first term of (3) on the right-hand side. $C_z(k)$ denotes the CO2 concentration of supply air, which can be estimated by

$$C_z(k) = \left(1 - d_r(k)\right)C_o(k) + d_r(k)C_{\mathrm{m}}(k),$$
$$\text{with } C_{\mathrm{m}}(k) = \frac{\sum_{i \in \mathcal{I}} m_i^z(k)C_i(k)}{\sum_{i \in \mathcal{I}} m_i^z(k)}, \ \forall k \in \mathcal{H} \tag{4}$$

where $d_r(k)$ ($0 \le d_r(k) \le 1$) denotes the fraction of return air (ventilation rate) delivered to AHU. $C_m(k)$ captures the CO2 concentration of the mixed return air from all the zones.

From (3)-(4), one may note that the zone CO2 dynamics are nonlinear and fully coupled through the recirculated air.

### D. Control Variables

The HVAC system is responsible for maintaining human comfort (i.e., temperature and CO2), which are closely related to its control inputs: *i)* the ventilation rate $d_r(k)$, *ii)* zone mass flow rate $m_i^z(k)$, and the inducted state trajectories: *iii)* zone temperature $T_i(k)$, *iv)* zone CO2 concentration $C_i(k)$.

### E. Objective Function

This paper seeks to reduce HVAC energy consumption that mainly caused by the cooling coil $P_f(k)$ and supply fan $P_f(k)$ within AHU, i.e.,

$$P_c(k) = c_p \eta (1 - d_r(k)) \sum_{i \in \mathcal{I}} m_i^z(k)(T_o(k) - T_c)$$
$$+ c_p \eta d_r(k) \sum_{i \in \mathcal{I}} m_i^z(k)(T_i(k) - T_c) \tag{5}$$
$$P_f(k) = \kappa_f (\sum_{i \in \mathcal{I}} m_i^z(k))^2$$

where $\eta$ is the reciprocal of the coefficient of performance (COP) of the chiller, which captures the ratio of provided cooling to the total consumed electrical power.

As the energy consumption is not easy to inspect in practice, we selected the total energy cost instead as the objective, which is calculated based on the electricity price $c_k$ (s$kW):

$$J = \sum_{k \in \mathcal{H}} c_t \big( P_c(k) + P_f(k) \big) \Delta_k \tag{6}$$

### F. System Constraints

The operation of the HVAC system should respect the thermal comfort and IAQ requirements, which are usually captured by some zone temperature bounds [1] and zone CO2 bounds [24] as described in (7) and (8), respectively.

$$T_i^{\min} \le T_i(k) \le T_i^{\max}, \quad \forall i \in \mathcal{I}, \ k \in \mathcal{H}. \tag{7}$$
$$C_i(k) \le C_i^{\max}, \quad \forall i \in \mathcal{I}, \ k \in \mathcal{H}. \tag{8}$$

where $T_i^{\min}$ and $T_i^{\max}$ represent the lower and upper zone temperature bound. $C_i^{\max}$ denotes the upper zone CO2 bound. One may note that the formulation allow to accommodate personalized zone comfort requirements by setting $T_i^{\min}$, $T_i^{\max}$ and $C_i^{\max}$ accordingly.

Additionally, the operation of the HVAC system should abide by the physical limits, i.e., *i)* the zone mass flow rate delivered by the local VAV box is bounded as (9); *ii)* the total mass flow rate supplied by the AHU is limited as (10).

$$m_i^{z,\min} \le m_i^z(k) \le m_i^{z,\max}, \quad \forall i \in \mathcal{I}, \ k \in \mathcal{H} \tag{9}$$

$$\sum_{i \in \mathcal{I}} m_i^z(k) \le m^{\max}, \quad \forall k \in \mathcal{H} \tag{10}$$

where $m_i^{z,\min}$ and $m_i^{z,\max}$ denote the lower and upper zone mass flow rate bounds. $m^{\max}$ denotes the maximum total mass flow rate that can be supplied by the AHU.

The ventilation rate is generally constrained by the upper and lower bound $d_r^{\min}$ and $d_r^{\max}$, i.e.,

$$d_r^{\min} \le d_r(k) \le d_r^{\max}, \quad \forall k \in \mathcal{H}. \tag{11}$$

### G. The Problem

Overall, the optimization problem at each decision epoch can be summarized as ($\mathcal{P}$).

$$\min_{\boldsymbol{m}_i^z, \boldsymbol{T}_i, \boldsymbol{C}_i, i \in \mathcal{I}, \boldsymbol{d}_r} J \tag{$\mathcal{P}$}$$
$$s.t. \ (2)-(4), (7)-(8), \ (9)-(10), \ (11).$$

where we have $\boldsymbol{m}_i^z = [m_i^z(k)]_{k \in \mathcal{H}}$, $\boldsymbol{T}_i = [T_i(k)]_{k \in \mathcal{H}}$, $\boldsymbol{C}_i = [C_i(k)]_{k \in \mathcal{H}}$ ($\forall i \in \mathcal{I}$) and $\boldsymbol{d}_r = [d_r(k)]_{k \in \mathcal{H}}$.

One may note that problem ($\mathcal{P}$) is non-linear and non-convex. The non-linearity and non-convexity both arise from the objective function and the constraints. The problem is NP-hard and even computationally intensive to search for a feasible solution while accounting for the two bundles of non-linear constraints for thermal comfort and IAQ. Indeed, such computationally challenges to solve problem ($\mathcal{P}$) directly have motivated our two-level method to be discussed.

## III. TWO-LEVEL DISTRIBUTED METHOD

To address ($\mathcal{P}$), this section develops a two-level distributed method (TLDM) by exploring the problem structures, i.e., the zone temperature and CO2 dynamics are independent. Specifically, we divide the problem into two levels: *upper* and *lower* level. Whereas the *upper* level controllers are mainly responsible for thermal comfort, and the *lower* level controllers will be activated to manage IAQ if zone CO2 concentration violations were detected. By using such two-level structure, it is possible to overcome the computational challenges by tackling the two bundles of nonlinear constraints for temperature and CO2, separately. Moreover, such two-level method can help reduce computation by allowing the *lower* level controllers to be activated only if necessary. Both the ULC and LLC use distributed methods with mainly zone level computation to achieve scalability and computation efficacy. The holistic framework of the TLDM is depicted in Fig. 2 with the details on the ULC and LLC to be discussed.
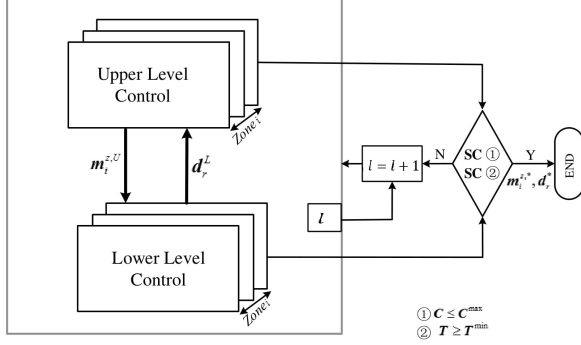
Fig. 2. The framework of two-level distributed method.

## A. The Upper Level Control (ULC)

The focus of the ULC is to achieve thermal comfort while minimizing the HVAC's total energy cost. Considering that *i)* the zone temperature is only affected by zone mass flow rate $m_i^z(k)$; *ii)* the HVAC's energy cost is non-decreasing w.r.t. the ventilation rate $d_r(k)$, we define the ULC problem as

$$
\begin{aligned}
\min_{\boldsymbol{m}_i^z, \boldsymbol{T}_i, i \in \mathcal{I}.} & \quad J^U \\
s.t. & \quad (2), (7), (9)-(10). \\
& \quad d_r(k) = d_r^L(k), k \in \mathcal{H}.
\end{aligned}
\quad (\mathcal{P}_U)
$$

where we have $J^U = J$ with fixed $\boldsymbol{d}_r$.

