DIP Assignment 2

Yang Yu

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Abstract

Spatial filtering is one of the main tools widely used in image processing. The spatial filter consists of predefined operations performed by a neighborhood on the image pixels surrounded by the neighborhood. The filter generates a new pixel whose coordinates are equal to the coordinates of the neighborhood center, and the value of the pixel is the result of the filtering operation. If a linear operation is performed on an image pixel, the filter is called a linear spatial filter. Otherwise, the filter is called a nonlinear spatial filter. When performing linear spatial filtering, there are two operations. One is the correlation, and the other is convolution. Correlation is the process in which the filter template moves over the image and calculates the sum of the product of each position. The mechanism of convolution is similar, but the filter must rotate 180° first.

1 Problem 1: Gaussian Noise

Gaussian noise is a kind of noise whose probability density function obeys Gaussian distribution (Normal distribution). The common Gaussian noise includes fluctuation noise, cosmic noise, thermal noise, and shot noise. In addition to the common noise suppression methods, the mathematical statistics method is often used to suppress Gaussian noise. In the digital image, the expression of Gaussian noise is variations in intensity draw from a Gaussian normal distribution

1.1 Question 1

1.1.1 Read and display the image

Figure 1: origin

From the Figure 1, we can find a lot of black and white noise spots. In this problem, we will use a spatial filter to remove noise from the image. The picture in this question is Gaussian noise, so the overall effect of the image can be seen as many black and white noise points and dense distribution.

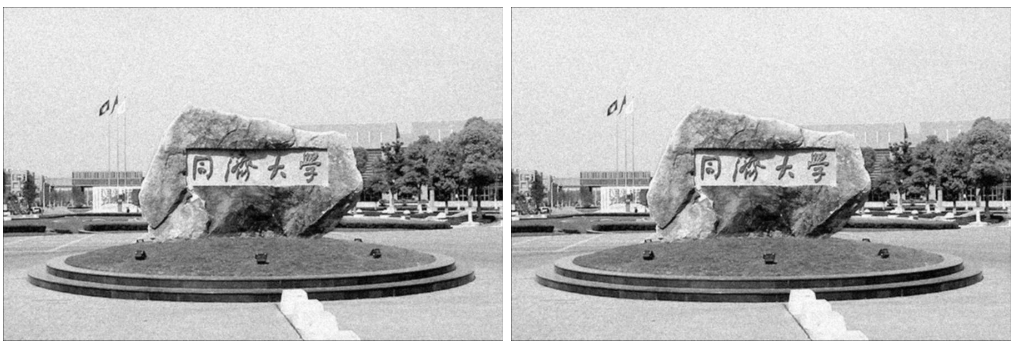
We can use the following matrix to describe the mask of all one:

$$
\frac{1}{n^2}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & \cdots & 1 \end{bmatrix}_{n\times n} \tag{1}
$$

where n is the size of the mask.

1.1.2 What's the difference between 'filter2' and 'conv2'?

The function 'filter2' and 'conv2' corresponds to correlation operation and convolution operation respectively. The correlation operation is to move the filter template over the image and calculate the sum of the product of each position. Specifically, the step is to first move the central element of the correlation core so that it is directly above the pixel to be processed in the input image, and then the pixel value of the input image is taken as the weight and multiplied by the correlation core. Finally, the results of the above steps are added as the output. The mechanism of convolution is similar, but the filter template needs to be rotated 180° before doing the above operation



(a) filter2 with size 3 (b) conv2 with size 3

Figure 2: Comparison of filter2 and conv2

In this case, the mask of all one filter is used. For the above two operations, the transformation is consistent. Because after the filter is transposed 180°, it is consistent with the original filter. Therefore, we can see from the Figure 2 that the results of 'filter2' and 'conv2' are consistent for masks of the same size.

