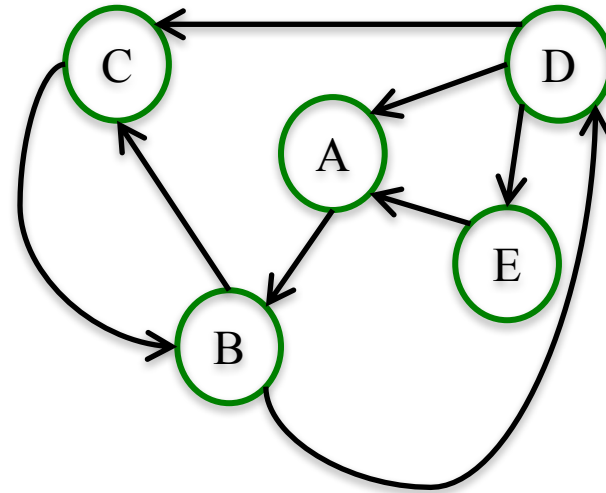


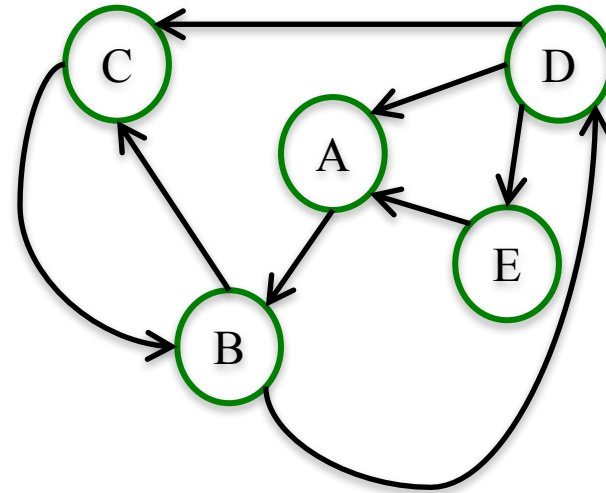
# Interpreting PageRank

The PageRank of a node at step  $k$  is the probability that a **random walker** lands on the node after taking  $k$  steps.



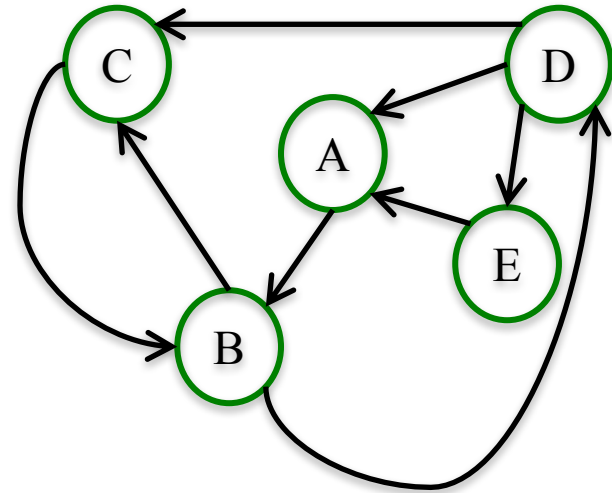
# Interpreting PageRank

**Random walk of k steps:** Start on a random node.



# Interpreting PageRank

**Random walk of k steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node.

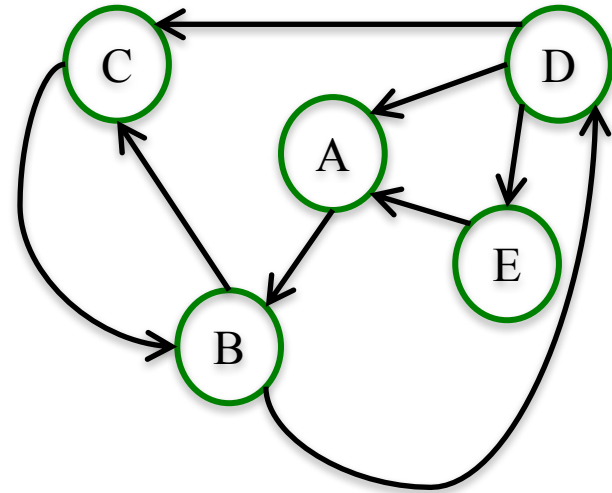


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

Step 1: Choose a random node.