As indicated in problem $(\mathcal{P}_U)$, the ULC attempts to reduce the energy cost by adopting the "minimum" ventilation rate $d_r^L(k)$ as required to achieve IAQ in the LLC (we can start with $d_r(k) = d_r^{\max}$ ($k \in \mathcal{H}$) as initial and updates it with the calculated value from the LLC afterwards). For problem $(\mathcal{P}_U)$, there already exists a number of scalable (distributed) solution methods. We refer the readers to [1, 4] for some options. This paper adopts a decentralized method proposed in our previous work [4], which has been demonstrated with favorable performance both in reducing HVAC energy cost and scalability. We place our main attention on IAQ in the LLC.

## B. The Lower Level Control (LLC)

As indicated in Fig. 2, the LLC only needs to be activated when zone CO2 concentrations violate the upper bounds (i.e., $C_i(k) \geq C_i^{\max}$). Generally, incorporating IAQ in HVAC control tends to increase the energy cost required to maintain thermal comfort. Therefore, to achieve IAQ while not compromising energy cost saving targets achieved in the ULC, the main ideas of the LLC is to achieve IAQ while minimizing the deviations of the control inputs inducted from the ULC.

From the zone CO2 dynamics (3)-(4), we note that both zone mass flow rates $m_i^z(k)$ and ventilation rate $d_r(k)$ influence zone CO2 concentrations. Particularly, by increasing zone mass flow rate, the zone CO2 concentration can generally be diluted. However, the lower bounds of zone temperature calculated in the ULC may be violated by only increasing zone mass flow rate to achieve IAQ. In fact, such case usually implies that by only regulating zone mass flow rate is in principle inadequate to satisfy zone temperature and CO2 bounds simultaneously and the outdoor fresh air infusion

needs to be increased (decrease $\boldsymbol{d}_r$). With such considerations, the LLC adopts a *two-phase* method. Specifically, the LLC first attempts to satisfy the user-defined zone CO2 bounds by regulating zone mass flow rate while achieving the *near-optimal* energy cost that computed in the ULC. As the HVAC's energy cost is increasing w.r.t the zone mass flow rate, this can be achieved by solving problem $(\mathcal{P}_L)$ with *i)* the objective defined as the deviations of zone mass flow rates, and *ii)* the constraints imposed by zone CO2 concentration (3)-(4), (8) and the operation limits of AHU (10). In particular, the LLC adopts the zone mass flow rates $m_i^{z,U}(k)$ computed in the ULC as the lower zone mass flow rate bounds. In this regard, one may note that by solving problem $(\mathcal{P}_L)$, only the upper zone temperature bounds that computed in the ULC can be maintained and the lower bounds may be violated if the fresh air infusion is intrinsically inadequate. In such situation, the other phase of LLC will be activated to increase fresh air infusion (decrease $\boldsymbol{d}_r$). The *two-phase* method in the *lower* control may need to be alternated to finally achieve both thermal comfort and IAQ as illustrated in in Fig. 2.

$$
\begin{aligned}
\min_{\boldsymbol{m}_i^z, \boldsymbol{C}_i, i \in \mathcal{I}.} & \quad J^L = \sum_{k \in \mathcal{H}} \sum_{i \in \mathcal{I}} \left( m_i^z(k) - m_i^{z,U}(k) \right)^2 \\
s.t. & \quad m_i^{z,U}(k) \leq m_i^z(k) \leq m_i^{z,\max}, \forall i \in \mathcal{I}. \\
& \quad (3)-(4), (8), (10).
\end{aligned}
\quad (\mathcal{P}_L)
$$

While in principle both the zone temperature and CO2 bounds can be satisfied by using the *two-phase* method, the remaining problem in the LLC is to solve the non-linear and non-convex $(\mathcal{P}_L)$. To address this issue, this section aims to develop a distributed method based on some relaxation techniques. As one may note, the main computational challenges of problem $(\mathcal{P}_L)$ arise from the fully coupled non-linear constraints of zone CO2. To handle it, we introduce a learning framework to iteratively estimate the CO2 concentration of supply air ($C_z(k)$). Generally, the main procedures to solve problem $(\mathcal{P}_L)$ capitalize on two-loop iteration, i. e., *Inner-loop*: computing zone mass flow rates $m_i^z(k)$ and CO2 concentration $C_i(k)$ by solving problem $(\mathcal{P}_L)$ with the estimated CO2 concentration of supply air ($C_z(k)$) in a distributed manner; and *Outer-loop*: iteratively estimating/updating the CO2 concentration of the supply air $C_z(k)$ based on the computed zone mass flow rates $m_i^z(k)$) and CO2 concentration $C_i(k)$. Such two procedures will be repeated until convergence.

*Inner-loop*: with the estimated CO2 concentration for supply air ($C_z(k)$), the zone CO2 dynamics can be restated in a decoupled manner:

$$
\begin{aligned}
C_i(k+1) = & C_i(k) + E_i(k)m_i^z(k) \\
& + F_i(k)m_i^z(k)C_i(k) + G_i(k)
\end{aligned}
\quad (12)
$$

where $E_i(k) = C_z(k)\Delta_k/m_i$, $F_i(k) = -\Delta_k/m_i$ and $G_i(k) = N_i(k)C_g\Delta_k/m_i$ are constant parameters with the estimated $C_z(k)$.

From (12), we note that the zone CO2 dynamics are bilinear w.r.t the zone mass flow rate $m_i^z(k)$ and CO2 concentration $C_i(k)$. To handle this, we use the *McCormick envelopes* [30] to

relax the bilinear terms by introducing some auxiliary decision variables $Z_i(k) = m_i^z(k)C_i(k)$, i.e.,

$$Z_i(0) = m_i^z(0)C_i(0). \tag{13a}$$

$$Z_i(k) \geq m_i^{z,\min}C_i(k) + m_i^z(k)C_i^{\min} - m_i^{z,\min}C_i^{\min}, \tag{13b}$$

$$Z_i(k) \geq m_i^{z,\max}C_i(k) + m_i^z(k)C_i^{\max} - m_i^{z,\max}C_i^{\max}, \tag{13c}$$

$$Z_i(k) \leq m_i^z(k)C_i^{\max} + m_i^{z,\min}C_i(k) - m_i^{z,\min}C_i^{\max}, \tag{13d}$$

$$Z_i(k) \leq m_i^{z,\max}C_i(k) + m_i^z(k)C_i^{\min} - m_i^{z,\max}C_i^{\min}, \tag{13e}$$

$$\forall k \in \mathcal{H} \setminus \{0\}. \tag{13}$$

where $C_i(0)$ denotes the zone CO2 concentration at the beginning of the planning horizon, which are supposed to be measured through sensors.

By involving the auxiliary decision variables $Z_i(k)$, we have the following relaxed problem for the LLC:

$$\min_{m_i^z, C_i, Z_i, \forall i \in \mathcal{I}.} J^L = \sum_{k \in \mathcal{H}} \sum_{i \in \mathcal{I}} (m_i^z(k) - m_i^{z,U}(k))^2$$

$$s.t. \quad C_i(k+1) = C_i(k) + E_i(k)m_i^z(k) + F_i(k)Z_i(k)$$
$$+ G_i(k), \quad \forall i \in \mathcal{I}, k \in \mathcal{H}. \quad (\mathcal{P}'_L)$$
$$m_i^{z,U}(k) \leq m_i^z(k) \leq m_i^{z,\max}, \forall i \in \mathcal{I}.$$
$$(8), (10), (13a) - (13e).$$