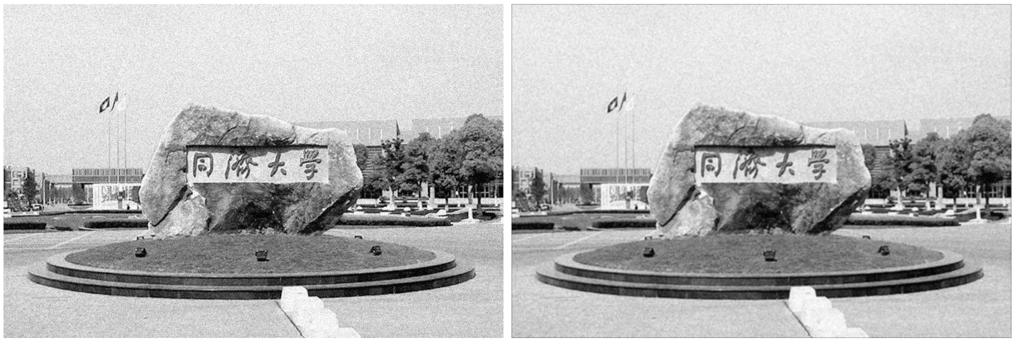
1.1.3 How dose the size of mask affect the blurring and noise reduction?

(a) conv2 with size 3 (b) conv2 with size 9

Figure 3: Comparison of the mask size 3 and 9 with conv2

As the size of the filter increases, we can see that the image becomes more and more blurred. This is because the current filter we use is to average the target pixels and the neighboring pixels. Therefore, in the Figure 3, we can see that with the increase of the filter size, the greater the effect of blur is especially for the edges. As a result, the overall picture is more blurred.

1.1.4 Which mask provides the best trade-off between blurring and noise reduction in this image



(a) origin (b) conv2 with size 3

Figure 4: The result of the smoothing spatial filters

From my point of view, I think a square average filter with the size of 3 provides a good enough balance between the blurring and noise reduction. After passing the filter, the Gaussian noise is reduced. At the same time, the original information of the image is not erased under such an operation. The edge of the image is not blurred. In the Figure 4, we can check the results.

1.2 Question 2

Gaussian filter is a kind of linear smoothing filter, which is suitable for eliminating Gaussian noise and is widely used in image processing. Generally speaking, Gaussian filtering is the process of the weighted average for the whole image. The value of each pixel is obtained by the weighted average of its own and other pixel values in the neighborhood. The specific operation of Gaussian filtering is to scan every pixel in the image with a mask and use the weighted average gray value of pixels in the neighborhood determined by the mask to replace the value of the central pixel of the mask.

We can use a Gaussian function (which also expresses the normal distribution in statistics) for calculating the transformation to apply to each pixel in the image. The formula of a Gaussian function in one dimension is as follows:

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \tag{2}$$

where x is the distance from the origin in the horizontal axis and σ is the standard deviation of the Gaussian distribution.

In two dimensions, it is the product of two such Gaussian functions, one in each dimension:

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} \tag{3}$$

where x is the distance from the origin in the horizontal axis, y is the distance from the origin in the vertical axis, and σ is the standard deviation of the Gaussian distribution.

Because the 2D Gaussian can be expressed as the product of two functions, we can produce the 2D Gaussian by convolution of an 1D Gaussian with its transpose.

In this question, we use the Gaussian filter to process the image with Gaussian noise. In the selection of parameters, the size of the Gaussian filter is 5 and the value of sigma is 1. From the Figure 5, we can

(a) origin (b) Gaussian filter with my function (c) Gaussian filter with imfilter

Figure 5: The result of the smoothing spatial filters with Gaussian filter

see that the effect is slightly better than the average filter mentioned above. The filter not only blurs the Gaussian noise but also retains more image information as much as possible. In this problem, we use the 'imfilter' function and compare it with our own function. We can find that the effect of the two functions is completely consistent.

2 Problem 2: Pepper and salt noise

Salt and pepper noise is a common noise in digital images, which is usually produced by image sensors, transmission channels, and decoding processing. Salt and pepper noise is often generated by image cutting. Salt and pepper noise refers to two kinds of noise: salt noise and pepper noise. Salt noise is generally white noise and pepper noise is generally black noise. The former is high gray noise and the latter belongs to low gray noise. Generally, two kinds of noise appear at the same time produces black and white spots on the image. The median filter is the most commonly used algorithm to remove impulse interference and salt and pepper noise in image simulation. Adding salt and pepper noise to image simulation is achieved by randomly obtaining pixel values and setting them as highlight points.

2.1 MinFilter

Minimum filtering is a relatively conservative image processing method. In the process of operation, it is necessary to sort the surrounding pixels and center pixel values, and then compare the central pixel value with the minimum pixel value. If the value is smaller than the minimum value, the center pixel is replaced by the minimum value. This is the operation of minimum filtering.

