HVAC Energy Cost Optimization for a Multi-zone Building via A Decentralized Approach

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***Abstract*—The control of heating, ventilation and air-conditioning (HVAC) systems has raised extensive attention due to their high energy consumption and their operation patterns far from being energy-efficient. However, most of the existing methods suffer limitations from scalability and computation efficiency due to the centralized structures for large buildings. To compensate for such defects, this paper studies the scalable control of multi-zone HVAC systems with the objective to reduce their energy cost while satisfying zone thermal comfort. In particular, the thermal couplings due to heat transfer among the adjacent zones are incorporated, which has been ignored or not well tackled due to complexity in the literature. To overcome the computational challenges of the non-linear and non-convex problem caused by the complex system dynamics, this paper proposed a tailored decentralized approach, which mainly contains three steps: *i)* relaxing the bilinear system dynamics; *ii)* solve the relaxed non-convex problem in a decentralized manner using the Accelerated Distributed Augmented Lagrangian (ADAL) method [1]; and *iii)* recover the recursive feasibility of the solution. Through comparison with the centralized method, this approach is demonstrated with satisfactory sub-optimality. Besides, this approach is contrasted with the existing Distributed Token-Based Scheduling Strategy (DTBSS) proposed in [2]. The numeric results imply that for buildings with relatively small number of zones (less than 20), the two methods can achieve comparable performance. However, the proposed approach demonstrates better performance for larger cases with a considerable reduction of energy cost and less computation time.**

***Note to Practitioners*—This paper designs a scalable controller for HVAC systems for multi-zone buildings which is able to minimize the total electricity cost while maintaining the comfortable temperature bands set by the occupants. The proposed control technique is scalable as it's implemented in a decentralized manner by each individual zone. Specifically, the zone local controllers computes a quadratic programming (QP) subproblems to determine the operations of local Variable Air Volume (VAV) boxes to regulate their zone temperature. Consequently, this controller can be realized on simple hardware and is suitable for large buildings. The performance (the HVAC's energy cost and the average computation time) of the controller is validated by compared with the centralized method and the DTBSS proposed in [2] through simulations.**

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***Index Terms*—HVAC system, multi-zone buildings, energy cost, decentralized methods, thermal couplings.**

NOMENCLATURE

c_p	The specific heat of air $[\mathrm{kJ}/(\mathrm{kg}\cdot\mathrm{K})]$;
T_t^i	Temperature of zone i at time t [°C];
$\underline{T}^i/\overline{T}^i$	The lower/upper bound of the comfortable temperature range for zone i [°C];
$\underline{m}^{zi}/\overline{m}^{zi}$	The lower/upper bound of the zone air flow rate for zone i $[\mathrm{kg\,s^{-1}}]$;
$\overline{m}$	The upper bound of supply air flow rate by the AHU $[\mathrm{kg\,s^{-1}}]$;
c_t	the electricity price at time t [s\$/kW];
Q_t^i	The internal heat generation of zone i at time t [kW];
R_{oi}	The thermal resistance between zone i and the outside [K/kW];
R_{ij}	The thermal resistance between zone i and zone j [K/kW];
$\mathcal{N}_i$	The collection of zone i's adjacent zones;
d_r	The fraction of the return air supplied to AHU;
T_c	The set-point temperature of AHU [°C];
P_t^f	The fan power of AHU at time t [kW];
P_t^c	The cooling power of AHU at time t [kW];
η	the reciprocal of coefficient of performance (COP) of the chiller;
κ_f	the efficiency of fan within the AHU;

I. INTRODUCTION

As we may know buildings are responsible for a large proportion of the world's energy consumption [3]. In particular, wherein about 40%-50% is attributed to heating, ventilation and air-conditioning (HVAC) [3], and the number is even bigger for tropical countries like Singapore due to the perennial demand [2]. Except for high energy consumption, the operation patterns of HVAC systems are still far from being energy-efficient, which reveal dramatic potential to save energy by improving their energy efficiency [2].

As a consequence, the control of HVAC systems has raised extensive investigations in the literature. The available methods include model predictive control (MPC) [4–7], mixed-integer linear programming (MILP) [8, 9], sequential quadratic programming (SQP) [10–12], intelligent control based on fuzzy logic or genetic algorithm [13–15], and rule-based methods [16, 17]. As most of these methods are performed in a centralized manner, they tend to be computationally prohibitive or not scalable to large buildings. Decentralized methods have been recognized as viable solutions to such issue and raised considerable concerns. However, most of the existing decentralized methods are focused on temperature regulation with the control of HVAC systems circumvented and not discussed due to the system complexity [18–20]. At present, the available results on scalable control of multi-zone HVAC system are fairly limited except for [2, 21–23]. However, in the above works, the thermal coupling among the neighboring zones were ignored [21, 22] or not well tackled [2, 23], which may result in performance degradation or implementation challenges in practice.

To compensate for such defects, this paper focuses on developing a decentralized method for multi-zone HVAC control with the objective to minimize their energy cost while satisfying thermal comfort. It's challenging and nontrivial to achieve such goal concerning the following complexity. *First*, there exist various couplings among the different zones. The couplings both arise from the heat transfer among adjacent zones and the operation limits of the Air Handling Unit (AHU), which lead to various temporally and spatially coupled constraints. *Second*, the problem is nonlinear and non-convex due to complex system behaviors. Consequently, most of the existing decentralized methods established for convex problems with linear dynamics (see [24–26] for examples) can't be applied to solve the problem.

Main contributions: To overcome the challenges, this paper develops a tailored decentralized approach, which mainly contains three steps: *i)* the first step relaxes the bilinear system dynamics by introducing some auxiliary decision variables; *ii)* the second step solves a relaxed non-convex problem in a decentralized manner based on an Accelerated Distributed Augmented Lagrangian (ADAL) method proposed in [1]; *iii)* the last step recovers the recursive feasibility of the solutions by a proposed heuristic method. As it allows the zones to compute their mass flow rates of local VAV boxes by solving small-scale QP problems in parallel, this method is scalable to large buildings. To evaluate its performance, the approach is compared with a centralized method, in which the optimal solutions can be obtained for small-scale cases and the results imply that the decentralized approach can achieve satisfactory sub-optimality. Besides, this approach is contrasted with a distributed Token-Based Scheduling Strategy (DTBSS) proposed in [2]. The numeric results demonstrate that for buildings with relatively small number of zones (less than 20), the decentralized approach slightly outperform DTBSS with about $2\%\sim4\%$ reduction in energy cost. However, for larger cases, the proposed method demonstrates better performance with a considerable reduction of energy cost and computation time.

The remainder of this paper is outlined. In Section II, the related works on decentralized methods for HVAC control are carefully reviewed. In Section III, the problem formulation is presented. In Section IV, the decentralized approach is introduced. In Section V, the performance and scalability of the decentralized approach is validated through case studies. In Section VI, we briefly conclude this paper.

II. Related Works

This part reviews the available decentralized methods for thermal comfort control. Generally, these methods can be broadly divided into two categories based on the control variables. The first category is mainly focused on zone temperature regulation [18–20]. Specifically, the zone temperature are regarded as the control variables. However, the control the HVAC systems (e.g., AHU, VAV boxes, etc.) to achieve the desired zone temperature has been circumvented or not discussed mainly due to the complex system dynamics. More practically, the other category directly studies the control of HVAC system. For such cases, the problems are generally challenging and nontrivial. *First*, the complex thermal dynamics need to be tackled, which makes the problem non-linear and non-convex. *Second*, there exist various couplings among different zones, which both arise from the thermal couplings among the adjacent zones due to heat transfer and the operation limits of the AHU. To simplify discussions, most of the existing works ignored the thermal couplings among the neighboring zones [21, 22] or regarded them as external disturbances that supposed be measured through sensors or learned from data [2, 23]. As a scarce attempt, [27] explicitly discussed the thermal couplings while developing a distributed MPC strategy for multi-zone HVAC system. To cope with the difficulties due to the nonlinearity and non-convexity, a distributed Alternating Direction Method of Multipliers (ADMM) method [26] was applied based on some convexity approximations.

