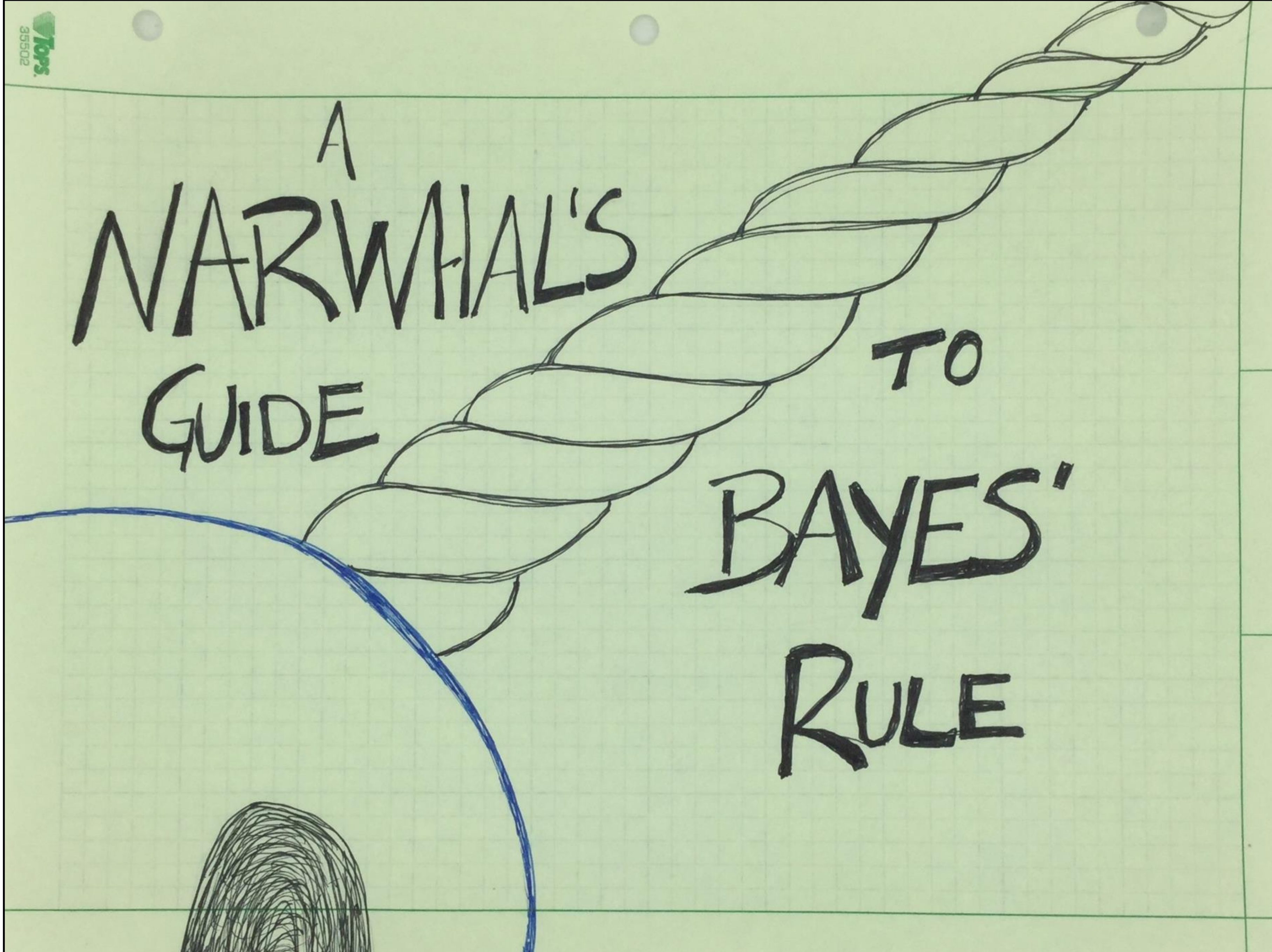


Welcome to a Narwhal's Guide to Bayes' Rule. If you don't know what a narwhal is, [wikipedia] can help you.