(a) minFilter with size 1 (b) minFilter with size 3 (c) minFilter with size 5 (d) minFilter with size 7

Figure 6: The result of the minFilter

Through the Figure 6, we can find out that after the minimum filtering, salt noise in the original image is eliminated. This is because salt noise is represented as white noise points, and the gray value of the pixel is much larger than the surrounding gray value. Therefore, this kind of noise will be eliminated after minimum filtering. However, the method of getting the minimum value will make the picture dark and lose information. The larger the filter size is, the more information is lost, the darker the picture is.

2.2 MaxFilter

Maximum filtering is also a relatively conservative image processing method. In the process of operation, it is necessary to sort the surrounding pixels and center pixel value, and then compare the center pixel value with the maximum pixel value. If the value is larger than the maximum value, the center pixel is replaced by the maximum value. This is the operation of maximum filtering.

(a) maxFilter with size 1 (b) maxFilter with size 3 (c) maxFilter with size 5 (d) maxFilter with size 7

Figure 7: The result of the maxFilter

Through the Figure 7, we can find out that after the maximum filtering, the pepper noise in the original image is eliminated. This is because pepper noise is represented as black noise points, and the gray value of the pixel is far less than the surrounding gray value. Therefore, this kind of noise will be eliminated after maximum filtering. However, the method of getting the maximum value will make the picture bright and lose information. The larger the filter size is, the more information is lost, the brighter the picture is.

2.3 MedianFilter

The median filter is also one of the most common means to eliminate image noise, especially to eliminate salt and pepper noise, the effect of the median filter is better than that of the mean filter. Median filtering is different from mean filtering in that it does not replace each pixel in the center with the mean intensity, but takes the median after sorting the surrounding pixels and central pixels

(a) MedianFilter with size 1 (b) MedianFilter with size 3 (c) MedianFilter with size 5 (d) MedianFilter with size 7

Figure 8: The result of the maxFilter

Through the Figure 8, we can find that after medium filtering, Pepper and Salt noise in the original image are all eliminated. This is because pepper and salt noise are black and white noise points, which correspond to the noise points with intensity higher than the surrounding intensity and lower than the surrounding value respectively. Therefore, after median filtering, the maximum intensity and minimum intensity will be replaced by the middle intensity around their field, so the image noise can be repaired. However, if the filter used is too large, it will also affect the information of the image, resulting in the corresponding fuzziness.

2.4 Question

2.4.1 Can we apply the max filter to 'p2d1' or apply the min filter to 'p2d2'?

No, we cannot.The noise corresponding to p2d1 is salt noise, which is expressed as white noise on the image. If maxfilter is used, there is no doubt that the maximum gray value will be selected as the gray value of our current target pixel. Obviously, this will result in the selection of noise points, which will cause more noise points to fill the screen. vice versa. For p2d2, using minfilter will result in more black noise.

2.4.2 Can you describe the scope of application of these three different filters

The minimum filter can be used on the image affected by salt noise with white noise, which produces good results. The maximum filter can be used on the image affected by pepper noise with black noise, which produces good results. The median filter can be applied to the above two kinds of noise, and at the same time, it can also be applied to the place where the two noises are combined. Median filters have excellent results.

3 Problem 3: Sharpen

The main purpose of sharpening is to highlight the transition part of the intensity. Image sharpening has a variety of uses, ranging from electronic printing and medical imaging to industrial detection and guidance of military systems. The above image blur can be realized by using the pixel neighborhood average method in the spatial domain. Because the mean processing is similar to integral processing, logically, we can conclude that sharpening can be realized by spatial differentiation. Basically, the response intensity of the differential operator is proportional to the mutation degree of the image at this point operated by the operator. In this way, the image differential enhances the edge and other abrupt changes (such as noise) and weakens the area with slow gray change.

3.1 Question 1

3.1.1 Sharpen the best result in Problem 1

In this part, we will use the following filter to sharpen the image in the Problem 1:

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \tag{4}$$

The purpose of using this filter is to enhance the edge information obtained by using the Laplacian filter to the original image, so as to obtain a clear image with the original image information retained and the edge enhanced.