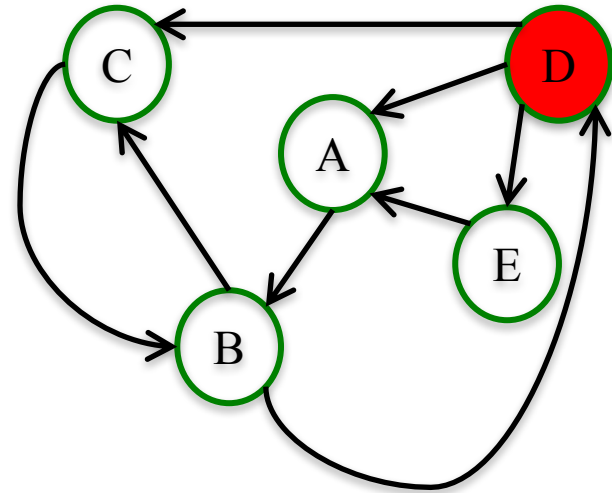


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

Step 1: Choose a random node.

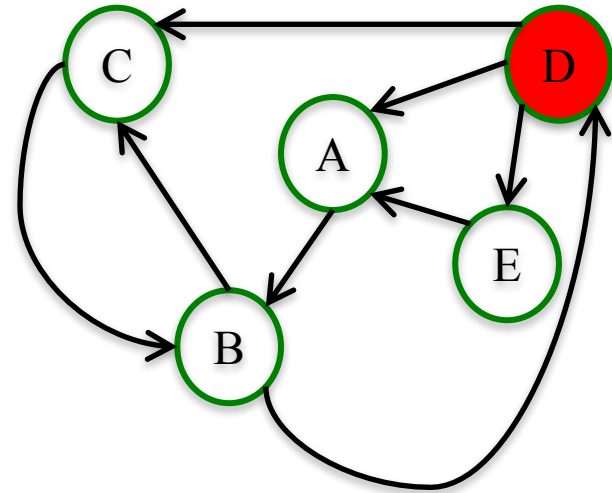


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

Step 1: Choose a random outgoing edge.

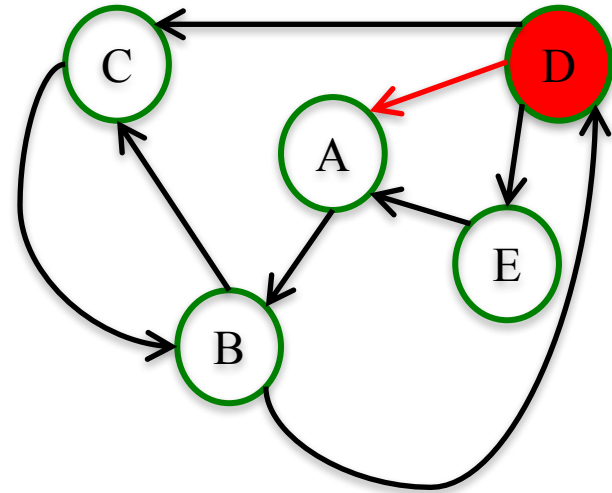


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

Step 1: Choose a random outgoing edge.

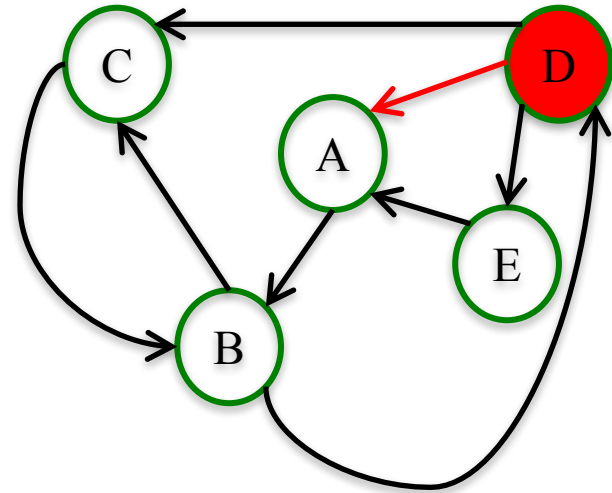


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

Step 1: Follow the edge to the next node.



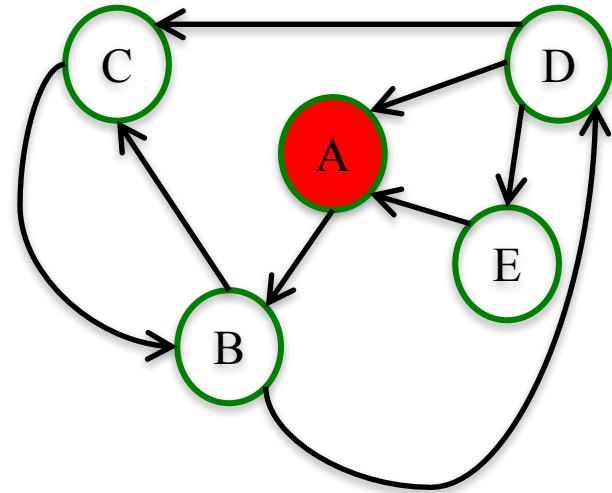


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

Step 1: Follow the edge to the next node.