We note problem $(\mathcal{P}'_L)$ is convex and characterized by *i)* a decomposable objective function w.r.t. the zones; *ii)* coupled linear constraints; and *iii)* local linear constraints. This problem can be efficiently tackled by the Accelerated Distributed Augmented Lagrangian (ADAL) method [31]. Before we illustrate the main procedures, we first recast problem $(\mathcal{P}'_L)$ into a standard form:

$$\min_{x_i, i \in \mathcal{I}} J^L = \sum_{i \in \mathcal{I}} J_i^L(x_i)$$

$$s.t. \quad \sum_{i=0}^I A_i^c x_i = b^c. \quad (\mathcal{P}''_L)$$
$$x_i \in \mathcal{X}_i, \forall i \in \mathcal{I} \cup \{0\}.$$

where $x_i = [(x_i(k))^T]_{k \in \mathcal{H}}^T$, $x_i(k) = (C_i(k), m_i^z(k), Z_i(k))^T$ collecting the decision variables of zone $i$. $J_i^L = \sum_{k \in \mathcal{H}} (m_i^z(k) - m_i^{z,U}(k))^2$ represents the local objective function of zone $i$. $\sum_{i=0}^I A_i^c x_i = b^c$ accounts for the coupled linear constraints (10) with an additional slack variables $x_0(k) \geq 0$ introduced at each stage (transform the inequality constraints to equality constraints). $\mathcal{X}_i$ indicates the collection of the local constraints in $(\mathcal{P}'_L)$ attached to zone $i$ ($\forall i \in \mathcal{I}$), and we have $\mathcal{X}_0 = \{x_0 | x_0 \geq \mathbf{0}\}$. We have the constant parameters
$$A_i^c = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \in \mathbb{R}^{H \times 3H}$$
($\forall i \in \mathcal{I}$), and $b^c = (m^{\max}, m^{\max}, \cdots, m^{\max})^T \in \mathbb{R}^H$.

Typically, the ADAL [31] capitalizes on an augmented Lagrangian function to tackle the coupled constraints in $(\mathcal{P}''_L)$,

which can be described as

$$\mathbb{L}_\rho(x_0, x_1, \cdots, x_I, \alpha) = \sum_{i \in \mathcal{I}} J_i^L + \alpha^T \left( \sum_{i=0}^I A_i^c x_i - b^c \right)$$
$$+ \frac{\rho}{2} \| \sum_{i=0}^I A_i^c x^i - b^c \|^2 \tag{14}$$

where $\alpha = (\alpha_0, \alpha_1, \cdots, \alpha_{H-1})^T$ are Lagrangian multipliers. $\rho (\rho > 0)$ is penalty parameter.

With given Lagrangian multipliers $\alpha$, we have the following primal problem:

$$\min_{x_0, x^i, \forall i \in \mathcal{I}} \mathbb{L}_\rho(x_0, x_1, \cdots, x_I, \alpha)$$
$$s.t. \quad x_i \in \mathcal{X}_i, \quad \forall i \in \mathcal{I}. \tag{15}$$
$$x_0 \geq \mathbf{0}.$$

Similar to the mots methods of multipliers (MMs), the main procedures of using ADAL to solve problem $(\mathcal{P}''_L)$ contains three main steps: *i)* tackling the primal problem (15) in a distributed manner, *ii)* updating the dual variables (Lagrangian multipliers) $\alpha$, and *iii)* updating the penalty factor $\rho$. While the last two procedures are standard, we illustrate how to solve the primal problem $(\mathcal{P}''_L)$ in a distributed fashion. Specifically, we define $I+1$ agents, where Agent $1 \sim I$ correspond to the $I$ zones and Agent 0 is a virtual agent responsible for managing the slack decision variable $x_0$. At each iteration $q$, the local objective function for Agent $i$ is defined as

$$\mathbb{L}_\rho^0(x_0, x_{-0}^q, \alpha) = \alpha^T A_0^c x_0 + \frac{\rho}{2} \| A_0^c x_0 + \sum_{i=1}^I A_i^c x_i^q - b^c \|^2$$

$$\mathbb{L}_\rho^i(x_i, x_{-i}^q, \alpha) = J_i^L + \alpha^T A_i^c x_i + \frac{\rho}{2} \| A_i^c x_i + \sum_{j \in \mathcal{I} \cup \{0\}, j \neq i} A_j^c x_j^q - b^c \|^2, \tag{16}$$

$$\forall i \in \mathcal{I}.$$

*Outer-loop*: by solving problem $(\mathcal{P}''_L)$, we can obtain the estimated zone mass flow rates $m_i^z(k)$ and CO2 concentration $C_i(k)$, thereby the CO2 concentration of the supply air can be iterated accordingly.

The details to solve problem $(\mathcal{P}''_L)$ based on ADAL [31] are gathered in **Algorithm** 1. In the the *inner-loop*, we use $x^q = [(x_j^q)^T]_{j \in \mathcal{I} \cup \{0\}}^T$ to represent the augmented control and state trajectories for all the agents at each iteration $q$, and $x_{-i}^q = [(x_j^q)^T]_{j \in \mathcal{I}\{0\}, j \neq i}^T$ denotes the collection of control and state trajectories for all agents except Agent $i$. With a little bit abuse of notation, we use the superscript $p$ to denote the results for the *outer-loop*. While the *inner-loop* uses the residual error of the coupled constraints, i.e.,

$$r_q(x) = \| \sum_{i=0}^I A_i^c x_i^q - b^c \| \leq \epsilon^{in}. \tag{17}$$

as the stopping indicator, the *outer-loop* adopts the deviation of $C_z = [C_z(k)]_{k \in \mathcal{H}}$ over two successive iterations as the stopping criterion, i.e.,

$$\| C_z^{p+1} - C_z^p \| \leq \epsilon^{out}, \tag{18}$$

where $\epsilon^{in}$ and $\epsilon^{out}$ are small positive thresholds.

While an optimal solution for problem $(\mathcal{P}'_L)$ can be attained by **Algorithm** 1, there still exists a remaining issue on

**Algorithm 1** Solve problem ($\mathcal{P}_L''$) based on ADAL

---

1: **Initialize** $p \leftarrow 0$, $\boldsymbol{C}_z^0$ and $\boldsymbol{d}_r$.                    ▷ Outer-loop
2:   **Initialize** $q \leftarrow 0$, $\boldsymbol{\alpha}^0$, and $\boldsymbol{x}_i^0$ ($\forall i \in \mathcal{I} \cup \{0\}$), $\boldsymbol{C}_z = \boldsymbol{C}_z^p$.
      ▷ Inner-loop
3:   **for** $i \in \mathcal{I} \cup \{0\}$, **do**

$$\boldsymbol{x}_i^{q+1} = \arg\min_{\boldsymbol{x}_i} \mathbb{L}_\rho^i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}^q, \boldsymbol{\alpha}^q) \qquad (19)$$
$$s.t. \quad \boldsymbol{x}_i \in \mathcal{X}_i.$$

4:   **end for**
5:   Update the dual variables:

$$\boldsymbol{\alpha}^{q+1} = \boldsymbol{\alpha}^q + \rho\Big(\sum_{i=0}^I \boldsymbol{A}_i^c \boldsymbol{x}_i^{q+1} - \boldsymbol{b}^c\Big) \qquad (20)$$

6:   If (17) is satisfied, stop with $\boldsymbol{x}^p = \boldsymbol{x}^{q+1}(\boldsymbol{C}_z^p)$, otherwise
      $q \rightarrow q+1$ and go to Step 3.                    ▷ Inner-loop
7: Estimate $\boldsymbol{C}_z^{p+1}$ according to

$$\boldsymbol{C}_z^{p+1} = (1 - \boldsymbol{d}_r)\boldsymbol{C}_o + \boldsymbol{d}_r \frac{\sum_{i \in \mathcal{I}} \boldsymbol{m}_i^{z,p} \boldsymbol{C}_i^p}{\sum_{i \in \mathcal{I}} \boldsymbol{m}_i^{z,p}}. \qquad (21)$$

8: If (18) is satisfied, stop with $\boldsymbol{x}^p$, otherwise set $p \rightarrow p+1$
   and go to Step 2.                    ▷ Outer-loop
**Output:** $\boldsymbol{x}^* = [(\boldsymbol{x}_i^p(k))^T]_{k \in \mathcal{H}}^T$, $\boldsymbol{x}_i^p(k) = \big(C_i^p(k), m_i^{z,p}(k), Z_i^p(k)\big)^T$.

