Many-Body Phys HW8

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1 Problem 1

The Hamiltonian in the collective space is constructed via

$$\hat{H}_{k,k'} = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta d\theta' H(\theta,\theta')\frac{e^{i(k\theta-k'\theta')}}{\sqrt{n_k n_{k'}}} \tag{1}$$

Where

$$H(\theta,\theta') = -\frac{\epsilon\Omega}{2}\cos^{\Omega}\left(\frac{\theta-\theta'}{2}\right) * \left[\frac{\cos\left(\frac{\theta+\theta'}{2}\right)}{\cos\left(\frac{\theta-\theta'}{2}\right)} + \frac{\chi}{2}\left[\frac{1+\sin^2\left(\frac{\theta+\theta'}{2}\right)}{\cos^2\left(\frac{\theta-\theta'}{2}\right)} - 1\right]\right] \tag{2}$$

And

$$n_k = \frac{2\pi}{2^{\Omega}}\binom{\Omega}{k+\frac{\Omega}{2}} \tag{3}$$

Where $k \in [-\frac{\Omega}{2}, \frac{\Omega}{2}]$ is in the multiplet of total quasi-spin $K = \frac{\Omega}{2}$. Numerically performing the integration in equation 1, and diagonalizing the subsequent $H_{k,k'}$, the eigenvalues for the total quasi-spin $K = \frac{\Omega}{2}$ block are extracted.

Please see the data file "Output_HW8.txt". The file also contains the energies and corresponding collective wavefunctions. The computed eigenvalues agree with the exact calculation seen in "Output_HW2.txt". Moreover, please see figure 1 comparing the GCM calculation to exact diagonalization over different interaction strengths.

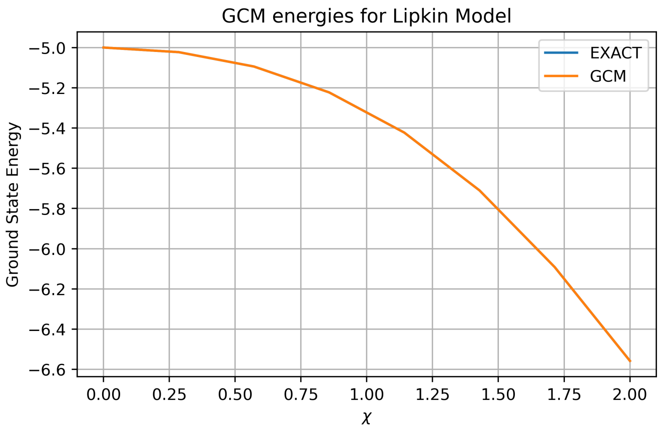

Figure 1: Parameters: $\Omega = 10$, $N = 10$, $\epsilon = 1$. Only $V = [0, 0.222]$ is varied. Here are the ground state energy calculations from both GCM and exact diagonalization. The GCM method works well over all values of the interaction. It really is equivalent to exact diagonalization of the Hamiltonian constructed in the basis of Slater determinants.