BAYES' RULE IS A TOOL FOR **UPDATING** YOUR **BELIEFS** IN THE FACE OF **EVIDENCE**

IT HELPS YOU MAKE BETTER **PREDICTIONS** ABOUT THE WORLD

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

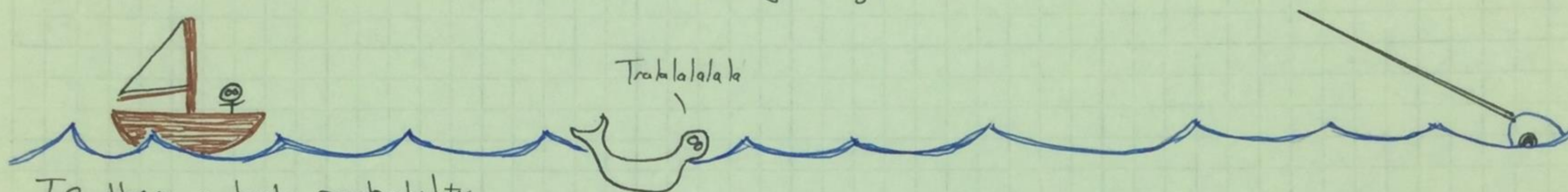
" the probability of A given that B is true

equals the probability of B given A times the probability of A divided by the probability of B "

THIS is probably not that **HELPFUL**...

HOW ABOUT A STORY??! **ONCE** UPON A TIME

ONCE UPON A TIME, there was a near sighted sailor who saw an animal... it breathed air and it swam and it sang such enchanting songs.



Is there a high probability the sailor has seen a MERMAID?

$$P\left(\text{mermaid} \,\middle|\, \begin{matrix}\text{Singing}\\ \text{Marine}\\ \text{Mammal}\end{matrix}\right) = \frac{P\left(\begin{matrix}\text{Singing}\\ \text{marine}\\ \text{Mammal}\end{matrix} \,\middle|\, \text{Mermaid}\right) P(\text{Mermaid})}{P\left(\begin{matrix}\text{singing}\\ \text{marine}\\ \text{mammal}\end{matrix}\right)}$$

that is to say:
the probability he's seen a MERMAID
given that he's seen a
singing marine mammal

is equal to
the probability that one would
see a singing marine mammal
if one was seeing a MERMAID

times
the probability of seeing a mermaid

divided by
the probability of seeing a
singing marine mammal

abbreviating that as SMM

$P(\text{SMM} \mid \text{mermaid}) =$ 1.0: If there are mermaids they are definitely singing marine mammals

$P(\text{mermaid}) =$ pretty low... maybe zero. Close to zero anyway. (No one's seen one so far. Sadly.)

$P(\text{SMM}) =$ actually quite high: Manatees dolphins, whales, etc

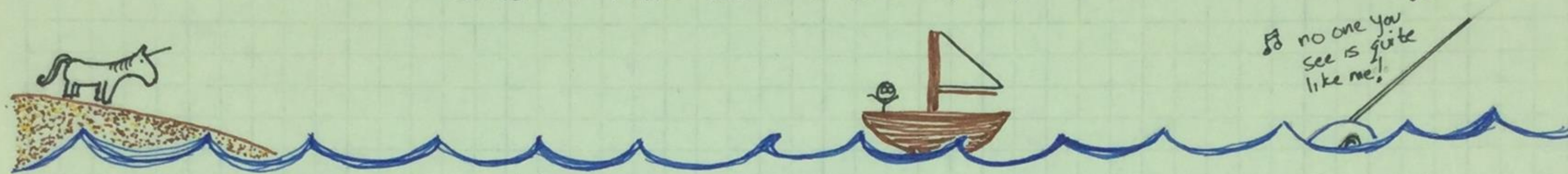
So

$$P(\text{mermaid} \mid \text{SMM}) = \frac{(1.0)(\text{close to zero})}{(\text{pretty high})}$$

$\boxed{= \text{probably not}}$

An simple example of its use with respect to mermaids and singing marine mammals.