---

recursive feasibility to be discussed. As the bilinear terms $Z_i(k) = m_i^z(k)C_i(k)$ in zone CO2 dynamics (3) are relaxed in ($\mathcal{P}_L'$), one may realize that the recursive zone CO2 dynamics may not be guaranteed. To address such issue, we propose a heuristic method (see, **Algorithm** 2) to estimate the zone temperature and CO2 trajectories over the current planning horizon that abide by their recursive dynamics for the next computing epoch based on the obtained solution. In **Algorithm** 2, the variables with a hat above indicate the recovered solution. The main idea of the algorithm is to assign high priority to the computed zone mass flow rates $\boldsymbol{m}_i^{z,*}$ which determine the HVAC's energy cost. In particular, as we have $Z_i(k) = m_i^z(0)C_i(0)$ at the beginning of each planning horizon, the CO2 bounds computed in **Algorithm** 1 can be maintained at each executed epoch, though that may be tentatively violated for the remaining look-ahead stages.

As discussed previously, it may occur that the lower temperature bounds computed in the ULC are violated in the LLC due to the regulation of zone mass flow rates to achieve IAQ. In fact, such situation implies that by only regulating zone mass flow rates is in principle inadequate to satisfy the user-defined temperature and CO2 bounds simultaneously and the fresh air infusion are required to be increased. This motivates the second phase of the LLC which is mainly responsible for regulating fresh air infusion delivered to AHU. Overall, the proposed TLDM is summarized in **Algorithm** 3 which embeds the ULC and LLC as discussed with $l$ as the iteration. $\mathbb{I}(A)$ is an indicator function. We have $\mathbb{I}(A) = 1$ with the condition $A$ true, otherwise $\mathbb{I}(A) = 0$. In particular, when lower zone temperature bounds violations are detected in LLC, the ventilation rate will be increased (decrease $\boldsymbol{d}_r$). We use $\Delta d_r$ to denote the regulating step-size of ventilation rate $\boldsymbol{d}_r$. In principle,

the step-size $\Delta d_r$ is supposed to be determined based on the amplitude of temperature violations. However, it's generally difficult to figure it out as the temperature and ventilation rates are in two different dimensions. Therefore, this paper selects a small positive constant step-size to guarantee the convergence.

---

**Algorithm 2** Estimate zone temperature and CO2 trajectories

---

**Input:** $\boldsymbol{x}^* = [(\boldsymbol{x}_i^p(k))^T]_{k \in \mathcal{H}}^T$, $\boldsymbol{x}_i^p(k) = \big(C_i^p(k), m_i^{z,p}(k), Z_i^p(k)\big)^T$
   (from **Algorithm** 1).
1: Set $\hat{C}^i(0) = C^i(0)$ and $\hat{T}^i(0) = T^i(0)$ ($\forall i \in \mathcal{I}$).
2: **for** $k \in \mathcal{H}$ **do**
3:   **for** $i \in \mathcal{I}$ **do**
4:     Set $\hat{m}_i^z(k) = m_i^{z,*}(k)$.
5:     Set $\hat{Z}_i(k) = \hat{m}_i^z(k)\hat{C}_i(k)$.
6:     Estimate $\hat{C}_i(k+1)$ and $\hat{T}_i(k+1)$ by

$$\hat{C}_i(k+1) = \hat{C}_i(k) + \hat{E}_i(k)\hat{m}_i^z(k) + F_i(k)\hat{Z}_i(k) + G_i(k),$$
$$\hat{T}_i(k+1) = A_{ii}\hat{T}_i(k) + \sum_{j \in \mathcal{N}_i} A_{ij}\hat{T}_j(k)$$
$$+ C_{ii}\hat{m}_i^z(k)(\hat{T}_i(k) - T_c) + D_i(k),$$
$$\text{with } \hat{E}_i^z(k) = \hat{C}_z(k)\Delta_t/m_i, \text{and } \hat{C}^z(k) = (1 - d_r^*(k))C_o(k)$$
$$+ d_r^*(k)\frac{\sum_{i \in \mathcal{I}} \hat{m}_i^z(k)\hat{C}_i(k)}{\sum_{i \in \mathcal{I}} \hat{m}_i^z(k)}, \forall i \in \mathcal{I}, k \in \mathcal{H}.$$

7:   **end for**
8: **end for**
**Output:** $\hat{\boldsymbol{m}}_i^z = [\hat{m}_i^z(k)]_{k \in \mathcal{H}}$, $\hat{\boldsymbol{C}}_i = [\hat{C}_i(k)]_{k \in \mathcal{H}}$, and $\hat{\boldsymbol{T}}_i = [\hat{T}_i(k)]_{k \in \mathcal{H}}$.

---

**Algorithm 3** Two-level Distributed Method (TLDM)

---

1: **Initialize** $l \leftarrow 0$ and $\boldsymbol{d}_r^0$.
2: $[\boldsymbol{m}_i^{z,L}]_{i \in \mathcal{I}} = ULC(\boldsymbol{d}_r^l)$ [4].
3: $[\hat{\boldsymbol{m}}_i^z, \hat{\boldsymbol{C}}_i, \hat{\boldsymbol{T}}_i]_{i \in \mathcal{I}} = LLC([\boldsymbol{m}_i^{z,L}]_{i \in \mathcal{I}}, \boldsymbol{d}_r^l)$(**Algorithm** 1-2).
4: If $\hat{\boldsymbol{T}}^i \geq T_i^{\min}$, then stop, otherwise continue.
5: Update ventilation rate $\boldsymbol{d}_r$:

$$\boldsymbol{d}_r^{l+1}(k) = \boldsymbol{d}_r^l(k) - \Delta d_r \mathbb{I}(\hat{\boldsymbol{T}}_i(k+1) < T_i^{\min}), \; \forall k \in \mathcal{H}.$$

6: Set $r \rightarrow r+1$ and go to **Step** 2.

---

## IV. APPLICATION

This section reports the the performance of the TLDM on multi-zone HVAC system control through simulations. Specifically, the sub-optimality and computational advantages of the method are illustrated through the *benchmark* (5-zone case study). After that the capability and scalability of the method to *medium* scale (10,20 zones) and *large* scale (50,100 zones) cases are discussed.

### A. Benchmark

This part studies the performance of the TLDM through a 5-zone case study. Without loss of generality, we set the comfortable zone temperature bounds as $[24, 26]°$C [32, 33] and the zone CO2 bounds as 800 ppm. The outlet air temperature of AHU is set as $T_c = 15°$C. We assume the zones are adjacent as $(5) \leftrightarrow 1 \leftrightarrow 2 \leftrightarrow 3 \leftrightarrow 4 \leftrightarrow 5 \leftrightarrow (1)$ ($i \leftrightarrow j$ indicates

zone $i$ and zone $j$ are adjacent and there exists heat transfer). The initial zone temperature is set as $[29, 30, 31, 30, 29]°C$ (zone 1 to zone 5). The outside air temperature and zone occupancy are shown in Fig. 3. The HVAC's energy cost is calculated according to the time-of-use (TOU) price [12]. The other parameters refer to TABLE I.

TABLE I
SIMULATION PARAMETERS

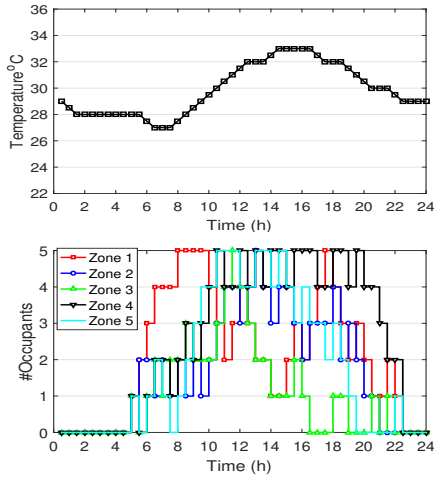| Param. | Value | Units |
|---|---|---|
| $C_i (i \in \mathcal{I})$ | $1.5 \times 10^3$ | $kJ\,K^{-1}$ |
| $c_p$ | 1.012 | $kJ/kg - K$ |
| $R_{oi}$ | 50 | $kW\,K^{-1}$ |
| $R_{ij}(i, j \in \mathcal{I})$ | 14 | $kW\,K^{-1}$ |
| $\kappa_f$ | 0.08 | - |
| $\eta$ | 1 | - |
| $C_g$ | 40 | $g\,h^{-1}$ |
| $\Delta d_r$ | 0.05 | - |
| $m_i^{z,\min}$ | 0 | $kg\,h^{-1}$ |
| $m_i^{z,\max}$ | 0.5 | $kg\,h^{-1}$ |



Fig. 3. (a) Outside air temperature. (b) Zone occupancy.