Generally speaking, the thermal couplings among the neighboring zones haven't been well studied due to the complexity while developing scalable control methods for HVAC systems, which may lead to performance deviations both in energy costs and thermal conditions in practice. Complementary to the existing works, this paper studies the decentralized control of multi-zone HVAC system, which incorporates the thermal couplings among the neighboring zones in the optimization.

III. Problem Formulation

A. HVAC Systems for Multi-zone Buildings

A typical schematic of a multi-zone HVAC system is shown in Fig. 1. The main parts include the AHU, the VAV boxes, and the chiller. The central AHU is shared by multi-zones, which is equipped with a damper, a cooling/heating coil and a supply fan. The damper is responsible for mixing the return air from inside and the fresh air from outside. The heating/cooling coil can cool down/heat up the mixed air to a set-point temperature. Without loss of generality, this paper investigates the cooling mode of the HVAC system. Generally, the temperature of the supply air out of AHU is 12-16°C. There usually exist a local VAV box related to each zone, which consists of a damper and an heating coil. This damper can regulate air flow rate supplied to the zone, while the heating coil can reheat the supply air before supplied to the zone if necessary (not discussed in this paper). We refer the readers to [10, 28] for more details on such systems.

For the control of multi-zone HVAC systems, this paper discusses minimizing their total energy cost while satisfying the zone thermal comfort over the optimization horizon. The

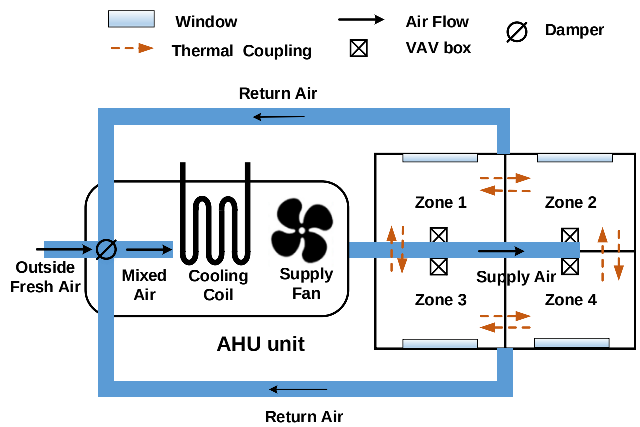

Fig. 1. The schematic of multi-zone HVAC systems.

problem is discussed in a discrete-time framework with the optimization horizon (one day) equally divided into $T=48$ stages, corresponding to a decision interval $\Delta_t = 30$ mins.

B. Zone Thermal Dynamics

This paper studies the control of a multi-zone building with I thermal zones indexed by $\mathcal{I}=\{1,2,\cdots,I\}$ over the optimization horizon $\mathcal{T}=\{0,1,\cdots,T-1\}$. The thermal dynamics of zone i ($\forall i \in \mathcal{I}$) can be described based on the Resistance-Capacitance (RC) network [29, 30], i.e.,

$$\begin{aligned} C_i(T^i_{t+1}-T^i_t) &= \sum_{j\in\mathcal{N}_i}\frac{T^j_t-T^i_t}{R_{ij}}\Delta_t \\ &+\frac{T^o_t-T^i_t}{R_{oi}}\Delta_t + c_p m^{zi}_t(T_c-T^i_t)\Delta_t + Q^i_t\Delta_t \end{aligned} \tag{1}$$

where C_i is the zone air heat capacity. The internal zone heat generation Q^i_t is affected by multiple factors, such as occupancy, devices and solar radiation, etc.

If we define $A^{ii}=1-(\sum_{j\in\mathcal{N}_i}\frac{\Delta_t}{R_{ij}C_i}+\frac{\Delta_t}{C_iR_{oi}})$, $A^{ij}=\frac{\Delta_t}{C_iR_{ij}}$, $C^{ii}=-\frac{\Delta_t\cdot c_p}{C_i}$, and $D^{ii}_t=\frac{\Delta_t T^o_t}{C_iR_{oi}}+\frac{\Delta_t\cdot Q^i_t}{C_i}$, the zone thermal dynamics in (2) can be compactly described as

$$T^i_{t+1}=A^{ii}T^i_t+\sum_{j\in\mathcal{N}_i}A^{ij}T^j_t+C^{ii}m^{zi}_t(T^i_t-T_c)+D^{ii}_t \tag{2}$$

C. The AHU

As introduced in Section II-A, the AHU is responsible for cooling down the mixed air to the set-point temperature. The main parameters related to the AHU include 1) d_r ($0\le d_r\le 1$): the fraction of the return air from inside. 2) T_c: the set-point temperature of the supply air. 3) T^r_t: the average temperature of the return air from inside. 4) T^m_t: the average temperature of the mixed air. The settings of d_r and T_c are usually fixed and determined by experiences, whereas T^r_t and T^m_t are dynamically changing with the control of the HVAC system. Specifically, the average temperature of the return air from inside at time t can be estimated as

$$T^r_t=\frac{\sum_{i=1}^I m^{zi}_tT^i_t}{\sum_{i=1}^I m^{zi}_t} \tag{3}$$

Accordingly, the average temperature of the mixed air at time t can be calculated as

$$\begin{aligned} T^m_t &= (1-d_r)T^o_t + d_rT^r_t \\ &= (1-d_r)T^o_t + d_r\frac{\sum_{i=1}^I m^{zi}_tT^i_t}{\sum_{i=1}^I m^{zi}_t} \end{aligned} \tag{4}$$

As the AHU is mainly composed of the cooling coil and the supply fan, its total energy consumption is mainly produced by the two parts. Specifically, the cooling power of the cooling coil can be calculated as

$$\begin{aligned} P^c_t &= c_p\eta(\sum_{i=1}^I m^{zi}_t)(T^m_t-T_c) \\ &= c_p\eta(1-d_r)\sum_{i=1}^I m^{zi}_t(T^o_t-T_c)+c_p\eta d_r\sum_{i=1}^I m^{zi}_t(T^i_t-T_c) \end{aligned} \tag{5}$$

As described in (5), the cooling power can be interpreted as two parts, i.e., *i)* the first part results from the proportion of outside fresh air, and *ii)* the second part caused by the proportion of return air from inside.

According to [22] and [27], the energy consumption of the supply fan within the AHU can be estimated as

$$P^f_t=\kappa_f(\sum_{i=1}^I m^{zi}_t)^3 \tag{6}$$

Thus, the total power consumption of the HVAC system at time t is gathered as

$$\begin{aligned} P^c_t+P^f_t =& c_p(1-d_r)\sum_{i=1}^I m^{zi}_t(T^o_t-T_c) \\ &+c_pd_r\sum_{i=1}^I m^{zi}_t(T^i_t-T_c)+\kappa_f(\sum_{i=1}^I m^{zi}_t)^3 \end{aligned} \tag{7}$$

From above, one may note that the power consumption of the HVAC system is a nonlinear function w.r.t. the control and state variables m^{zi}_t and T^i_t ($i\in\mathcal{I}$).