ON ANOTHER DAY, the sailor sighted Land! Upon Land, he saw there was a single horned mammal.
Could it have been a UNICORN?



$$P\left(\text{unicorn} \,\middle|\, \begin{array}{c}\text{single}\\ \text{horned}\\ \text{land}\\ \text{mammal}\end{array}\right) = \frac{\overset{\text{definite!}}{P(\text{SHUM} \mid \text{unicorn})}\, P(\text{unicorn})}{P(\text{SHUM})}$$

SHUM

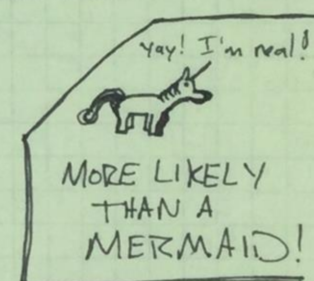
← umm...zero? really close to zero.

← Very small as only the **ENDANGERED** Indian Rhino is a naturally occurring SHUM (and its territory doesn't ~~too~~ border the ocean)

$$P(\text{unicorn} \mid \text{SHUM}) = \frac{(1.0)(\text{close to zero})}{(\text{pretty low})}$$

NEAT! The lack of other SHUMS makes it <u>more</u> likely that (since our sailor saw a SHUM) he saw a unicorn

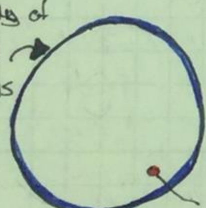
$$= \boxed{\text{Probably not...but maaaybe}}$$



Consider this Another Way

CIRCLES

Probability of Singing marine mammals

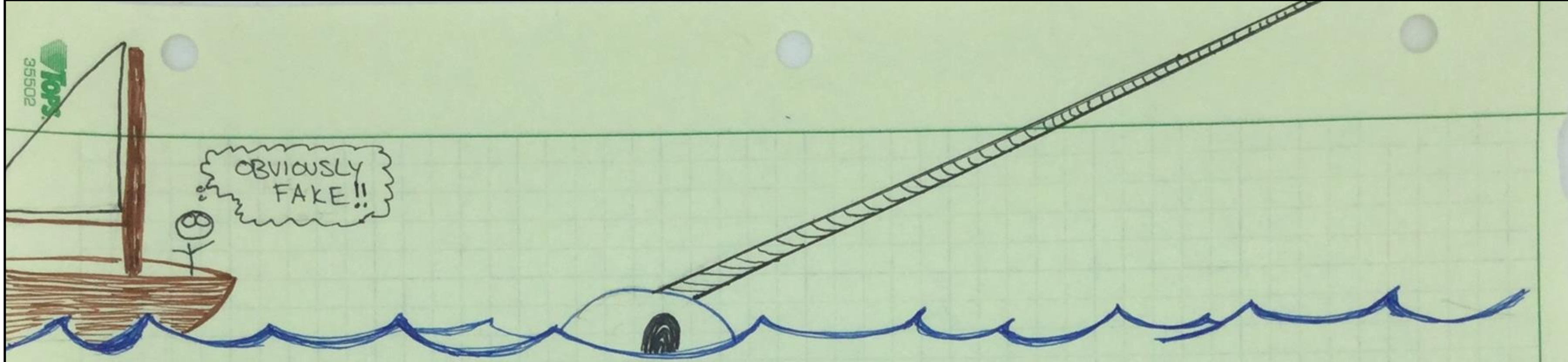


Probability of a mermaid is a tiny speck in a big circle

Probability of single horned land mammals

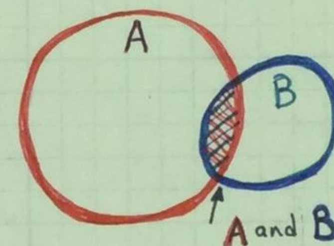
on the other hand the probability of a unicorn is a tiny speck in a tiny circle

NOTE: All non-unicorn single horned land mammals are rhinos and there aren't many of those



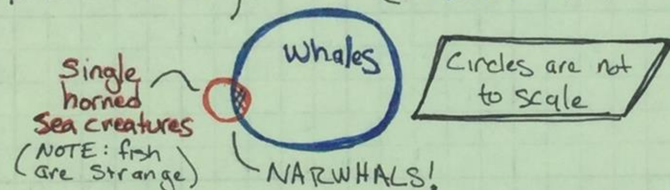
$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

BAYES' RULE is really all about whether you expect to see **A** AND **B** together.



