Yaw, pitch, and roll rotations

A 3D body can be rotated about three orthogonal axes, as shown in Figure 3.8. Borrowing aviation terminology, these rotations will be referred to as yaw, pitch, and roll:

1. A *yaw* is a counterclockwise rotation of α about the z-axis. The rotation matrix is given by

$$R_z(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3.39}$$

   Note that the upper left entries of $R_z(\alpha)$ form a 2D rotation applied to the x and y coordinates, whereas the z coordinate remains constant.

2. A *pitch* is a counterclockwise rotation of β about the y-axis. The rotation matrix is given by

$$R_y(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} . \tag{3.40}$$

3. A *roll* is a counterclockwise rotation of γ about the x-axis. The rotation matrix is given by

$$R_x(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix} . \tag{3.41}$$

Each rotation matrix is a simple extension of the 2D rotation matrix, (3.31). For example, the yaw matrix, $R_z(\alpha)$, essentially performs a 2D rotation with respect to the x and y coordinates while leaving the z

coordinate unchanged. Thus, the third row and third column of $R_z(\alpha)$ look like part of the identity matrix, while the upper right portion of $R_z(\alpha)$ looks like the 2D rotation matrix.

The yaw, pitch, and roll rotations can be used to place a 3D body in any orientation. A single rotation matrix can be formed by multiplying the yaw, pitch, and roll rotation matrices to obtain

$$\begin{split} R(\alpha,\beta,\gamma) = R_z(\alpha)\,R_y(\beta)\,R_x(\gamma) = \\ \begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{pmatrix} . \end{split} \tag{3.42}$$

It is important to note that $R(\alpha, \beta, \gamma)$ performs the roll first, then the pitch, and finally the yaw. If the order of these operations is changed, a different rotation matrix would result. Be careful when interpreting the rotations. Consider the final rotation, a yaw by α. Imagine sitting inside of a robot $\mathcal{A}$ that looks like an aircraft. If $\beta = \gamma = 0$, then the yaw turns the plane in a way that feels like turning a car to the left. However, for arbitrary values of β and γ, the final rotation axis will not be vertically aligned with the aircraft because the aircraft is left in an unusual orientation before α is applied. The yaw rotation occurs about the z-axis of the world frame, not the body frame of $\mathcal{A}$. Each time a new rotation matrix is introduced from the left, it has no concern for original body frame of $\mathcal{A}$. It simply rotates every point in $\mathbb{R}^3$ in terms of the world frame. Note that 3D rotations depend on three parameters, α, β, and γ, whereas 2D rotations depend only on a single parameter, θ. The primitives of the model can be transformed using $R(\alpha, \beta, \gamma)$, resulting in $\mathcal{A}(\alpha, \beta, \gamma)$.

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