D. System Constraints

The operation of the HVAC system is subject to *i)* the zone thermal comfort requirements in (8a); *ii)* the operation limits of the local VAV boxes in (8b) ; and *iii)* the operation limits of the AHU in (8c).

$$\underline{T}^i \le T^i_t \le \overline{T}^i, \quad \forall i\in\mathcal{I},\ t\in\mathcal{T}. \tag{8a}$$

$$\underline{m}^{zi} \le m^{zi}_t \le \overline{m}^{zi}, \quad \forall i\in\mathcal{I},\ t\in\mathcal{T}. \tag{8b}$$

$$\sum_{i=1}^I m^{zi}_t \le \overline{m},\ \forall t\in\mathcal{T}. \tag{8c}$$

As in (8a), this paper uses zone temperature ranges to describe the zone thermal comfort requirements. To accommodate personalized thermal comfort, $\underline{T}^i$ and $\overline{T}^i$ may be customized. The lower/upper bounds of zone air flow rates ($\underline{m}^{zi}/\overline{m}^{zi}$) depend on the pressure supplied by the fan in the duct system [2].

E. The Optimization Problem

To improve energy efficiency, the optimization problem to minimize the HVAC's energy cost while guaranteeing zone thermal comfort can be summarized as

$$\min_{m_t^{zi}, T_t^i} J = \sum_{t=0}^{T-1} c_t \cdot \left\{ P_t^c + P_t^f \right\} \cdot \Delta_t \qquad \textbf{(P1)}$$

Constraints: (2), $(8a) - (8c)$

One may note that problem (**P1**) is non-linear and non-convex, and the nonlinearity and non-convexity both arise from the objective function and the constraints. Moreover, the objective function is non-separable and non-decomposable w.r.t. the zones. Those make it challenging and nontrivial to find a scalable solution method.

IV. Decentralized Approach

To cope with computational challenges, this section proposes a tailored decentralized approach to solve problem (**P1**), which mainly contains three steps: *i)* the first step relaxes problem (**P1**) by introducing some auxiliary decision variables; *ii)* the second step solves a relaxed non-convex problem in a decentralized manner using the ADAL method [1]; and *iii)* the third step recovers the recursive feasibility of the solution based on a heuristic method. The details of the three steps are detailed in the remainder of this section.

Step 1: to deal with the nonlinear (bilinear) constraints in problem (**P1**), we first introduce the following auxiliary decision variables, i.e., $X_t^i = m_t^{zi}(T_t^i - T_c) \geq 0$ $(\forall i \in \mathcal{I}, t \in \mathcal{T})$ and $Y_t = \sum_{i=1}^{I} m_t^{zi}$ $(\forall t \in \mathcal{T})$. We note that X_t^i can be interpreted as the "cooling power" supplied to zone i at time t, whereas Y_t can be regarded as the total zone air flow rate supplied by the AHU at time t.

According to [31] and [27], these auxiliary decision variables X_t^i $(\forall i \in \mathcal{I},\ t \in \mathcal{T})$ are bounded by their convex and concave envelopes, i.e.,

$$\begin{aligned}
X_t^i &= m_t^{zi}(T_t^i - T_c) \\
&\geq \max\Big\{ \underline{m}^{zi}(T_t^i - T_c) + m_t^{zi}(\underline{T}^i - T_c) - \underline{m}^{zi}(\underline{T}^i - T_c), \\
&\qquad \overline{m}^{zi}(T_t^i - T_c) + m_t^{zi}(\overline{T}^i - T_c) - \overline{m}^{zi}(\overline{T}^i - T_c) \Big\}, \\
X_t^i &= m_t^{zi}(T_t^i - T_c) \\
&\leq \min\Big\{ m_t^{zi}(\overline{T}^i - T_c) + \underline{m}^{zi}(T_t^i - T_c) - \underline{m}^{zi}(\overline{T}^i - T_c), \\
&\qquad \overline{m}^{zi}(T_t^i - T_c) + m^{zi}(\underline{T}^i - T_c) - \overline{m}^{zi}(\underline{T}^i - T_c) \Big\}, \\
&\qquad \forall i \in \mathcal{I},\ t \in \mathcal{T}.
\end{aligned} \tag{9}$$

Therefore, by introducing these auxiliary decision variables, we can obtain the relaxed problem (**P2**) for problem (**P1**):

$$\begin{aligned}
\min_{m_t^{zi}, T_t^i, X_t^i, Y_t} J = \sum_{t=0}^{T-1} c_t \cdot \Big\{ & c_p \eta (1 - d_r)(T_t^o - T_c) Y_t \\
& + \kappa_f (Y_t)^3 + c_p \eta d_r \sum_{i=1}^{I} X_t^i \Big\} \cdot \Delta_t
\end{aligned} \qquad \textbf{(P2)}$$

subject to

$(8a) - (8c)$

$$T_{t+1}^i = A^{ii} T_t^i + \sum_{j \in \mathcal{N}_i} A^{ij} T_t^j + C_{ii} X_t^i + D_t^{ii}, \tag{10a}$$

$$X_t^i \geq \underline{m}^{zi}(T_t^i - T_c) + m_t^{zi}(\underline{T}^i - T_c) - \underline{m}^{zi}(\underline{T}^i - T_c), \tag{10b}$$

$$X_t^i \geq \overline{m}^{zi}(T_t^i - T_c) + m_t^{zi}(\overline{T}^i - T_c) - \overline{m}^{zi}(\overline{T}^i - T_c), \tag{10c}$$

$$X_t^i \leq m_t^{zi}(\overline{T}^i - T_c) + \underline{m}^{zi}(T_t^i - T_c) - \underline{m}^{zi}(\overline{T}^i - T_c), \tag{10d}$$

$$X_t^i \leq \overline{m}^{zi}(T_t^i - T_c) + m^{zi}(\underline{T}^i - T_c) - \overline{m}^{zi}(\underline{T}^i - T_c), \tag{10e}$$

$$\forall i \in \mathcal{I},\ t \in \mathcal{T}.$$

$$\sum_{i=1}^{I} m_t^{zi} = Y_t, \quad \forall t \in \mathcal{T}. \tag{10f}$$

where constraints (10a) inherit constraints (2) in problem (**P1**). Constraints (10b)-(10e) and (10f) related to the auxiliary decision variables X_t^i and Y_t are acquired by constraints (9) and the definition.

Step 2: observe the relaxed problem (**P2**), we note that it's still non-convex due to the global objective function but only with linear (coupled) constraints. This problem can be tackled by the ADAL method proposed in [1] in a decentralized manner. More specifically, we assume that there are $I+1$ agents indexed by $\mathcal{I} \cup \{0\}$, wherein the collection of agents in $\mathcal{I}$ corresponds to the I zones, and the virtual Agent 0 is defined to manage the total zone air flow rates supplied by the AHU.

For Agent i corresponding to zone i $(\forall i \in \mathcal{I})$, the local decision variables at time t can be gathered as

$$\boldsymbol{x}_t^i = \left(T_t^i, m_t^{zi}, X_t^i\right)^T, \ \forall i \in \mathcal{I}, t \in \mathcal{T}. \tag{11}$$

For Agent 0, the local decision variable can be defined as

$$\boldsymbol{x}_t^0 = Y_t, \ \forall t \in \mathcal{T}. \tag{12}$$

For notation, we use the vector $\boldsymbol{x}^i = [(\boldsymbol{x}_0^i)^T, (\boldsymbol{x}_1^i)^T, \cdots, (\boldsymbol{x}_{T-1}^i)^T]^T$ to represent the local decision variables of Agent i $(\forall i \in \mathcal{I} \cup \{0\})$ over the optimization horizon $\mathcal{T}$.