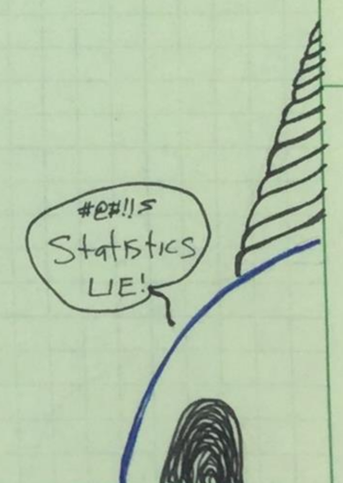
$P(A|B)$ means given you are in **B** what is the probability you are also in **A**.

$$P(\text{single horned sea creature AND whale}) = P(\text{whale AND single horned sea creature})$$



So! $$P(\text{single horned sea creature} \mid \text{whale}) = \frac{P(\text{whale} \mid \text{single harked sea creature}) \cdot P(\text{single horned sea creature})}{P(\text{whale})}$$

$\therefore$ Really, there is no chance of narwhals being real.



Another way to think of what the equation means.

ONE DAY the sailor discovered **TREASURE**

The sailor saw a chest filled with:
50 beautiful PEARLS
10 GOLDEN doubloons
40 rusty iron coins (worthless)

Let's make some probabilities! Yay!

$P(\text{pearl} \mid \text{spherical}) = 1.0$ $P(\text{spherical} \mid \text{pearl}) = 1.0$

$P(\text{coin} \mid \text{iron}) = 1.0$ $P(\text{coin} \mid \text{gold}) = 1.0$

$P(\text{gold if coin}) = \frac{\text{gold}}{\text{all coins}} = \frac{10}{10+40} = \frac{10}{50} = 0.2$

$P(\text{iron if coin}) = \boxed{}$

TRY ME!

BEWARE!
You may only take **ONE** treasure AND you must choose with your eyes CLOSED or **SUFFER THE CURSE!**

What should the sailor choose? Something **SPHERICAL** or **COIN**-shaped?

$P(\text{gold}) = \frac{\text{gold}}{\text{everything}} = \frac{10}{100} = 0.1$ $P(\text{iron}) = \frac{40}{100} = 0.4$

$P(\text{coin}) = \frac{10+40}{\text{everything}} = \frac{50}{100} = 0.5$ $P(\text{spherical}) = \frac{50}{100} = 0.5$

$P(\text{pearl}) = \boxed{}$

TRY ME!

$P(\text{pearl} \mid \text{spherical}) = 1.0$

$$P(\text{gold} \mid \text{coin}) = \frac{\overset{1.0}{P(\text{coin} \mid \text{gold})}\ \overset{0.1}{P(\text{gold})}}{\underset{0.5}{P(\text{coin})}} = \underline{\underline{0.2}}!$$

We didn't need **BAYES' RULE** as everything was known ahead of time and didn't change. NOTE that the answer was just P(gold if coin) ... Good practice though...

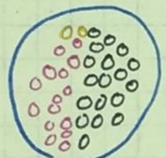
SPHERICAL
as there is a **100%** chance the sailor will get a PEARL but only 20% of getting GOLD if a COIN is chosen

HO HUM! Another day, another cursed treasure...

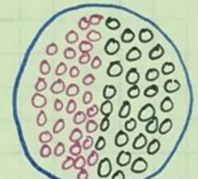
SAME CURSE

only one and eyes closed

CONTENTS
- 35 spherical pearls
- 15 coin pearls
- 2 gold doubloons
- 30 glass marbles
- 18 iron coins