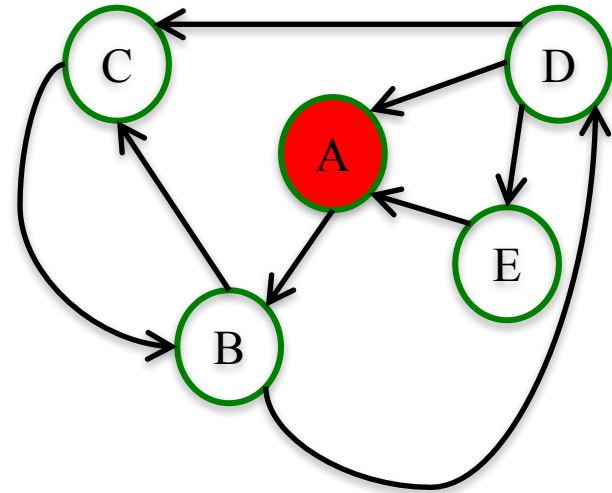


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

Step 2: Choose a random outgoing edge and follow it.

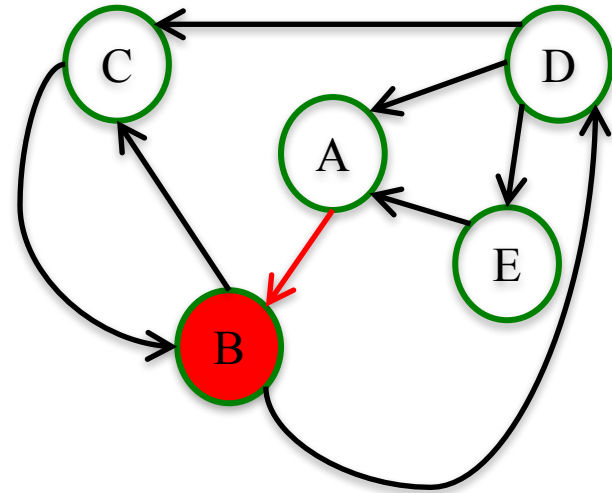


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

Step 2: Choose a random outgoing edge and follow it.

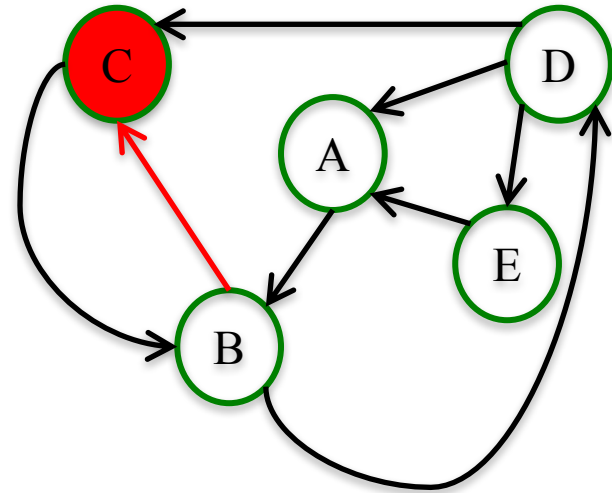


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

Step 3: Choose a random outgoing edge and follow it.

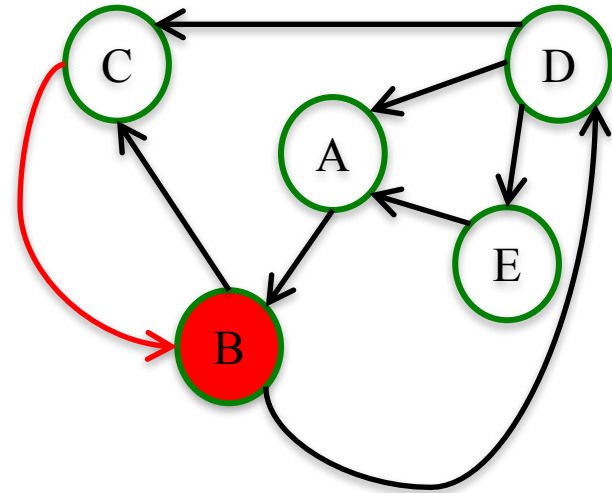


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

Step 4: Choose a random outgoing edge and follow it.