In this case, the global objective function in (**P2**) is decomposable w.r.t. the agents $\mathcal{I} \cup \{0\}$, i.e.,

$$\begin{aligned}
J_0(\boldsymbol{x}^0) &= \sum_{t=0}^{T-1} c_t \cdot \Big\{ c_p \eta (1 - d_r)(T_t^o - T_c) Y_t + \kappa_f (Y_t)^3 \Big\} \cdot \Delta_t. \\
J_i(\boldsymbol{x}^i) &= \sum_{t=0}^{T-1} c_t \cdot \Big\{ c_p \eta d_r X_t^i \Big\} \cdot \Delta_t, \ \forall i \in \mathcal{I}.
\end{aligned} \tag{13}$$

where $J_i(\boldsymbol{x}^i)$ $(\forall i \in \mathcal{I} \cup \{0\})$ can be regarded as the local objective function of Agent i.

And problem (**P2**) can be written in a compact form as (**P3**). The details of the transformation can refer to **Appendix** A.

$$
\begin{aligned}
&\min_{\boldsymbol{x}^i, i=0,1,2,\cdots,I} \sum_{i=0}^{I} J_i(\boldsymbol{x}_i) \\
s.t. \quad & \boldsymbol{A}_d^{ii}\boldsymbol{x}^i + \sum_{j\in\mathcal{N}_i} \boldsymbol{A}_d^{ij}\boldsymbol{x}^j = \boldsymbol{b}_d^i, \forall i \in \mathcal{I}. \\
& \sum_{i=0}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i = \boldsymbol{0}. \\
& \sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i \leq \boldsymbol{c}_d. \\
& \boldsymbol{x}^i \in \mathcal{X}^i, \forall i \in \mathcal{I}.
\end{aligned}
\tag{P3}
$$

where we have

$$
\boldsymbol{A}_d^{ii} = \begin{pmatrix} \overline{A}^{ii} & -\mathbf{I}_1 & \mathbf{0} & \cdots & \cdots & \cdots \\ \mathbf{0} & \overline{A}^{ii} & -\mathbf{I}_1 & \mathbf{0} & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{A}^{ii} & -\mathbf{I}_1 & \mathbf{0} & \cdots \end{pmatrix} \in \mathbb{R}^{(T-1)\times 3T},
$$

$$
\boldsymbol{A}_d^{ij} = \begin{pmatrix} \overline{A}^{ij} & \mathbf{0} & \cdots & \cdots & \cdots \\ \mathbf{0} & \overline{A}^{ij} & \mathbf{0} & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{A}^{ij} & \mathbf{0} & \cdots \end{pmatrix} \in \mathbb{R}^{(T-1)\times 3T},
$$

and $\boldsymbol{b}_d^i = (\overline{D}_0^{ii}, \overline{D}_1^{ii}, \cdots, \overline{D}_{T-1}^{ii})^T \in \mathbb{R}^{T-1}$, with $\overline{A}^{ii} = (A^{ii} \ \ 0 \ \ C^{ii})$, $\overline{A}^{ij} = (A^{ij} \ \ 0 \ \ 0)$ $(i,j \in \mathcal{I})$. $\mathbf{I}_1 = (1 \ 0 \ 0)$. $\overline{D}_t^{ii} = -D_t^{ii}$. $\boldsymbol{B}_d^i = \begin{pmatrix} \overline{B}^i & \mathbf{0} & \cdots & \cdots & \cdots \\ \mathbf{0} & \overline{B}^i & \mathbf{0} & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{B}^i & \mathbf{0} & \cdots \end{pmatrix} \in \mathbb{R}^{T\times 3T}$, with $\overline{B}^i = (0 \ \ 1 \ \ 0)$ $(\forall i \in \mathcal{I})$. $\boldsymbol{B}_d^0 = \begin{pmatrix} \overline{B}^0 & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \overline{B}^0 & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{B}^0 & \cdots \end{pmatrix} \in \mathbb{R}^{T\times T}$ with $\overline{B}^0 = (-1)$, $\boldsymbol{c}_d = (\overline{m} \ \ \overline{m} \ \ \cdots \ \ \overline{m})^T \in \mathbb{R}^T$. The sets $\mathcal{X}^i$ $(i \in \mathcal{I})$ represent the collection of admissible control trajectories for Agent i, which is constructed by the local constraints (8a)-(8c) and (10b)-(10e) related to zone i.

As the ADAL method [1] can not efficiently tackle the coupled inequality constraints in (**P3**), we introduce some other auxiliary decision variables $\boldsymbol{s}$ as discussed in [32]. In this case, (**P3**) is equivalent to the following problem (**P4**):

$$
\begin{aligned}
&\min_{\boldsymbol{x}^i, i=0,1,2,\cdots,I, \boldsymbol{s}^1, \boldsymbol{s}^2} \sum_{i=0}^{I} J_i(\boldsymbol{x}_i) \\
s.t. \quad & \boldsymbol{A}_d^{ii}\boldsymbol{x}^i + \sum_{j\in\mathcal{N}_i} \boldsymbol{A}_d^{ij}\boldsymbol{x}^j = \boldsymbol{b}_d^i, \quad \forall i \in \mathcal{I}. \\
& \sum_{i=0}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i = \boldsymbol{0}. \\
& \sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i - \boldsymbol{c}_d + \boldsymbol{s} = \boldsymbol{0}. \\
& \boldsymbol{x}^i \in \mathcal{X}^i, \ \forall i \in \mathcal{I}. \\
& \boldsymbol{s} \geq \boldsymbol{0}.
\end{aligned}
\tag{P4}
$$

When the ADAL method is applied to tackle problem (**P4**), we can define the following augmented Lagrangian function to eliminate the coupled equality constraints, i.e.,

$$
\begin{aligned}
&\mathbb{L}_\rho(\boldsymbol{x}^0, \boldsymbol{x}^1, \cdots, \boldsymbol{x}^I, \boldsymbol{s}, \boldsymbol{\lambda}, \boldsymbol{\gamma}, \boldsymbol{\eta}) = \sum_{i=0}^{I} J_i \\
&+ \sum_{i=1}^{I} (\boldsymbol{\lambda}^i)^T \Big(\boldsymbol{A}_d^{ii}\boldsymbol{x}^i + \sum_{j\in\mathcal{N}_i} \boldsymbol{A}_d^{ij}\boldsymbol{x}^j - \boldsymbol{b}_d^i \Big) \\
&+ \sum_{i=1}^{I} \frac{\rho}{2} \Big\| \boldsymbol{A}_d^{ii}\boldsymbol{x}^i + \sum_{j\in\mathcal{N}_i} \boldsymbol{A}_d^{ij}\boldsymbol{x}^j - \boldsymbol{b}_d^i \Big\|^2 \\
&+ \boldsymbol{\gamma}^T \Big(\sum_{i=0}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i + \boldsymbol{s}^1 \Big) + \frac{\rho}{2} \Big\| \sum_{i=0}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i + \boldsymbol{s}^1 \Big\|^2 \\
&+ \boldsymbol{\eta}^T \Big(\sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i - \boldsymbol{c}_d + \boldsymbol{s}^2 \Big) + \frac{\rho}{2} \Big\| \sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i - \boldsymbol{c}_d + \boldsymbol{s}^2 \Big\|^2
\end{aligned}
\tag{14}
$$

where $\boldsymbol{\lambda} = [(\boldsymbol{\lambda}^1)^T, (\boldsymbol{\lambda}^2)^T, \cdots, (\boldsymbol{\lambda}^I)^T]^T$ with $\boldsymbol{\lambda}^i \in \mathbb{R}^{(T-1)\times 3T}$, and $\boldsymbol{\gamma}, \boldsymbol{\eta} \in \mathbb{R}^T$ are Lagrangian multipliers. $\rho > 0$ is penalty parameter.