In the *benchmark*, we compare the proposed method with *(i)* Distributed Token-based Scheduling Strategy (DTBSS) [1], *(ii)* centralized method, *(iii)* the commonly-used DCV strategies [2, 3], and *(iv)* sequential quadratic programming (SQP) [34]. In particular, similar to most existing distributed methods for HVAC control, the DTBSS has mainly focused on thermal comfort and hasn't been able to involve the complex nonlinear constraints on IAQ [1]. Therefore, we fix the ventilation rate $\boldsymbol{d}_r$ of DTBSS as $d_r(k) = d_r^{max}$ ($\boldsymbol{k} \in \mathcal{H}$) in our comparison. In the centralized method, we try to obtain the optimal solution of the *benchmark* by solving the nonlinear optimization problem $(\mathcal{P})$ using the Ipopt solver embed in Matlab [35]. As the commonly-used methods to account for IAQ, the DCV strategies generally calculate the required fresh air for each zone based on the occupancy and the zone area [2, 3], i.e.,

$$m_i^{z,\text{fresh}}(k) = N_i(k)R_p + A_iR_a, \quad \forall k \in \mathcal{H}. \quad (22)$$

where $A_i$ denotes the area of zone $i$. $R_p$ and $R_a$ denote the average occupancy and space ventilation rate. This paper investigates two DCV strategies [2, 3], i.e., *(i) DCV I*: calculating

zone fresh air flow rates based on zone occupancy ($R_p \geq 0$, $R_a = 0$, ASHRAE Standard 62-1989-2001), and *(ii) DCV II*: computing zone fresh air flow rates by zone occupancy and space ($R_p \geq 0$, $R_a \geq 0$, ASHRAE Standard 62.1 2004-2010).

For single-zone building, the ventilation rate $(d_r)$ of DCV strategies can be easily determined provided with the fresh air and mass flow rates. However, for multi-zone cases, we need to make a balance regarding the different zone requirements and the ventilation rate $(\boldsymbol{d}_r)$ can be determined by [3]

$$m^{z,\text{fresh}}(k) = \sum_{i\in\mathcal{I}} m_i^{z,\text{fresh}}(k), \quad m^z(k) = \sum_{i\in\mathcal{I}} m_i^z(k),$$
$$Z(k) = \max_{i\in\mathcal{I}} \left\{ \frac{m_i^{z,\text{fresh}}(k)}{m_i^z(k)} \right\}, \quad X(k) = \frac{m^{z,\text{fresh}}(k)}{m^z(k)}, \quad (23)$$
$$Y(k) = \frac{X(k)}{1 + X(k) - Z(k)}, \quad d_r(k) = 1 - Y(k), \quad \forall k \in \mathcal{H}.$$

As the DCV strategies only determine fresh air for IAQ, we calculate the zone mass flow rates $m_i^z(k)$ in (23) using the same decentralized method as the ULC of TLDM to make a fair comparison. Similarly, the DCVs start with $d_r(k) = d_r^{\max}$ ($\forall k \in \mathcal{H}$) while computing zone mass flow rates $m_i^z(k)$ by solving problem $(\mathcal{P}_U)$. After that $d_r(k)$ is amended according to (23). The above two procedures are repeated until convergence.

TABLE II
PERFORMANCE COMPARISONS

| Method | $R_p$ (L/p) | $R_a$ (L/m²) | Cost (L/m²) | Time (s) | TC | IAQ |
|---|---|---|---|---|---|---|
| DTBSS | - | - | 245.03 | 2.11 | Y | N |
| Centralized | - | - | 247.15 | 575.07 | Y | Y |
| SQP | - | - | 275.91 | 151.26 | Y | Y |
| DCV I | 21 | 0 | 276.23 | - | Y | Y |
| DCV II | 16 | 0.04 | 274.99 | - | Y | Y |
| TLDM | - | - | 257.02 | 5.21 | Y | Y |

N=No, Y=Yes.

For the *benchmark*, the numeric results on energy cost, average stage computation time (using MATLAB R2016a on PC with Intel(R) Core(TM) i7-5500U CPU @2.40GHZ processor), satisfaction of thermal comfort (TC) and IAQ inducted by the different methods are contrasted in TABLE II. In particular, as the DCVs generally depend on *off-line* regulation of the occupancy ($R_p$) and space ($R_a$) ventilation rate for the DCVs, the average stage computation time for the other methods are investigated and compared in this paper. Accordingly, the zone mass flow rates, zone temperature, zone CO2 concentration, and ventilation rates $\boldsymbol{d}_r$ are displayed in Fig. 4-7. First of all, we find that the thermal comfort can be achieved by each of the methods. However, the requirements on IAQ may be violated for the DTBSS as shown in Fig. 6. As aforementioned, this is caused by the fact that DTBSS hasn't been able to involve IAQ in HVAC control. Therefore, the method prefers a lower zone mass flow rates and ventilation rate (large $\boldsymbol{d}_r$) to achieve the energy cost saving target as indicated in Fig. 4 and 7. That means that the HVAC energy cost may be saved at the expense of awful IAQ.

In the subsequent, we report our results both from energy cost and computational efficacy. First, by investigating the energy cost inducted by the other methods, we observe that the proposed TLDM provides lower energy cost compared

with the DCVs (i.e., DCV I, II) and the SQP while both maintaining thermal comfort and IAQ. Specifically, the energy cost has been reduced by about $7.0\%$ compared with the other three methods. In particular, we note from Fig. 6 that the peak levels of zone CO2 concentrations are very close under these methods. This implies the fairness of our comparison as the HVAC energy cost are calculated under the same IAQ. By inspecting the computation time, we find that the average computation time of TLDM for each individual zone at each decision epoch is about $5.21s$ (in parallel). We see a slight growth in computation time versus DTBSS, which may be attributed to the LLC to achieve IAQ in TLDM. Besides, we see that the TLDM outperforms *SQP* both with lower energy cost and less computation time required. Finally, through comparisons with centralized method, we imply that the sub-optimality (energy cost) of TLDM is around $4\%$. This is negligible compared to the computational advantages gained as shown in TABLE II.
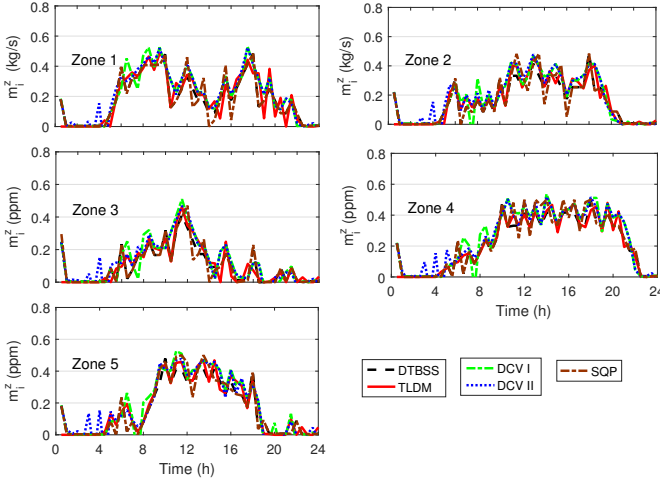


Fig. 6.  Zone CO2 (*Benchmark*).



Fig. 4.  Zone mass flow rate (*Benchmark*).



Fig. 7.  Ventilation rate $\boldsymbol{d}_r$ (*benchmark*).