Coins



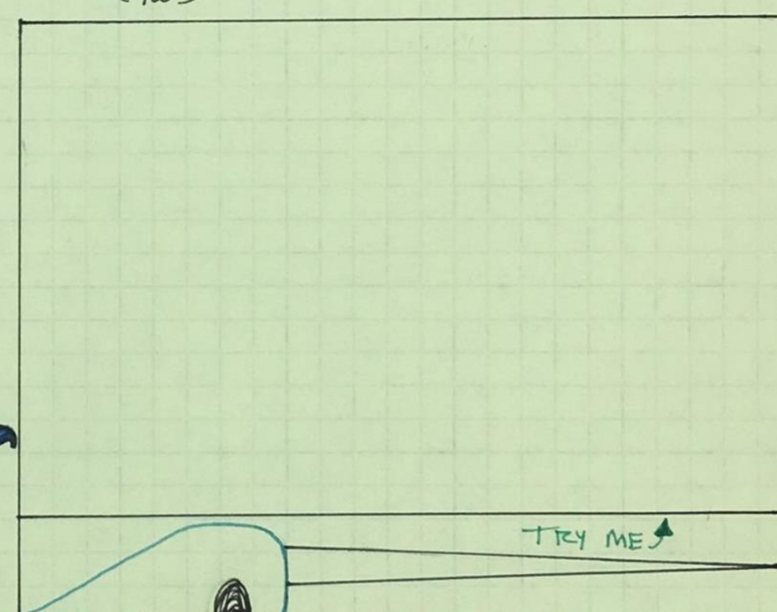
Spherical

$$P\left(\text{good stuff} \mid \text{spherical}\right) = \frac{P\left(\text{spherical} \mid \text{good stuff}\right) P\left(\text{good stuff}\right)}{P\left(\text{spherical}\right)}$$

$$= \left(\frac{35 \text{ spherical pearls}}{35 \text{ s pearls} + 15 \text{ coin pearls} + 2 \text{ gold}}\right)\left(\frac{35 \text{ spherical pearls} + 15 \text{ coin pearls} + 2 \text{ gold coins}}{\text{everything} = 100}\right)$$

$$= \frac{\left(\frac{35}{52} \cdot \frac{52}{100}\right)}{\left(\frac{65}{100}\right)} = \frac{35}{65} = \underline{\underline{0.54}}$$