Thus, the primal problem of (**P4**) can be described as

$$
\begin{aligned}
&\min_{\boldsymbol{x}^0, \boldsymbol{x}^1, \cdots, \boldsymbol{x}^I, \boldsymbol{s}} \mathbb{L}_\rho(\boldsymbol{x}^0, \boldsymbol{x}^1, \cdots, \boldsymbol{x}^I, \boldsymbol{s}^1, \boldsymbol{s}^2, \boldsymbol{\lambda}, \boldsymbol{\gamma}, \boldsymbol{\eta}) \\
&\quad s.t. \quad \boldsymbol{x}^i \in \mathcal{X}^i, \forall i \in \mathcal{I}. \\
&\qquad\quad\ \ \boldsymbol{s} \geq \boldsymbol{0}.
\end{aligned}
\tag{15}
$$

Similar to standard Method of Multipliers (MMs), such as ADMM[26], the ADAL method mainly includes two steps when applied to solve problem (**P4**): *i)* solving the primal problem (15) in a decentralized manner; and *ii)* updating the Lagrangian multipliers. The details of the algorithm are displayed in **Algorithm** 1. The superscript k represents the iteration. We use $\boldsymbol{x}^k = ((\boldsymbol{x}^{0,k})^T, (\boldsymbol{x}^{1,k})^T, \cdots, (\boldsymbol{x}^{I,k})^T)^T$ to represent the collection of control trajectories for all zones at iteration k, and $\boldsymbol{x}^{-i,k} = \big((\boldsymbol{x}^{0,k})^T, \cdots, (\boldsymbol{x}^{i-1,k})^T, (\boldsymbol{x}^{i+1,k})^T, \cdots, (\boldsymbol{x}^{I,k})^T\big)^T$ denotes the collection of control trajectories for all zones except zone i. The local objective functions for Agent i $(i \in \mathcal{I} \cup \{0\})$ at iteration k is defined as

$$
\begin{aligned}
&\mathbb{L}_\rho^0(\boldsymbol{x}^0, \boldsymbol{s}, \boldsymbol{x}^{-0,k}, \boldsymbol{\lambda}^k, \boldsymbol{\gamma}^k, \boldsymbol{\eta}^k) = J_0(\boldsymbol{x}^0) \\
&+ \boldsymbol{\gamma}^T(\boldsymbol{B}_d^0\boldsymbol{x}^0) + \frac{\rho}{2} \Big\| \boldsymbol{B}_d^0 \boldsymbol{x}^0 + \sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^{i,k} \Big\|^2 \\
&+ \boldsymbol{\eta}^T \boldsymbol{s} + \frac{\rho}{2} \Big\| \sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^{i,k} - \boldsymbol{c}_d + \boldsymbol{s} \Big\|^2
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&\mathbb{L}_\rho^i(\boldsymbol{x}^i, \boldsymbol{x}^{-i,k}, \boldsymbol{s}^k, \boldsymbol{\lambda}^k, \boldsymbol{\gamma}^k, \boldsymbol{\eta}^k) = J_i(\boldsymbol{x}^i) \\
&+ \sum_{j\in\mathcal{N}_i\cup\{i\}} (\boldsymbol{\lambda}_j^k)^T \boldsymbol{A}_d^{ji} \boldsymbol{x}^i \\
&+ \frac{\rho}{2} \sum_{j\in\mathcal{N}_i\cup\{i\}} \Big\| \boldsymbol{A}_d^{ji}\boldsymbol{x}^i + \sum_{l\in\mathcal{N}_j} \boldsymbol{A}^{jl}\boldsymbol{x}^{l,k} - \boldsymbol{b}_d^j \Big\|^2 \\
&+ (\boldsymbol{\gamma}^k)^T \boldsymbol{B}_d^i \boldsymbol{x}^i + \frac{\rho}{2} \Big\| \boldsymbol{B}_d^i \boldsymbol{x}^i + \sum_{l=0, l\neq i}^{I} \boldsymbol{B}_d^l \boldsymbol{x}^{l,k} \Big\|^2 \\
&+ (\boldsymbol{\eta}^k)^T \boldsymbol{B}_d^i \boldsymbol{x}^i + \frac{\rho}{2} \Big\| \boldsymbol{B}_d^i \boldsymbol{x}^i + \sum_{l=1, l\neq i}^{I} \boldsymbol{B}_d^l \boldsymbol{x}^l - \boldsymbol{c}_d + \boldsymbol{s}^k \Big\|^2, \\
&\qquad \forall i \in \mathcal{I}.
\end{aligned}
\tag{17}
$$

The residual error of all the coupled constraints is selected as the stopping criterion with ϵ a constant threshold, i.e.,

$$\begin{aligned} r(\boldsymbol{x}^{\boldsymbol{k}},\boldsymbol{s}^k)=&\sum_{i=1}^{I}\big\|\boldsymbol{A}_d^{ii}\boldsymbol{x}^{i,k}+\sum_{j\in\mathcal{N}_i}\boldsymbol{A}_d^{ij}\boldsymbol{x}^{j,k}-\boldsymbol{b}_d^i\big\|_2 \\ +\big\|&\sum_{i=0}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^{i,k}\big\|_2+\big\|\sum_{i=1}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^{i,k}-\boldsymbol{c}_d+\boldsymbol{s}^k\big\|_2\leq\epsilon. \end{aligned} \tag{18}$$

We note that the subproblems in $\mathcal{I}$ are quadratic programming (QP) problems, which can be tackled efficiently by many existing toolboxes, such as CPLEX, and Subproblem 0 is a small-scale nonlinear optimization problem, which also can be solved efficiently.

Algorithm 1 Accelerated Distributed Augmented Lagrangian (ADAL)

1: **Initialization:** $k \leftarrow 0$, set $\boldsymbol{\lambda}^0$, $\boldsymbol{\gamma}^0$, $\boldsymbol{\eta}^0$, $\boldsymbol{x}^{i,0}$ $(\forall i\in\{0\}\cup\mathcal{I})$ and $s^{1,0}$, $\boldsymbol{s}^{2,0}$.
2: **Iteration:**
3: **Solve the primal problem** (15)**:**
4: Solve Subproblem 0, i.e.,

$$\begin{aligned} (\boldsymbol{x}^{0,k+1},\boldsymbol{s}^{k+1}) &= \arg\min_{\boldsymbol{x}^0,\boldsymbol{s}}\mathbb{L}_\rho^0(\boldsymbol{x}^0,\boldsymbol{s},\boldsymbol{x}^{-0,k},\boldsymbol{\lambda}^k,\boldsymbol{\gamma}^k,\boldsymbol{\eta}^k) \\ s.t.\ \ &\boldsymbol{x}^0\in\mathbb{R}. \\ &\boldsymbol{s}\geq\boldsymbol{0}. \end{aligned} \tag{19}$$

5: Solve the subproblems in $\mathcal{I}$, i.e.,

$$\begin{aligned} \boldsymbol{x}^{i,k+1}=&\arg\min_{\boldsymbol{x}^i}\mathbb{L}_\rho^i(\boldsymbol{x}^i,\boldsymbol{x}^{-i,k},\boldsymbol{s}^k,\boldsymbol{\lambda}^k,\boldsymbol{\gamma}^k,\boldsymbol{\eta}^k) \\ s.t.\ \ &\boldsymbol{x}^i\in\mathcal{X}^i. \end{aligned} \tag{20}$$

6: Check the stopping criterion in (18). If Yes, then stop and obtain the solution $\boldsymbol{x}^{k+1,*}$, othewise continue.
7: **Update the Lagrangian multipliers:**

$$\boldsymbol{\lambda}^{i,k+1}=\boldsymbol{\lambda}^{i,k}+\rho\Big(\boldsymbol{A}_d^{ii}\boldsymbol{x}^{i,k+1}+\sum_{j\in\mathcal{N}_i}\boldsymbol{A}_d^{ij}\boldsymbol{x}^{j,k+1}-\boldsymbol{b}_d^i\Big),\quad \forall i\in\mathcal{I}.$$

$$\boldsymbol{\gamma}^{k+1}=\boldsymbol{\gamma}^k+\rho\Big(\sum_{i=0}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^{i,k+1}\Big)$$

$$\boldsymbol{\eta}^{k+1}=\boldsymbol{\eta}^k+\rho\Big(\sum_{i=1}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^{i,k+1}-\boldsymbol{c}_d+\boldsymbol{s}^{k+1}\Big)$$

8: Set $k\rightarrow k+1$ and go to **Step** 3.