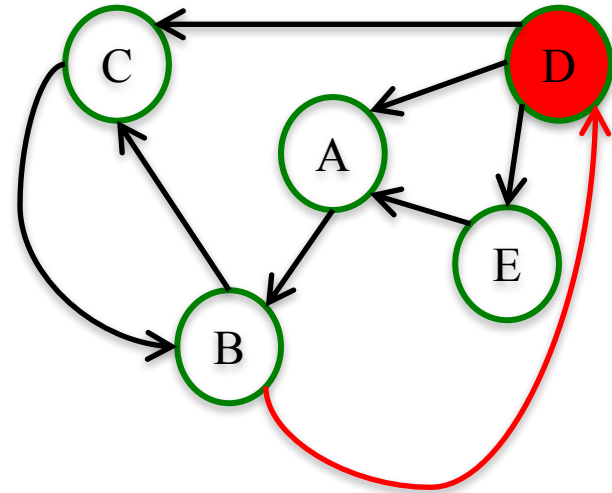


# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

For example, a random walk of 5 steps on this graph looks like this:

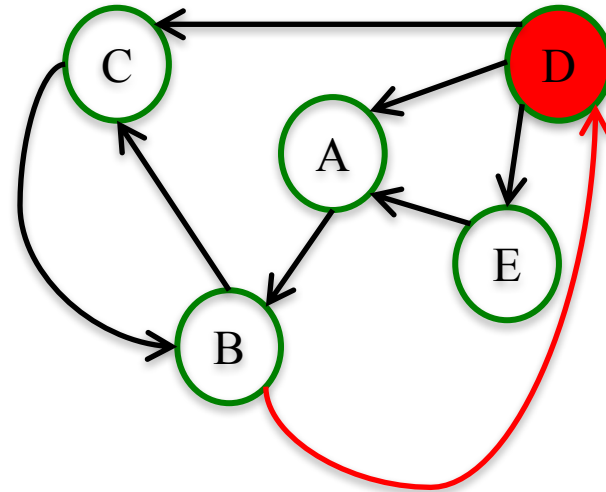
Step 5: Choose a random outgoing edge and follow it.



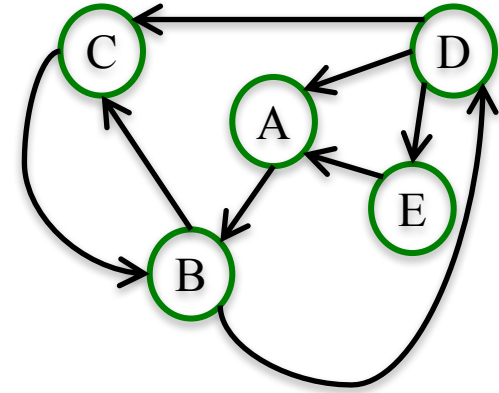
# Interpreting PageRank

**Random walk of  $k$  steps:** Start on a random node. Then choose an outgoing edge at random and follow it to the next node. Repeat  $k$  times.

	Page Rank				
	A	B	C	D	E
$k=\infty$	.12	.38	.25	.19	.06



# PageRank Problem





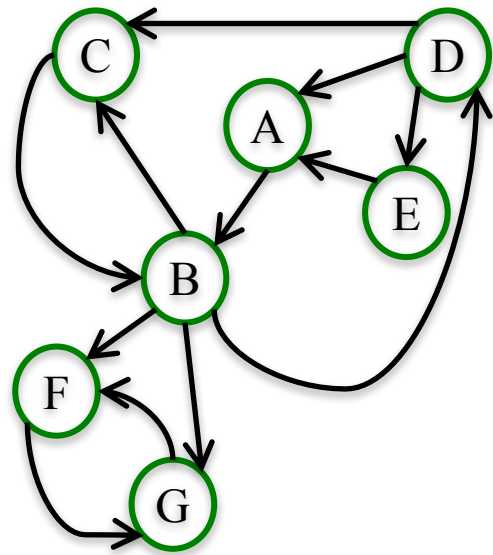
# PageRank Problem

What's the PageRank of the nodes in this network?  
[Hint: think about the random walk interpretation]

For a large enough  $k$ : F and G each have PageRank of  $\frac{1}{2}$  and all the other nodes have PageRank 0.

Why? Imagine a random walk on this network.  
Whenever the walk lands on F or G, it is “stuck” on F and G.

This seems problematic!