(a) origin

(b) Gaussian filter

(c) Sharpen filter based on Gaussian filter

Figure 9: The result of the sharpen images

In the Figure 9, we can see the result that not only remove the noise but also sharpen the edges in the image.

3.1.2 Does this operation affect the blurring and the noise reduction

The sharpening operation has little effect on blurring and noise reduction. This is because the sharpening operation will add the original image information to the image after the operation. The 5 in the middle of the mask ensures the implementation of this operation. If the mask is changed to 4 in the middle, only the edge information of the original image can be extracted, and the edge can not be deepened while the content of the image is kept.

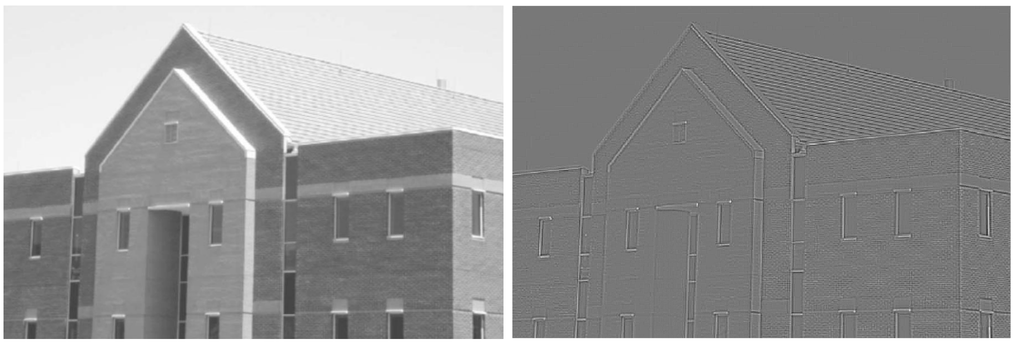
3.2 Question 2

3.2.1 Filter the image 'p3' using Laplacian mask

In this part, we will use the following Laplacian filter to sharpen the image in the Problem 2:

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \tag{5}$$

The purpose of using this filter is to get the edge information.



(a) origin (b) Sharpen filter

Figure 10: The result of the Laplacian filter

In this image, we successfully extract the edge information of the image and display it to an image with negative gray level.

By observing Figure 10, we find that Laplacian sharpening image is related to the mutation degree of the surrounding pixels of an image pixel to this pixel, that is to say, it is based on the change degree of image pixels. Second order differential describes the speed of image change, rapid growth and decline or gentle growth and decline. Therefore, when the gray level of the neighborhood center pixel is lower than the average gray level of other pixels in its field, the gray level of the center pixel should be further reduced. When the gray level of the neighborhood center pixel is higher than the average gray level of other pixels in its neighborhood, the gray level of the central pixel should be further improved to realize the image sharpening processing.

3.2.2 Apply laplacian of gaussian operator to 'p3

Laplace operator is a kind of high pass filter, which is the sum of the second partial derivatives of image gray function in two vertical directions. In the case of a discrete digital image, the second-order difference of image gray level is directly used to replace the second-order partial derivative in the continuous case, which is very sensitive to noise, and the pseudo edge response often appears in edge extraction. In order to overcome the shortcomings of Laplacian operator, it is necessary to carry out low-pass filtering to suppress noise. Gaussian function is a good normalized low-pass filter, which can be used for low-pass filtering of digital images to reduce the impact of noise. On this basis, Laplacian operator is used to extract edge, which is called log (Laplacian of Gaussian) operator. The formula is as following:

$$LoG(x, y) = -\frac{1}{\pi\sigma^4}\left[1 - \frac{x^2 + y^2}{2\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{6}$$

where x is the distance from the origin in the horizontal axis, y is the distance from the origin in the vertical axis, and σ is the standard deviation of the Gaussian distribution.

In the Figure 11, we use the $\sigma = 0.3$. Because of the influence of the parameter σ of the Gaussian function on the Gaussian operator, if the σ is selected very small, the fuzzy effect in the early stage is very weak, and it can be approximately equal to the Laplacian operator. Therefore, Laplace operator can also be regarded

(a) origin (b) Log filter

Figure 11: The result of the LoG filter

as a kind of limit case of Gauss Laplacian operator. If σ is selected very large, the scale of the edge will be larger and larger, and the edge details will become more and more indistinct.