Step 3: as introduced, problem (**P4**) is an relaxed problem of the original optimization problem (**P1**). It's not difficult to note that both the zone thermal comfort and the operation limits of HVAC system can be guaranteed by solving problem (**P4**). However, the recursive feasibility of the solution can't be assured due to the introduction of the auxiliary decision variables X_t^i $(i\in\mathcal{I})$. To solve this problem, this part proposes a heuristic method to recover the recursive feasibility of the solution while still guaranteeing a satisfactory performance (the HVAC's energy cost and zone thermal comfort) by exploring the special structures of the problem. Specifically, we note that the decision variables X_t^i $(i\in\mathcal{I})$ not only "dominate" the HVAC's cost (compared with the decision variable m_t^{zi}, κ_f is relatively small) but also determine the zone temperature. Therefore, if high priority is distributed to X_t^i when recovering the recursive feasibility of the solution, the performance of the recovered solution can be retained. Following the above ideas, a heuristic method is proposed in **Algorithm** 2. We use $\hat{m}_t^{zi}$, $\hat{T}_t^i$ and $\hat{X}_t^i$ to represent the recovered solution for (**P1**), which is obtained stage by stage. Specifically, at each time t, the zone air flow rate $\hat{m}_t^{zi}$ is first determined by approaching $X_t^{i,*}$ while complying with the upper and lower bounds of zone air flow rates (Step 5). After that the zone temperature is updated accordingly (Step 7). The control sequence ($\hat{T}_t^i$, m_t^{zi} and X_t^i) $(i\in\mathcal{I})$ can be obtained by repeating this process until the end of the optimization horizon $\mathcal{T}$.

Algorithm 2 Heuristic Method to Recover the Recursive Feasibility of the Solution

1: Obtain the optimal solution $\boldsymbol{x}^*=((\boldsymbol{x}^{0,*})^T,(\boldsymbol{x}^{1,*})^T,\cdots,(\boldsymbol{x}^{I,*})^T)^T$, $\boldsymbol{X}^*=((\boldsymbol{X}^{0,*})^T,(\boldsymbol{X}^{1,*})^T,\cdots,(\boldsymbol{X}^{I,*})^T)^T$ of the relaxed problem (**P4**) according to **Algorithm** 1.
2: Obtain the initial zone temperature $\hat{T}_0^i=T_0^i$ $(\forall i\in\mathcal{I})$.
3: **for** $t\in\mathcal{T}$ **do**
4: **for** $i\in\mathcal{I}$ **do**
5: Determine the air flow rate of zone i by

$$\begin{aligned} \overline{m}_t^{zi}&=\min\big(\overline{m}^{zi},X_t^{i,*}/(\hat{T}_t^i-T_c)\big), \\ \hat{m}_t^{zi}&=\max\big(\overline{m}_t^{zi},\underline{m}^{zi})\big). \end{aligned} \tag{21}$$

6: Determine the auxiliary variable $\hat{X}_t^i$ of zone i by

$$\hat{X}_t^i=\hat{m}_t^{zi}(\hat{T}_t^i-T_c) \tag{22}$$

7: Update the temperature of zone i by

$$\hat{T}_{t+1}^i=A^{ii}\hat{T}_t^i+\sum_{j\in\mathcal{N}_i}A^{ij}\hat{T}_t^j+C^ii\hat{X}_t^i+D_t^{ii} \tag{23}$$

8: **end for**
9: **end for**
10: Output the recovered solution $\hat{m}_t^{zi}$ and $\hat{T}_t^i$ $(\forall i\in\mathcal{I},t\in\mathcal{T})$ of the original optimization problem (**P1**).

V. Numeric Results

This chapter investigates the performance of the decentralized approach through applications. *First*, we compare it with a centralized method to capture the sub-optimality of the solutions in Section V-A. *Second*, the scalability of the decentralized approach is demonstrated through comparison with the DTBSS method [2] in Section V-B.

A. Performance Evaluation

We first consider small-scale case studies with 2 and 5 zones. Similar to [33, 34], the comfortable temperature ranges are set as 24-26°C for all zones. The set-point temperature of the AHU is $T_c=15$°C. Considering that the internal zone heat generation is affected by various factors, we randomly generate thermal load curves for all zones according to a uniform

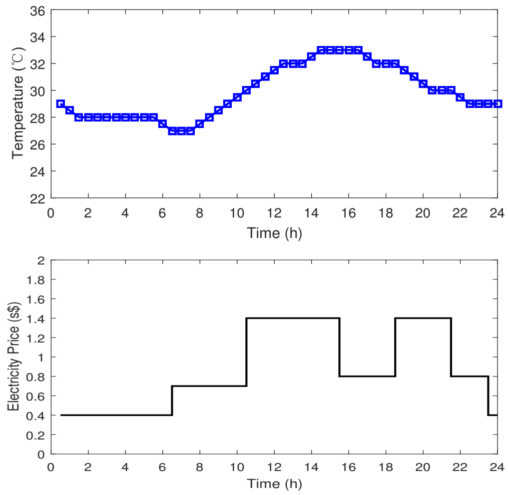

Fig. 2. (a) the outside air temperature. (b) the TOU electricity price.

distribution, i.e., $Q_t^i \sim U[0,1]kW$ $(i \in \mathcal{I}, t \in \mathcal{I})$. As an example, the thermal load scenarios for the 5-zone case study are exhibited in Fig. 3. We assume that the initial zone temperature is $[26, 28]$°C (2-zone case) and $[26, 28, 28, 27, 24]$°C (5-zone case) and there exist heat transfer among each pair of zones. The zone air flow rate bounds are set as $[0, 0.5]$ $\mathrm{kg\,s^{-1}}$ (2-zone case and 5-zone case). As an exception, the local VAV box for zone 5 is forced to be tclosed ($m_t^{zi} = 0$) over the optimization horizon in the 5-zone case study. The outside air temperature fluctuates over the time as shown in Fig. 2(a). The time-of-use (TOU) electricity price is shown in Fig. 2(b), which refers to [8]. The other parameters are gathered in TABLE I.

TABLE I
SYSTEM PARAMETERS

Param.	Value	Units
$C_i (i \in \mathcal{I})$	1.375×10^3	kJ/K
c_p	1.012	kJ/(kg · K)
R^{oi}	50	K/kW
$R^{ij} (i, j \in \mathcal{I})$	14	K/kW
k_f	0.08	-
η	1	-

TABLE II
DECENTRALIZED APPROACH VS. CENTRALIZED METHOD

#Zones	Centralized Method Cost (s$)	Centralized Method Computation (s)	Decentralized Approach Cost (s$)	Decentralized Approach Computation (s)
2	24.01	52.71	24.20	3.09
5	51.75	-	53.44	6.95

We compare the decentralized approach with a centralized method, in which the optimal solution for such two small cases can be obtained by solving small-scale nonlinear problems described in (**P1**) using the IPOPT solver. Both the HVAC's total energy cost and the average computation time for each stage incurred by the two methods are contrasted in TABLE II. One may note that the total energy cost under the two methods are comparable and there only exist a slight performance degradation (less than 3.3%) for the decentralized approach. However, we observe a substantial improvement in computation efficiency as the average computation time is apparently reduced.