# PageRank Problem

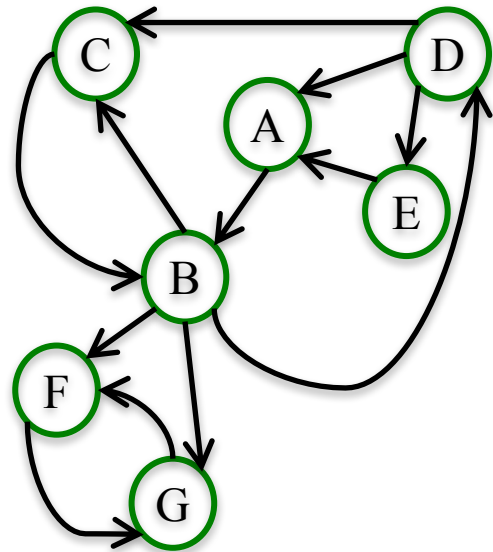
To fix this, we introduce a “damping parameter”  $\alpha$ .

**Random walk of  $k$  steps with damping parameter  $\alpha$ :** Start on a random node. Then:

- **With probability  $\alpha$ :** choose an outgoing edge at random and follow it to the next node.
- **With probability  $1 - \alpha$ :** choose a node at random and go to it.

Repeat  $k$  times.

The random walk is no longer “stuck” on nodes F and G.

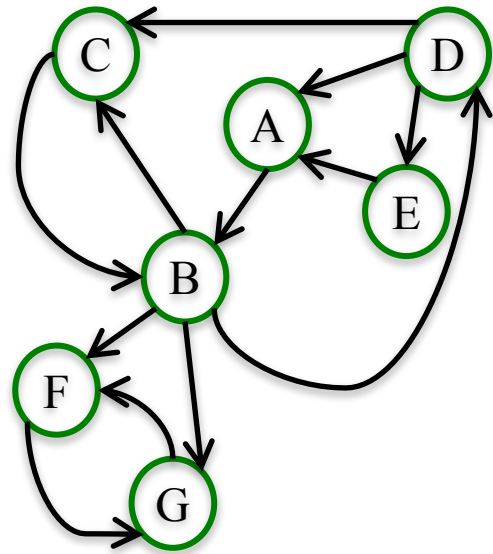


# Scaled PageRank

The **Scaled PageRank** of  $k$  steps and damping factor  $\alpha$  of a node  $n$  is the probability that a random walk with damping factor  $\alpha$  lands on a  $n$  after  $k$  steps.

For most networks, as  $k$  gets larger, Scaled PageRank converges to a unique value, which depends on  $\alpha$ .

In practice, we use a parameter of  $\alpha$  between 0.8 and 0.9.



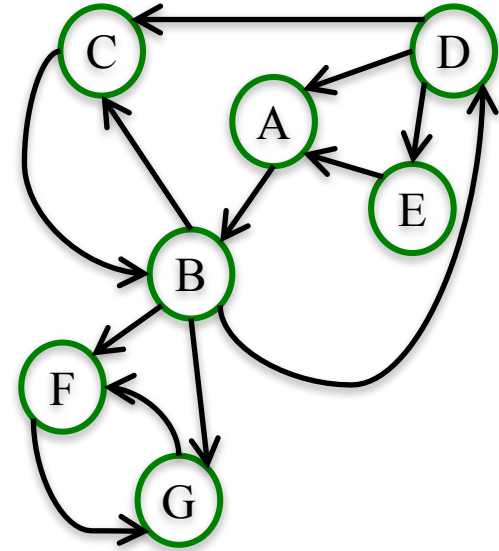
# Scaled PageRank

Scaled PageRank ( $\alpha = .8, k$ large)						
A	B	C	D	E	F	G
.08	.17	.1	.08	.05	.27	.25

F and G still have high PageRank, but not all the PageRank.

Damping factor works better in very large networks like the Web or large social networks.

You can use NetworkX function `pagerank(G, alpha=0.8)` to compute Scaled PageRank of network G with damping parameter alpha.



# Summary

- The Basic PageRank of a node can be interpreted as the probability that a random walk lands on the node after  $k$  random steps.
- Basic PageRank has the problem that, in some networks, a few nodes can “suck up” all the PageRank from the network.
- To fix this problem, Scaled PageRank introduces a parameter  $\alpha$ , such that the random walker chooses a random node to jump to with probability  $1 - \alpha$ .
- Typically we use  $\alpha$  between 0.8 and 0.9
- NetworkX function `pagerank(G, alpha=0.8)` computes Scaled PageRank of network  $G$  with damping parameter  $\alpha=0.8$ .