Considering that centralized method and *SQP* (centralized) are computationally intractable for such cases, we compare our method with the other three methods (i.e., DTBSS, and DCV I, II). For each case, we randomly generate some networks to describe the thermal connectivity among the different zones. In particular, the maximum number of adjacent zones for each zone is set as $4$ considering the common practice. In particular, to maintain the same level of IAQ (indicated by the peak level of CO2 concentration) and guarantee a fair comparison, both the occupancy ($R_p$) and space ventilation rate ($R_a$) are adjusted and recorded in TABLE III. The other parameters refer to the *benchmark* in Section IV-A.

TABLE III
OCCUPANCY AND SPACE VENTILATION RATES IN DCV I, II

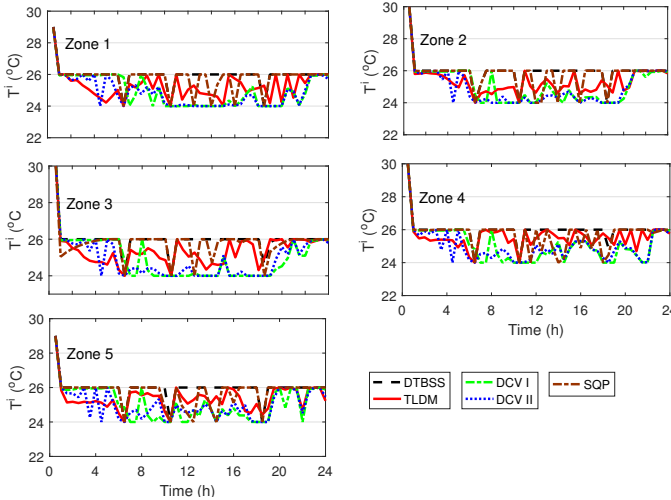| #zones | DCV I | DCV II | |
|---|---|---|---|
| | $R_p$ (L/p) | $R_p$ (L/p) | $R_a$ (L/m²) |
| 10 | 19 | 15 | 0.03 |
| 20 | 20 | 19 | 0.03 |
| 50 | 21 | 19 | 0.03 |
| 100 | 23 | 21 | 0.03 |



Fig. 5.  Zone temperature (*Benchmark*).

## B. Scalability

In this part, the proposed TLDM is applied to *medium* scale (10,20 zones) and *large* scale (50,100 zones) cases.
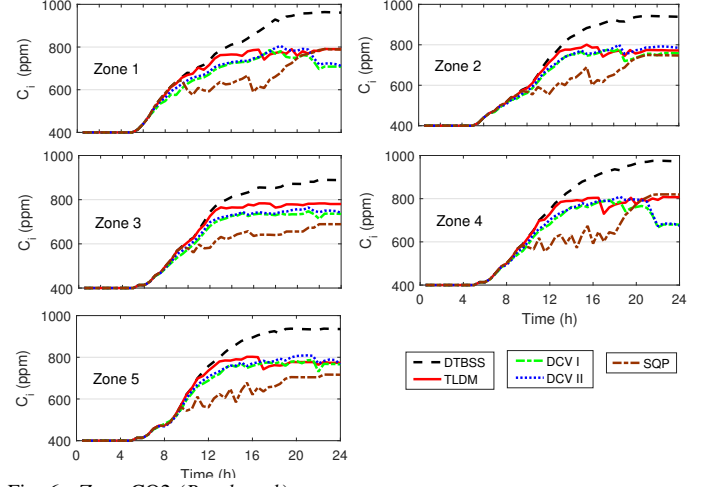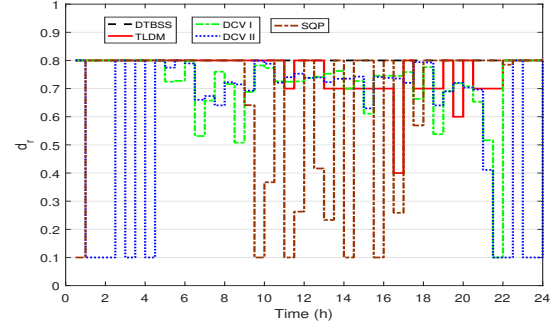
We mainly investigate the energy cost and average stage computation time of the different methods under the different cases as shown in TABLE IV. Compared with *DTBSS*, we observe a minor increase (4-5%) in energy cost and average stage computation time. Similar to the *benchmark*, this is mainly caused by the LLC in TLDM to regulate the ventilation rates to achieve IAQ, which can be perceived from Fig. 12.

However, the proposed method is still scalable and computationally efficient considering the average stage computation time (e.g., 21.68s for 100-zone case) *versus* the decision epoch 30mins. Besides, we imply similar results of TLDM in energy cost savings compared with the DCVs (i.e., DCV I, II). Particularly, while both maintaining thermal comfort and IAQ, the proposed TLDM can reduce the energy cost by about 8.0-9.8% (DCV I) and 8.1-10.2% (DCV II). Except for the capability in energy cost reduction, the proposed TLDM is supposed to be more preferable in application compared with the DCVs. Specifically, as aforementioned, the *incompatible* occupancy or space ventilation rate ($R_a$, $R_p$) of the DCVs for different cases. This implies that the practical deployments of DCVs for different cases depend on the repeated regulation of occupancy ($R_p$) and ventilation rate ($R_a$) *off-line*. On the contrary, the TLDM is amenable to different scenarios as it allows to dynamically adjust the ventilation rate ($\boldsymbol{d}_r$) *on-line*.

TABLE IV
PERFORMANCE COMPARISONS

| Method | *Medium* | | | | | | TC | IAQ |
|---|---|---|---|---|---|---|---|---|
| | 10 | | | 20 | | | | |
| | Cost (s$) | +/- (%) | Time (s) | Cost (s$) | +/- (%) | Time (s) | | |
| TLDM | 407.32 | - | 6.55 | 939.83 | - | 8.03 | Y | Y |
| DTBSS | 387.07 | -4.97 | 2.35 | 887.12 | -5.61 | 2.71 | Y | N |
| DCV I | 447.42 | +9.84 | - | 1015.50 | +8.05 | - | Y | Y |
| DCV II | 440.42 | +8.13 | - | 1020.60 | +8.59 | - | Y | Y |

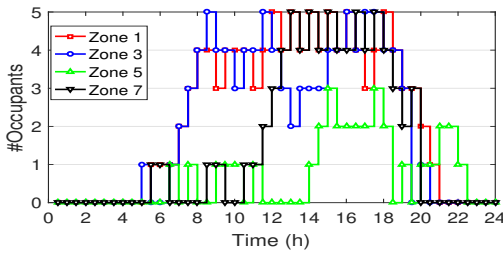| Method | *Large* | | | | | | TC | IAQ |
|---|---|---|---|---|---|---|---|---|
| | 50 | | | 100 | | | | |
| | Cost (s$)×10³ | +/- (%) | Time (s) | Cost (s$)×10³ | +/- (%) | Time (s) | | |
| TLDM | 2.65 | - | 13.28 | 5.80 | - | 21.68 | Y | Y |
| DTBSS | 2.54 | -4.15 | 4.47 | 5.49 | -5.34 | 6.90 | Y | N |
| DCV I | 2.91 | +9.81 | - | 6.31 | +8.79 | - | Y | Y |
| DCV II | 2.92 | +10.19 | - | 6.36 | +9.66 | - | Y | Y |

**N**=No, **Y**=Yes.


Fig. 8. The dynamic occupancy for the three zones among $I = 50$ zones.