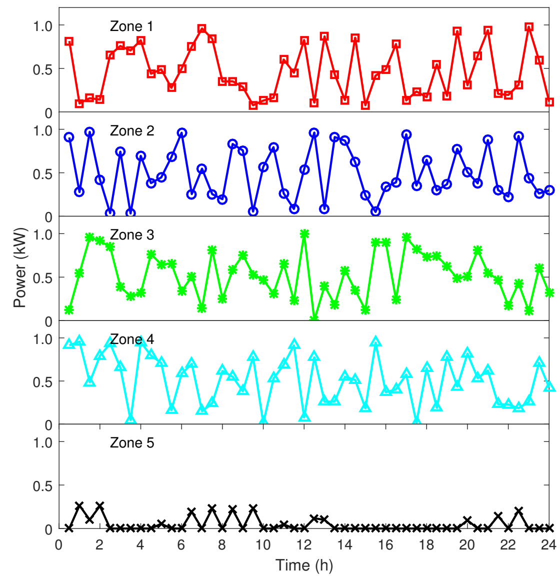

Fig. 3. The zone thermal load curves.

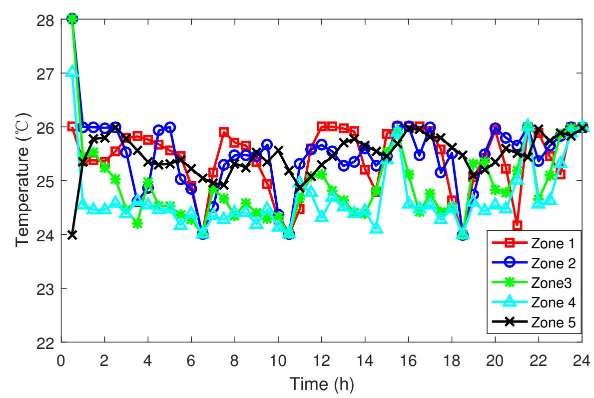

Fig. 4. The zone temperature.

Besides, the zone temperature (Fig. 4) and zone air flow rate (Fig. 5) are inspected under the decentralized approach for the 5-zone case study. We see that both the zone temperature and zone air flow rates are maintained in the desired ranges 24-26°C and $[0, 0.5]\mathrm{kg\,s^{-1}}$, respectively. In particular, the air flow rates of zone 5 are kept at *zero* as prescribed. This implies that proposed method can be applied to the general cases with customized zone settings in practice. Exceptionally, one may note some coincident valley points on the zone temperature curve the zones tend to pre-cool the area before the rise of the electricity price, thus saving their energy cost.

Further, to improve computation efficiency, we explore the convergence rate of the decentralized approach under the different penalty parameter ρ. In the 5-zone case study, the convergence rates of primal cost and the residual error of the coupled constraints under the different penalty parameters , i.e., $\rho = 1$, $\rho = 3$, $\rho = 5$, $\rho = 10$, $\rho = 15$ and $\rho = 20$ are

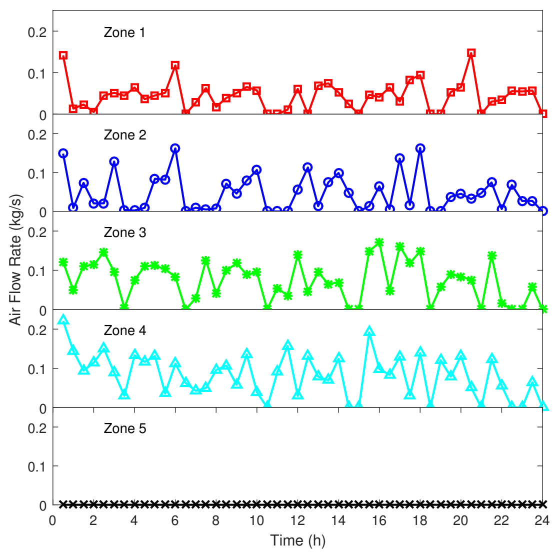

Fig. 5. The zone air mass flow rates.

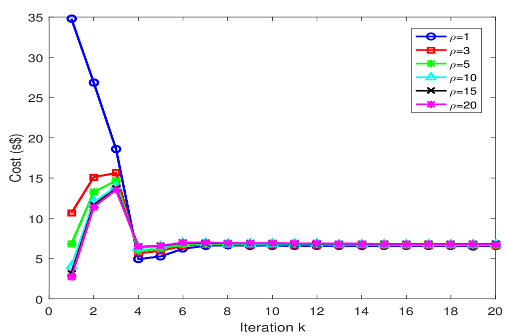

Fig. 6. The convergence rate of primal cost under the different penalty parameters ρ.

studied as shown in Fig. 6 and Fig. 7. The results imply that a larger penalty parameter ρ lead to a faster convergence rate. Whereas the convergence rate seems to saturate with $\rho = 15$. Therefore, in the following case studies, the penalty parameter is set as $\rho = 15$.

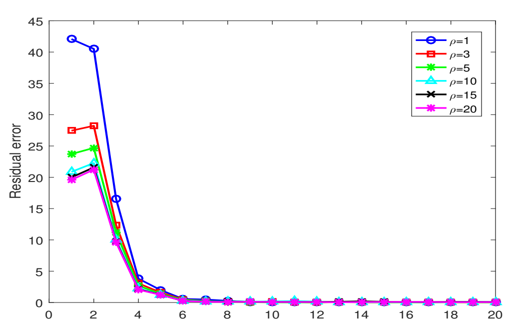

Fig. 7. The convergence rate of residual error of the coupled constraints with different penalty parameters ρ.

B. Scalability

The scalability of the proposed method is demonstrated through comparison with the DTBSS [2] developed for multi-zone HVAC control. Generally speaking, DTBSS is a heuristic hierarchical distributed method, in which problem (**P1**) is divided into three-level subproblems, which manage each part of the overall cost function and constraints. We refer readers to [2] for more details on DTBSS. To guarantee a fair comparison, the constraints caused by duct pressure and duct pressure and chiller efficiency mentioned in [2] are circumvented in DTBSS. Besides, the two methods are both carried out under model predictive control (MPC) framework with the convenience to estimate the disturbances caused by thermal couplings among neighboring zones as required by DTBSS. Specifically, over each planning horizon, the disturbances are estimated based on the control trajectories computed over the previous one in DTBSS. For the DTBSS in [2], the first and third step are retained for problem (**P1**). However, the second step can be skipped as the constraints due to duct pressure and chiller efficiency are not discussed in this paper. In these case studies, we select the planning horizon for the decentralized approach and the first step of DTBSS as $H = 10$ (replace T by H in problem (**P1**). However, the planning horizon is shorted to $H' = 5$ in the third step of DTBSS to reduce computation of the centralized problem as suggested in [2].

We consider a number of case studies with 5, 20, 50, 100, 200, 300 and 500 zones in this part. We resort to some randomly generated networks to describe the thermal coupling among the different zones and the maximum number of adjacent zones for each zone is set as 4. The zone air flow rare bounds are set as $[0, 0.5]\mathrm{kg\,s^{-1}}$. The other parameters can refer to TABLE I.

