$$Cov(X,Y) = E(X,Y) - E(X)E(Y)$$

証明：

$$\begin{aligned} Cov(X,Y) &= E\{(X - E(X))(Y - E(Y))\} \\ &= E(XY - E(X)Y - E(Y)X + E(X)E(Y)) \\ &= E(XY) - E(X)E(Y) - E(X)E(Y) + E(X)E(Y) \\ &= E(XY) - E(X)E(Y) \end{aligned}$$

$$Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X,Y)$$

証明：

$$\begin{aligned} Var(aX \pm bY) &= E[(aX \pm bY) - E(aX \pm bY)^2] \\ &= E\left[\left((aX - E(aX)) \pm (bY - E(bY))\right)^2\right] \\ = E\left[(aX - E(aX))^2\right] + E\left[(aY - E(aY))^2\right] &\pm 2E\{[aX - E(aX)][bY - E(bY)]\} \\ &= a^2Var(X) + b^2Var(Y) + 2abCov(X,Y) \end{aligned}$$