As an instance, we further investigate the numeric results of the 50-zone case study. Specifically, the zone occupancy (Fig. 8), zone mass flow rates, zone temperature, and zone CO2 for three randomly selected zones are inspected as displayed in Fig. 8-11. From Fig. 10 and Fig. 11, we see that both the proposed *TLDM* and the DCV strategies (DCV I, II) can achieve the user-defined zone temperature ($[24, 26]°$C) and CO2 bounds ($\leq 800$ ppm). In particular, we observe that the peak level of zone CO2 concentrations are very close under the TLDM and the DCV strategies (DCV I, II). This implies our fair comparison as the numeric results of the three methods are investigated under the same level of IAQ. By observing
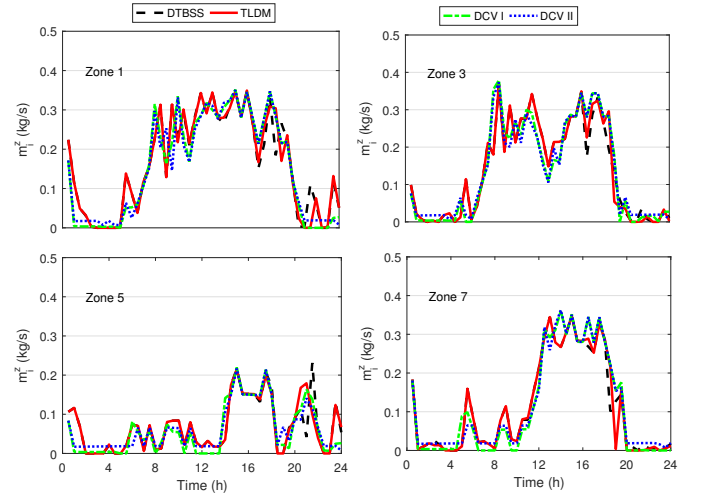

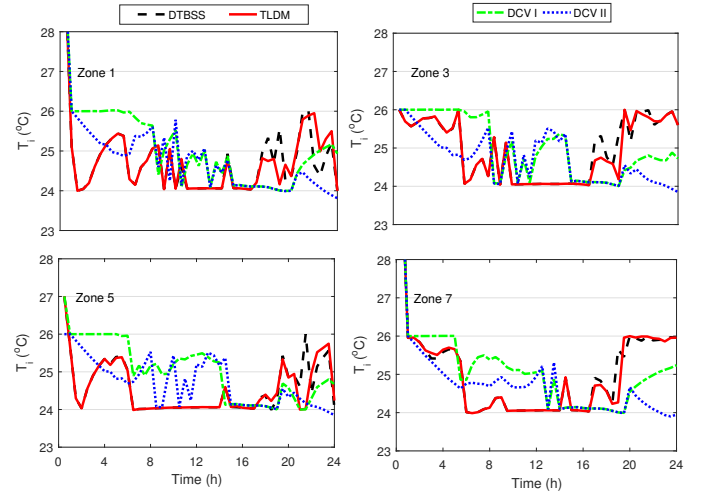Fig. 9. The zone air flow rates for the three zones among $I = 50$ zones.


Fig. 10. The temperature for the three zones among $I = 50$ zones.

the results of TLDM in Fig. 10, one may observe some particular phenomena that can reflect some characteristics of the proposed method. Specifically, we see that the zone temperature is close to the lower zone temperature bounds during the working hours with high occupancy. This phenomenon is caused by the *two-phase* method in the *lower* level of TLDM. As aforementioned, the LLC will first attempt to adjust zone mass flow rates to achieve the user-defined zone IAQ and thermal comfort simultaneously. If impossible, the ventilation rate ($d_r$) is increased afterwards.

Besides, the ventilation rate ($\boldsymbol{d}_r$) are compared in Fig. 12. For *TLDM*, we see that in the off-working hours with the relatively low zone occupancy, the computed ventilation rate (larger $\boldsymbol{d}_r$) is relatively low. However, during the working hours with high occupancy, the required ventilation rate (smaller $\boldsymbol{d}_r$) is apparently increased. This phenomenon is rational as the energy cost has been selected as the objective. When the indoor occupancy is low, it is possible to dilute the generated CO2 by only regulating zone mass flow rates. There is usually no need to increase fresh air infusion (decrease $d_r$), which tends to yield high energy cost. However, for the time periods with high occupancy, it's generally inadequate by
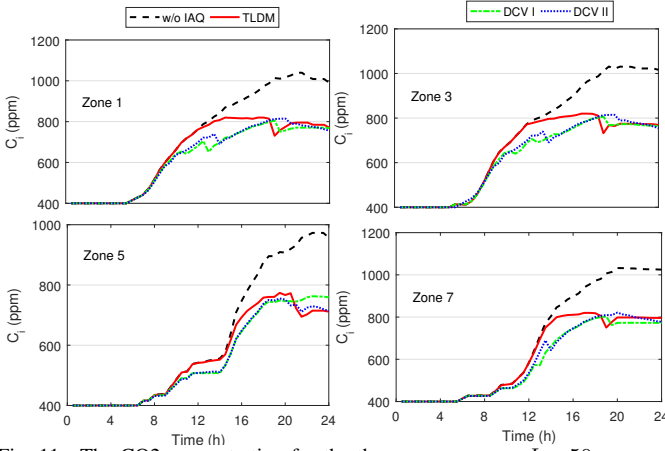
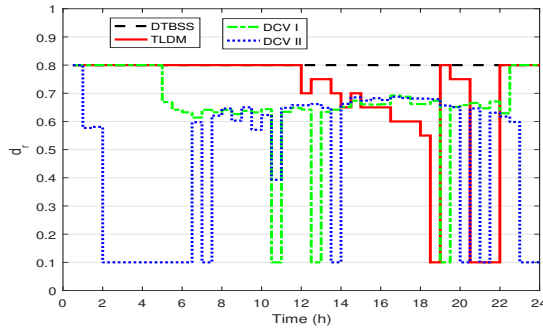Fig. 11. The CO2 concentration for the three zones among $I = 50$ zones



Fig. 12. The ventilation rate ($\boldsymbol{d}_r$) of the HVAC system

and non-convex (NP-hard problem) due to the complex HVAC system behaviors. To cope with the computational difficulties, we proposed a two-level distributed method (TLDM) based on the special structure of the problem: the zone temperature and CO2 dynamics are independent Specifically, we divided the problem into two levels: *upper* and *lower* control. The ULC first computes the "optimal" zone mass flow rates to guarantee thermal comfort and the LLC regulates zone mass flow rates and the ventilation rate (fresh air infusion) to achieve IAQ based on zone CO2 dynamics. As both the *upper* and *lower* level control use distributed methods with mainly zone level computation, this method was demonstrated capable and scalable compared with centralized method, distributed Token-based scheduling strategy (DTBSS), and the widely-used demand controlled ventilation strategies (DCVs). The numeric results implies that the sub-optimality of the proposed method is around $4\%$, which is acceptable compared with the computational benefits gained from the proposed method. Different from DTBSS which only considers thermal comfort, the proposed TLDM can maintain thermal comfort and IAQ simultaneously with a minor increase of energy cost. Besides, compared with the DCVs, we conclude that about $8\% - 10\%$ of energy cost for HVAC system can be reduced while maintaining both the same thermal comfort and IAQ.

only regulating zone mass flow rates to achieve both thermal comfort and IAQ and the fresh air infusion is required to be increased (decrease $\boldsymbol{d}_r$). However, the results for the DCV strategies are different. For DCV I, we see that the ventilation rates ($\boldsymbol{d}_r$) correspond well to the occupancy, which is opposed to that for *DCV II*. Particularly, for *DCV II*, the ventilation rate (smaller $d_r$) is relatively high during off-working hours with low occupancy. Conversely, we observe relatively low ventilation rate (larger $d_r$) during the working hours with high occupancy. The above phenomena are attributed to the rules that used to determine zone fresh air flow rates in the two DCV strategies as discussed earlier. For DCV I, the zone fresh air flow rates totally depend on the occupancy and that results in the consistent pace of ventilation rate ($d_r$) and occupancy as observed. However, for DCV II, the zone mass flow rates are jointly determined by the occupancy and space. In other word, the *DCV II* adopts an extra constant space ventilation rate except for the dynamic occupancy ventilation rate. Therefore, during the off-working hours with low occupancy, we can observe higher ventilation rate surprisingly as the zone fresh air flow rates dominate in the low zone mass flows rates that required to guarantee thermal comfort.