$$P\left(\text{good stuff} \mid \text{coin}\right) =$$

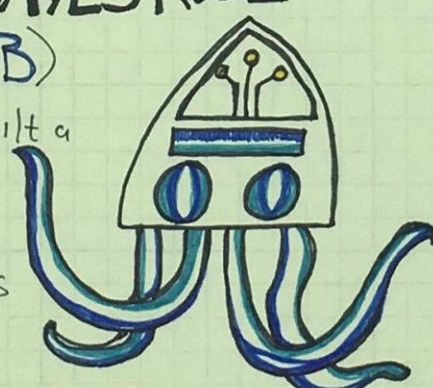


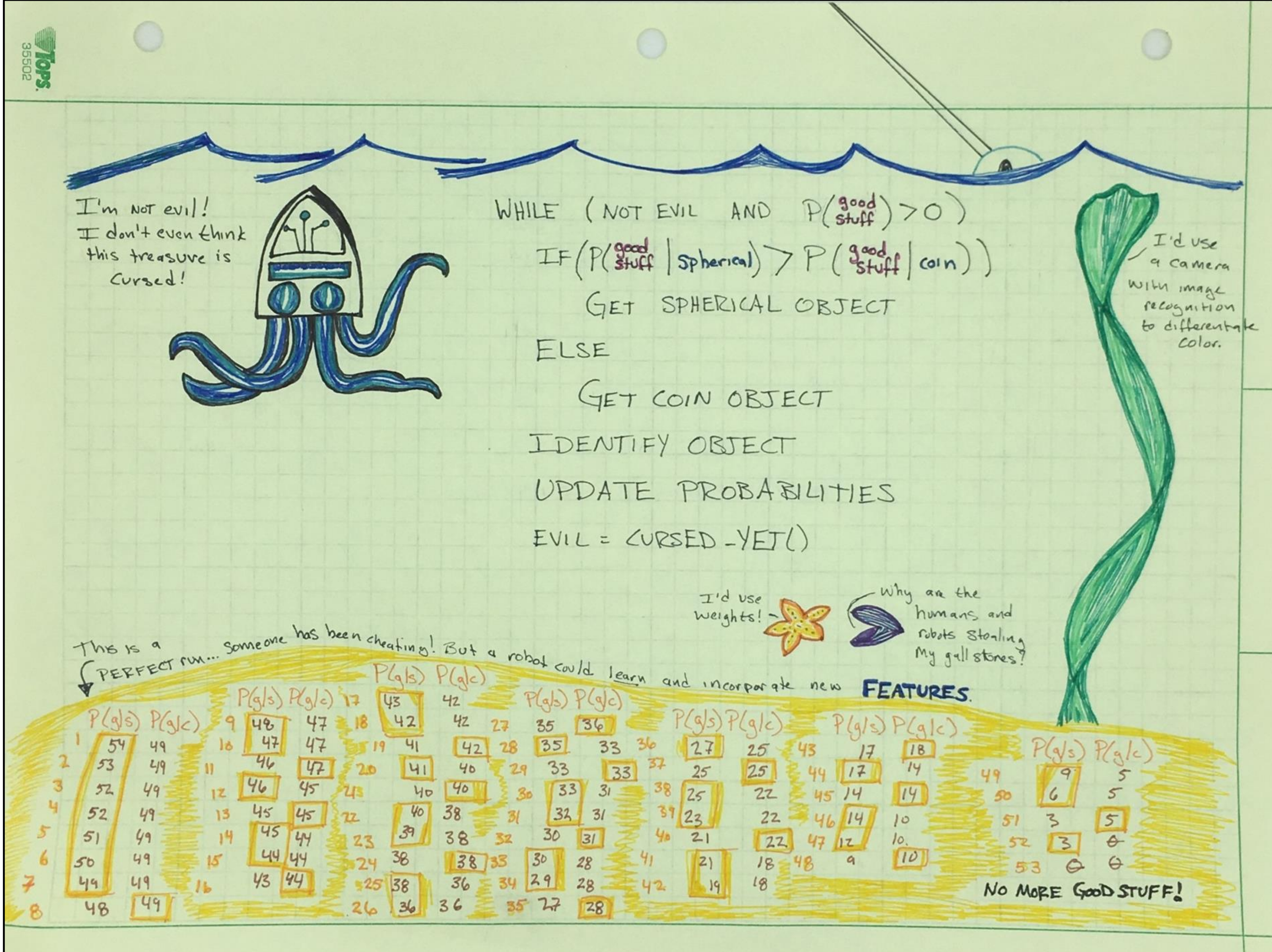
This is still a fixed problem where nothing changes!

We don't even need **BAYES' RULE** because

$$P(A \mid B) = P(A \text{ AND } B)$$

WHAT IF our sailor built a **ROBOT** ~ to choose the treasures until the curse turns the robot **EVIL**?





I'm NOT evil!
I don't even think this treasure is Cursed!
WHILE (NOT EVIL AND P(good stuff) > 0)
IF (P(good stuff | spherical) > P(good stuff | coin))
GET SPHERICAL OBJECT
ELSE
GET COIN OBJECT
IDENTIFY OBJECT
UPDATE PROBABILITIES
EVIL = CURSED_YET()
I'd use a camera with image recognition to differentiate color.
I'd use weights!
Why are the humans and robots stealing my gall stones?
This is a PERFECT run... Someone has been cheating! But a robot could learn and incorporate new FEATURES.
P(g|s) P(g|c)
1 54 49
2 53 49
3 52 49
4 52 49
5 51 49
6 50 49
7 49 49
8 48 49
P(g|s) P(g|c)
9 48 47
10 47 47
11 46 47
12 46 45
13 45 45
14 45 44
15 44 44
16 43 44
P(g|s) P(g|c)
17 43 42
18 42 42
19 41 42
20 41 40
21 40 40
22 40 38
23 39 38
24 38 38
25 38 36
26 36 36
P(g|s) P(g|c)
27 35 36
28 35 33
29 33 33
30 33 31
31 32 31
32 30 31
33 30 28
34 29 28
35 27 28
P(g|s) P(g|c)
36 27 25
37 25 25
38 25 22
39 23 22
40 21 22
41 21 18
42 19 18
P(g|s) P(g|c)
43 17 18
44 17 14
45 14 14
46 14 10
47 12 10.
48 9 10
P(g|s) P(g|c)
49 9 5
50 6 5
51 3 5
52 3 0
53 0 0
NO MORE GOOD STUFF!

I'm so RICH I'm going to write a BOOK!

