Many-Body Phys HW8

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1 Problem 1

The Hamiltonian in the collective space is constructed via

$$\hat{H}_{k,k'} = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta d\theta' H(\theta,\theta')\frac{e^{i(k\theta-k'\theta')}}{\sqrt{n_k n_{k'}}} \tag{1}$$

Where

$$H(\theta,\theta') = -\frac{\epsilon\Omega}{2}\cos^{\Omega}\left(\frac{\theta-\theta'}{2}\right) * \left[\frac{\cos\left(\frac{\theta+\theta'}{2}\right)}{\cos\left(\frac{\theta-\theta'}{2}\right)} + \frac{\chi}{2}\left[\frac{1+\sin^2\left(\frac{\theta+\theta'}{2}\right)}{\cos^2\left(\frac{\theta-\theta'}{2}\right)} - 1\right]\right] \tag{2}$$

And

$$n_k = \frac{2\pi}{2^{\Omega}}\binom{\Omega}{k+\frac{\Omega}{2}} \tag{3}$$

Where $k \in [-\frac{\Omega}{2},\ \frac{\Omega}{2}]$ is in the multiplet of total quasi-spin $K = \frac{\Omega}{2}$. Numerically performing the integration in equation 1, and diagonalizing the subsequent $H_{k,k'}$

$$\sum_{k'} \hat{H}_{k,k'} g_{k'} = E_k g_k \tag{4}$$

The eigenvalues for the total quasi-spin $K = \frac{\Omega}{2}$ block are extracted. Moreover, the collective wavefunctions

$$g(\Theta) = \sum_{k'} \frac{g_{k'}}{\sqrt{2\pi}} e^{-ik'\Theta} \tag{5}$$

Are plotted in figures 2 and 3.

Please see the data file "Output_HW8.txt". The file contains the energies and corresponding collective wavefunctions g_k for $\Omega = 10$, $\epsilon = 1$, $V = 0.1$, and $\chi = 0.9$. The computed eigenvalues agree with the exact calculation seen in "Output_HW2.txt". Moreover, please see figure 1 comparing the GCM calculation to exact diagonalization over different interaction strengths. All relevant files are stored under "HW8" on my Github.

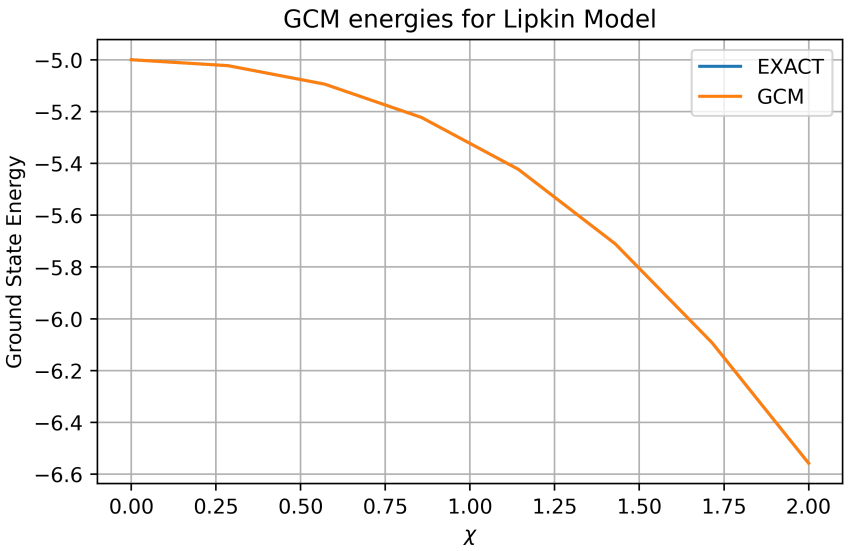

Figure 1: Parameters: $\Omega = 10$, $N = 10$, $\epsilon = 1$. Only $V = [0, 0.222]$ is varied. Here are the ground state energy calculations from both GCM and exact diagonalization. The GCM method works well over all values of the interaction. It really is equivalent to exact diagonalization of the Hamiltonian constructed in the basis of Slater determinants.

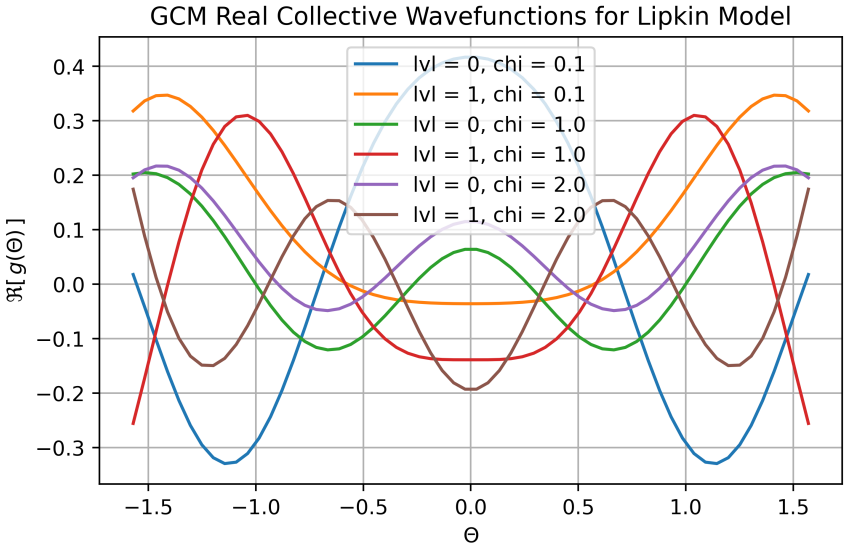

Figure 2: Real component of the ground and first excited state wavefunctions over multiple interactions.

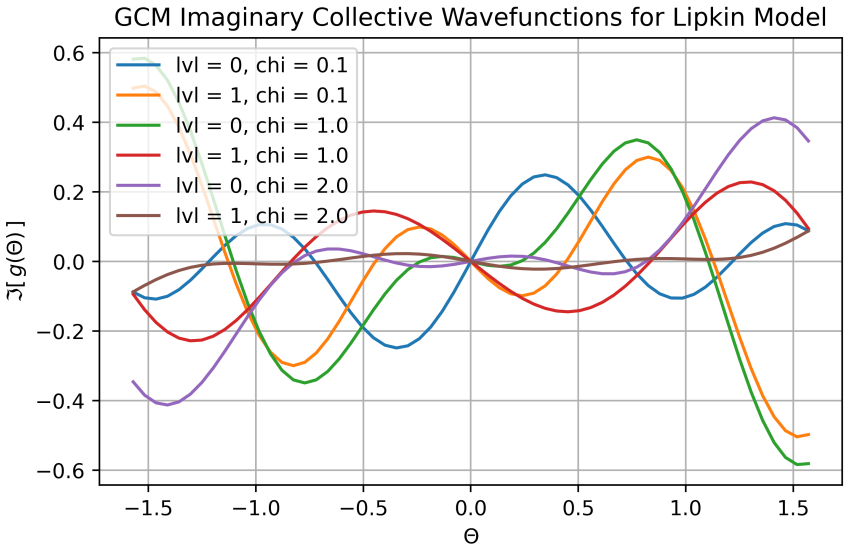

Figure 3: Imaginary component of the ground and first excited state wavefunctions over multiple interactions.