## V. CONCLUSION

This paper investigated scalable control method for commercial HVAC systems with the objective to reduce energy cost while maintaining thermal comfort and IAQ (represented by CO2) simultaneously. This problem is intrinsically non-linear

## REFERENCES

[1] N. Radhakrishnan, Y. Su, R. Su, and K. Poolla, "Token based scheduling for energy management in building HVAC systems," *Applied energy*, vol. 173, pp. 67–79, 2016.
[2] Z. Sun, S. Wang, and Z. Ma, "In-situ implementation and validation of a CO2-based adaptive demand-controlled ventilation strategy in a multi-zone office building," *Building and Environment*, vol. 46, no. 1, pp. 124–133, 2011.
[3] K. Shan, Y. Sun, S. Wang, and C. Yan, "Development and in-situ validation of a multi-zone demand-controlled ventilation strategy using a limited number of sensors," *Building and environment*, vol. 57, pp. 28–37, 2012.
[4] Y. Yang, G. Hu, and C. J. Spanos, "HVAC Energy cost optimization for a multi-zone building via a decentralized approach." Available [online]: https://arxiv.org/abs/1905.10934, 2019. Accessed Feb. 17, 2020.
[5] K. Ku, J. Liaw, M. Tsai, and T. Liu, "Automatic control system for thermal comfort based on predicted mean vote and energy saving," *IEEE Transactions on Automation Science and Engineering*, vol. 12, no. 1, pp. 378–383, 2015.
[6] H. Dasi, F. Xiaowei, and C. Daisheng, "On-line control strategy of fresh air to meet the requirement of IAQ in office buildings," in *2010 5th IEEE Conference on Industrial Electronics and Applications*, pp. 845–848, IEEE, 2010.
[7] W. Li, C. Koo, S. H. Cha, T. Hong, and J. Oh, "A novel real-time method for HVAC system operation to improve indoor environmental quality in meeting rooms," *Building and Environment*, vol. 144, pp. 365–385, 2018.

[8] H. Mirinejad, K. C. Welch, and L. Spicer, "A review of intelligent control techniques in HVAC systems," in *2012 IEEE Energytech*, pp. 1–5, IEEE, 2012.

[9] A. Afram and F. Janabi-Sharifi, "Theory and applications of HVAC control systems–a review of model predictive control (MPC)," *Building and Environment*, vol. 72, pp. 343–355, 2014.

[10] A. Kelman and F. Borrelli, "Bilinear model predictive control of a HVAC system using sequential quadratic programming," in *Ifac world congress*, vol. 18, pp. 9869–9874, 2011.

[11] Y. Yang, G. Hu, and C. J. Spanos, "Stochastic optimal control of HVAC system for energy-efficient buildings," *arXiv preprint arXiv:1911.00840*, 2019.

[12] Z. Xu, G. Hu, C. J. Spanos, and S. Schiavon, "PMV-based event-triggered mechanism for building energy management under uncertainties," *Energy and Buildings*, vol. 152, pp. 73–85, 2017.

[13] C. C. Okaeme, S. Mishra, and J. T.-Y. Wen, "Passivity-based thermohygrometric control in buildings," *IEEE Transactions on Control Systems Technology*, vol. 26, no. 5, pp. 1661–1672, 2017.

[14] C. C. Okaeme, S. Mishra, and J. T. Wen, "A comfort zone set-based approach for coupled temperature and humidity control in buildings," in *2016 IEEE International Conference on Automation Science and Engineering (CASE)*, pp. 456–461, IEEE, 2016.

[15] N. Nassif, S. Kajl, and R. Sabourin, "Ventilation control strategy using the supply CO2 concentration setpoint," *Hvac&R Research*, vol. 11, no. 2, pp. 239–262, 2005.

[16] T. R. Nielsen and C. Drivsholm, "Energy efficient demand controlled ventilation in single family houses," *Energy and buildings*, vol. 42, no. 11, pp. 1995–1998, 2010.

[17] A. Hesaraki, J. A. Myhren, and S. Holmberg, "Influence of different ventilation levels on indoor air quality and energy savings: A case study of a single-family house," *Sustainable cities and society*, vol. 19, pp. 165–172, 2015.

[18] Z. Wang and L. Wang, "Intelligent control of ventilation system for energy-efficient buildings with CO2 predictive model," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 686–693, 2013.

[19] N. Nassif, "A robust CO2-based demand-controlled ventilation control strategy for multi-zone HVAC systems," *Energy and buildings*, vol. 45, pp. 72–81, 2012.

[20] M. Marinov, T. Djamiykov, B. Ganev, and V. Zerbe, "Sensor-based multi-zone demand-controlled ventilation,"

[21] A. Parisio, M. Molinari, D. Varagnolo, and K. H. Johansson, "A scenario-based predictive control approach to building HVAC management systems," in *2013 IEEE International Conference on Automation Science and Engineering (CASE)*, pp. 428–435, IEEE, 2013.

[22] A. Parisio, D. Varagnolo, D. Risberg, G. Pattarello, M. Molinari, and K. H. Johansson, "Randomized model predictive control for HVAC systems," in *Proceedings of the 5th ACM Workshop on Embedded Systems For Energy-Efficient Buildings*, pp. 1–8, ACM, 2013.

[23] A. Parisio, D. Varagnolo, M. Molinari, G. Pattarello, L. Fabietti, and K. H. Johansson, "Implementation of a scenario-based MPC for HVAC systems: an experimental case study," *IFAC Proceedings Volumes*, vol. 47, no. 3, pp. 599–605, 2014.

[24] L. Yu, D. Xie, C. Huang, T. Jiang, and Y. Zou, "Energy Optimization of HVAC Systems in Commercial Buildings Considering Indoor Air Quality Management," *IEEE Transactions on Smart Grid*, 2018.

[25] Hattersley, "Variable flow vs constant flow." Available [online]: https://www.hattersley.com/page/hvac/variable-flow-systems. Accessed Dec. 20, 2019.

[26] X. Zhang, W. Shi, B. Yan, A. Malkawi, and N. Li, "Decentralized and distributed temperature control via HVAC systems in energy efficient buildings," *arXiv preprint arXiv:1702.03308*, 2017.

[27] Y. Lin, T. Middelkoop, and P. Barooah, "Issues in identification of control-oriented thermal models of zones in multi-zone buildings," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*, pp. 6932–6937, IEEE, 2012.

[28] M. Maasoumy, A. Pinto, and A. Sangiovanni-Vincentelli, "Model-based hierarchical optimal control design for HVAC systems," in *ASME 2011 Dynamic Systems and Control Conference and Bath/ASME Symposium on Fluid Power and Motion Control*, pp. 271–278, American Society of Mechanical Engineers, 2011.

[29] I. Korolija, L. Marjanovic-Halburd, Y. Zhang, and V. I. Hanby, "Influence of building parameters and hvac systems coupling on building energy performance," *Energy and Buildings*, vol. 43, no. 6, pp. 1247–1253, 2011.

[30] G. P. McCormick, "Computability of global solutions to factorable nonconvex programs: Part Iconvex underestimating problems," *Mathematical programming*, vol. 10, no. 1, pp. 147–175, 1976.

[31] N. Chatzipanagiotis, D. Dentcheva, and M. M. Zavlanos, "An augmented lagrangian method for distributed optimization," *Mathematical Programming*, vol. 152, no. 1-2, pp. 405–434, 2015.

[32] R. Jia, R. Dong, S. S. Sastry, and C. J. Sapnos, "Privacy-enhanced architecture for occupancy-based HVAC control," in *Cyber-Physical Systems (ICCPS), 2017 ACM/IEEE 8th International Conference on*, pp. 177–186, IEEE, 2017.

[33] S. Nagarathinam, A. Vasan, V. Ramakrishna P, S. R. Iyer, V. Sarangan, and A. Sivasubramaniam, "Centralized management of HVAC energy in large multi-ahu zones," in *Proceedings of the 2nd ACM International Conference on Embedded Systems for Energy-Efficient Built Environments*, pp. 157–166, ACM, 2015.

[34] J. Nocedal and S. J. Wright, "Sequential quadratic programming," *Numerical optimization*, pp. 529–562, 2006.

[35] Y. Kawajir, C. Laird, and A. Wachter, "Introduction to IPOPT: A tutorial for downloading, installing, and using IPOPT," 2006.