TABLE III
PERFORMANCE OF THE DECENTRALIZED APPROACH AND THE DTBSS

#Zones	DTBSS		Decentralized Approach		Cost
	Cost(s\$)	Time(s)	Cost(s\$)	Time(s)	Reduced (%)
5	63.57	2.11	62.05	1.39	2.39
20	266.09	2.71	258.79	1.38	2.74
50	713.07	4.47	687.23	2.35	3.62
100	1.69×10^3	6.90	1.61×10^3	2.56	4.73
200	5.73×10^3	10.32	5.10×10^3	5.04	11.00
300	1.51×10^4	17.89	1.27×10^4	8.80	15.89
500	5.57×10^4	28.01	4.49×10^4	13.46	19.39

Both the HVAC's energy cost and average computation time for each zone at each stage incurred by the two methods are contrasted in TABLE III. We see that for the cases with relatively small numbers of zones (less than 20), the performance (energy cost and computation time) of two methods are comparable except for a slight superiority for the decentralized approach. However, for large numbers of zones, the decentralized approach outperforms DTBSS both in reducing energy cost and improving computational efficiency. In particular, the total energy cost is cut down by around 19.39% for the 500-zone case study. The superior performance of the decentralized approach in reducing energy cost is attributed to the fact that the cooling power and fan power of AHU are coordinated in one problem in contrast to DTBSS where they are successively optimized in different levels of subproblems. This also illustrates the slight difference on energy cost incurred by the two methods for small case studies. Specifically, for small numbers of zones, the fan power of the AHU (depends on the cube of the total zone air flow rate) is negligible compared with the cooling power. Therefore, there only exist a small performance degradation in cost for DTBSS where the fan power is regulated (third step) after the management of cooling power (first step). However, for increasing numbers of zones, the part of energy cost caused by fan power increases rapidly, which results in an apparent difference on energy cost for the two methods.

Moreover, from TABLE III, we observe that the proposed decentralized approach is more computationally efficient (less average computation time) than DTBSS. The preferable performance can be attributed to that: *i)* the convergence rate of ADAL employed by the decentralized approach is fast (see Fig. 6); *ii)* the subproblems in the decentralized approach are QP instead of non-linear and non-convex as in DTBSS; *iii)* the decentralized approach is performed totally in decentralized manner by solving small-scale subproblems without the need to deal with centralized problem as the third step of DTBSS.

VI. Conclusion

This paper studies the scalable control of multi-zone HVAC systems with the objective to reduce their energy cost while guaranteeing zone thermal comfort. As centralized methods are usually computationally intensive or prohibitive for large buildings, a tailored decentralized approach based on the Accelerated Distributed Augmented Lagrangian (ADAL) method [1] was developed. Through comparison with the centralized method, we found the decentralized approach can achieve satisfactory sub-optimality. Besides, the scalability of the method was demonstrated through comparison with the Distributed Token-Based Scheduling Strategy (DTBSS) method [2] developed for multi-zone HVAC control. The results implied that when the numbers of zones are relatively small (less than 20), there exist a narrow performance superiority regarding the energy cost ($2\% \sim 4\%$) for the proposed decentralized approach. However, with increasing numbers of zones, the proposed method outperforms the DTBSS both in reducing the HVAC's energy cost and improving computational efficiency.

Appendix A
Transformation of Problem (P2) to (P3)

The temperature dynamics of zone i can be described as

$$
\begin{aligned}
T_{t+1}^{i} = & \begin{pmatrix} A^{ii} & 0 & C^{ii} \end{pmatrix} \begin{pmatrix} T_t^i \\ m_t^{zi} \\ X_t^i \end{pmatrix} \\
& + \sum_{j \in \mathcal{N}_i} \begin{pmatrix} A^{ij} & 0 & 0 \end{pmatrix} \begin{pmatrix} T_t^j \\ m_t^{zj} \\ X_t^j \end{pmatrix} + D_t^{ii}
\end{aligned} \tag{24}
$$

If we define $\overline{A}^{ii} = \begin{pmatrix} A^{ii} & 0 & C^{ii} \end{pmatrix}$, $\overline{A}^{ij} = \begin{pmatrix} A^{ij} & 0 & 0 \end{pmatrix}$ and $\overline{D}_t^{ii} = -D_t^{ii}$, and the decision variable $\boldsymbol{x}^i = ((\boldsymbol{x}_0^i)^T, (\boldsymbol{x}_1^i)^T, \cdots (\boldsymbol{x}_{T-1}^i)^T)^T$ $(\forall i \in \mathcal{I})$ with $\boldsymbol{x}_t^i = (T_t^i, m_t^{zi}, X_t^i)$, the temperature dynamics for all the thermal zones can be combined by

$$
\begin{aligned}
& \begin{pmatrix} \overline{A}_{ii} & -\boldsymbol{I}_1 & \cdots & \cdots & \cdots \\ \mathbf{0} & \overline{A}_{ii} & -\boldsymbol{I}_1 & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{A}_{ii} & -\boldsymbol{I}_1 & \cdots \end{pmatrix} \boldsymbol{x}^i \\
& + \sum_{j \in \mathcal{N}_i} \begin{pmatrix} \overline{A}_{ij} & \mathbf{0} & \cdots & \cdots & \cdots \\ \mathbf{0} & \overline{A}_{ij} & \mathbf{0} & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{A}_{ij} & \mathbf{0} & \cdots \end{pmatrix} \boldsymbol{x}^j \\
& + \begin{pmatrix} \overline{D}_0^{ii} \\ \vdots \\ \overline{D}_{T-1}^{ii} \end{pmatrix} = \mathbf{0}
\end{aligned} \tag{25}
$$

Further, we define

$$
\boldsymbol{A}_d^{ii} = \begin{pmatrix} \overline{A}^{ii} & -\boldsymbol{I}_1 & \mathbf{0} & \cdots & \cdots & \cdots \\ \mathbf{0} & \overline{A}^{ii} & -\boldsymbol{I}_1 & \mathbf{0} & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{A}^{ii} & -\boldsymbol{I}_1 & \mathbf{0} & \cdots \end{pmatrix} \in \mathbb{R}^{(T-1) \times 3T},
$$

$$
\boldsymbol{A}_d^{ij} = \begin{pmatrix} \overline{A}^{ij} & \mathbf{0} & \cdots & \cdots & \cdots \\ \mathbf{0} & \overline{A}^{ij} & \mathbf{0} & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{A}^{ij} & \mathbf{0} & \cdots \end{pmatrix} \in \mathbb{R}^{(T-1) \times 3T},
$$

and $\boldsymbol{b}_d^i = (\overline{D}_0^{ii}, \ \overline{D}_1^{ii}, \cdots, \overline{D}_{ii}^{T-1})^T \in \mathbb{R}^{(T-1)}$

Thus, the dynamics in (25) is equivalent to

$$
\boldsymbol{A}_d^i \boldsymbol{x}^i + \sum_{j \in \mathcal{N}_i} \boldsymbol{A}_d^{ij} \boldsymbol{x}^j = \boldsymbol{b}_d^i \tag{26}
$$

Similarly, the coupled constraints in (10f) can be written as

$$
\sum_{i=0}^{I} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} T_t^i \\ m_t^{zi} \\ X_t^i \end{pmatrix} - Y_t = 0 \tag{27}
$$

If we define $\overline{B}^i = (0 \ \ 1 \ \ 0)$ $(\forall i \in \mathcal{I})$ and $\overline{B}^0 = (-1)$. The coupled constraints in (27) over the optimization horizon $\mathcal{T}$ can be collected as

$$
\sum_{i=0}^{I} \begin{pmatrix} \overline{B}^i & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \overline{B}^i & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{B}^i & \cdots \end{pmatrix} \boldsymbol{x}^i = \mathbf{0} \tag{28}
$$

If we define $\boldsymbol{B}_d^i = \begin{pmatrix} \overline{B}^i & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \overline{B}^i & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{B}^i & \cdots \end{pmatrix} \in \mathbb{R}^{T \times 3T}$ $(\forall i \in \mathcal{I})$, (28) is equivalent to

$$
\sum_{i=0}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i = 0 \tag{29}
$$

Accordingly, the constraints (8c) can be described as

$$
\sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i \leq \boldsymbol{c}_d \tag{30}
$$

where we have $\boldsymbol{c}_d = (\overline{m}, \overline{m} \cdots \overline{m})^T \in \mathbb{R}^T$

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