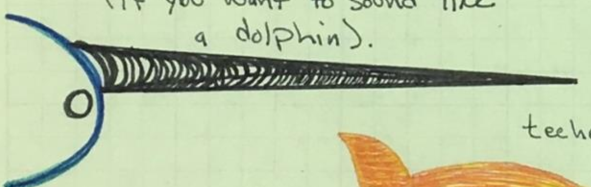
AFTER A YEAR OF TREASURE HUNTING THE SAILOR LOOKED AT THE DATA AND IDENTIFIED SOME TRENDS

PRIOR PROBABILITY

60% OF TREASURES WERE "GOOD STUFF"

But there was a mix of JEWELS, ROCKS, COINS, MARBLES, AND PEARLS.

This is also called A PRIORI PROBABILITY (if you want to sound like a dolphin).



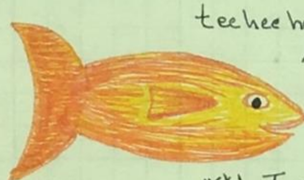
$$P\left(\text{good stuff} \,\middle|\, \begin{bmatrix}\text{shape}\\ \text{weight}\\ \text{color}\end{bmatrix}\right)$$

our goal is to determine the

POSTERIOR PROBABILITY

teeheeheeheeheehee ARE YOU SERIOUSLY LOOKING FOR YOUR POSTERIOR?!? teeheehee SNORT heehee gulp CHOKE

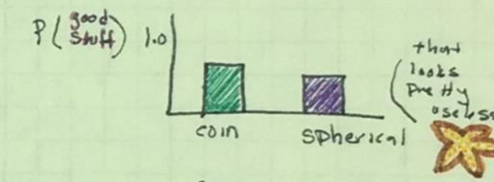
urk! I just got water up my nose! WAIT... THAT'S NORMAL...

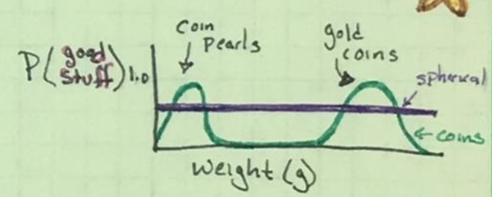


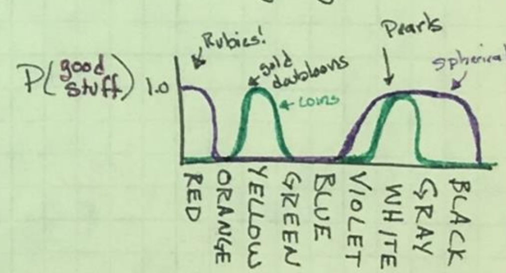
FEATURE VECTOR

No need to be limited to COIN or SPHERICAL

Using all available information is more useful:







$$P\left(\text{good stuff} \,\middle|\, \begin{bmatrix}\text{shape}\\ \text{weight}\\ \text{color}\end{bmatrix}\right) = \frac{P\left(\begin{bmatrix}s\\ w\\ c\end{bmatrix} \,\middle|\, \text{good stuff}\right) P(\text{good stuff})}{P\left(\begin{bmatrix}s\\ w\\ c\end{bmatrix}\right)}$$

FORGET FISH for a moment.

there are **ENDLESS APPLICATIONS**

I AM NOT A FISH!

Airplanes?!? Sounds MYTHICAL... only FISH should fly!

SEARCH

$$P\left(\text{person will click link} \mid \text{search term}\right) = \frac{P\left(\text{search term} \mid \text{others clicked on link}\right) P\left(\text{link clicked}\right)}{P\left(\text{term searched}\right)}$$

BETTER SEARCH → MORE FEATURES!

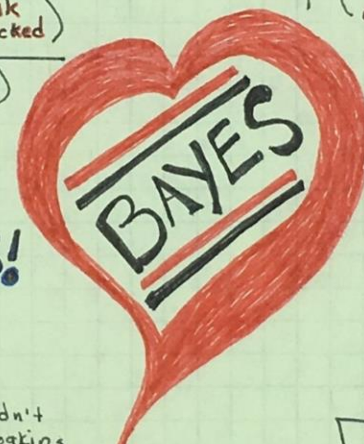
$$P\left(\text{person will click link} \mid \left[\text{search term, browser history, user age}\right]\right)$$

SPAM DETECTION

$$P\left(\text{this is spam} \mid \left[\text{"date a STARfish"}, \text{domain .sea}, \text{"get rich quick"}\right]\right)$$

why wouldn't aliens use cloaking or some sort of camoflauge? why would they skulk? why not say "hi"?

Exciting! But is seems like an awful lot of work

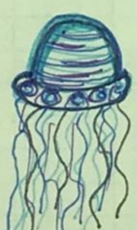


ALIENS

$$P\left(\text{Aliens visit} \mid \text{UFO}\right) = \frac{P\left(\text{UFO} \mid \text{Aliens visit}\right) P\left(\text{Aliens visit}\right)}{P\left(\text{UFO}\right)}$$

~ weather balloons or Japanese lanterns or shooting stars or all the other things I don't know about

$$= \frac{\text{low? low?}}{\text{high}} = \text{v.v. low}$$



"MAN OF WAR" is not a jelly fish at all!

NERVOUS FLYER

$$P\left(\text{death by crashing} \mid \text{on airplane}\right) = \frac{P\left(\text{airplane} \mid \text{crash}\right) P\left(\text{crash}\right)}{P\left(\text{airplane}\right)}$$

$$= \frac{\left(\text{airplanes v.v. very seldom crash}\right) \times \left(\text{other things crash: helicopters, cars, bikes, etc.}\right)}{\left(\text{time spent on airplane}\right)}$$

$$= \frac{\left(\text{vvv low}\right) \times \left(\text{vv low}\right)}{\left(\text{v low}\right)} = \text{vvvv low}$$

(less likely to die in airplane crash than in other ways

DISEASE ITIS

$$P\left(\text{have disease} \mid \text{positive test result}\right) = \frac{P\left(\text{positive result} \mid \text{have disease}\right) P\left(\text{have disease}\right)}{P\left(\text{positive result}\right)}$$

= disease testing is IMPERFECT and knowing how imperfect may be vitally important

You don't need to do math to use Bayes. It is a tool for thinking for humans or machines.

BAYES' RULE is a TOOL for UPDATING your BELIEFS using EVIDENCE

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

POSTERIOR PROBABILITY →

$$P\left(\text{hypothesis is TRUE} \,\middle|\, \text{evidence you see}\right) = \frac{P\left(\text{Seeing this evidence} \,\middle|\, \text{hypothesis is TRUE}\right) P\left(\text{hypothesis is TRUE}\right)}{P(\text{Seeing this evidence})}$$

This is the **PRIOR PROBABILITY**. It is the probability of your **hypothesis** being true if you have **NO EVIDENCE** at all.

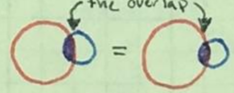
This is the **LIKELIHOOD** of the **EVIDENCE** SUPPORTING THE **HYPOTHESIS**

"MARGINAL LIKELIHOOD" is often ignored ("NORMALIZED")

How to REMEMBER the EQUATION:

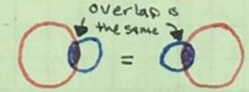
we only care about the overlap

$P(A \text{ AND } B) = P(A \text{ AND } B)$



overlap is the same

$P(A \text{ AND } B) = P(B \text{ AND } A)$



$P(A|B)P(B) = P(B|A)P(A)$

WHAT?!? How did that change?

let me explain that...

We only want the **OVERLAP** so let's stop looking at the whole circles!

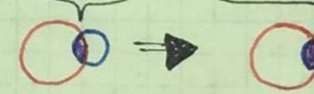
$P(A|B)\,P(B)$

is like saying

$P(A)$ given $P(B)$ is TRUE

THUS

$$\underbrace{P(A \text{ AND } B)}\ \rightarrow\ \underbrace{P(A|B)P(B)}$$



$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

This part is just a little ALGEBRA.

You don't need me anymore.

By ELECIA WHITE see http://embedded.fm/blog/narwhal for complete original **GOOD JOB!**

If you flip back to slide 2, remember how that looked scary? I hope it is not anymore.

The End

Any mistakes are mine. However, there are fewer than there might be thanks to the efforts of Linda Patton, Christopher Svec, Elizabeth Brenner, Andrei Chichak, and Christopher White.

If you want to know more, take a look at my favorite Bayes visualization method and this great warehouse of explanation techniques.