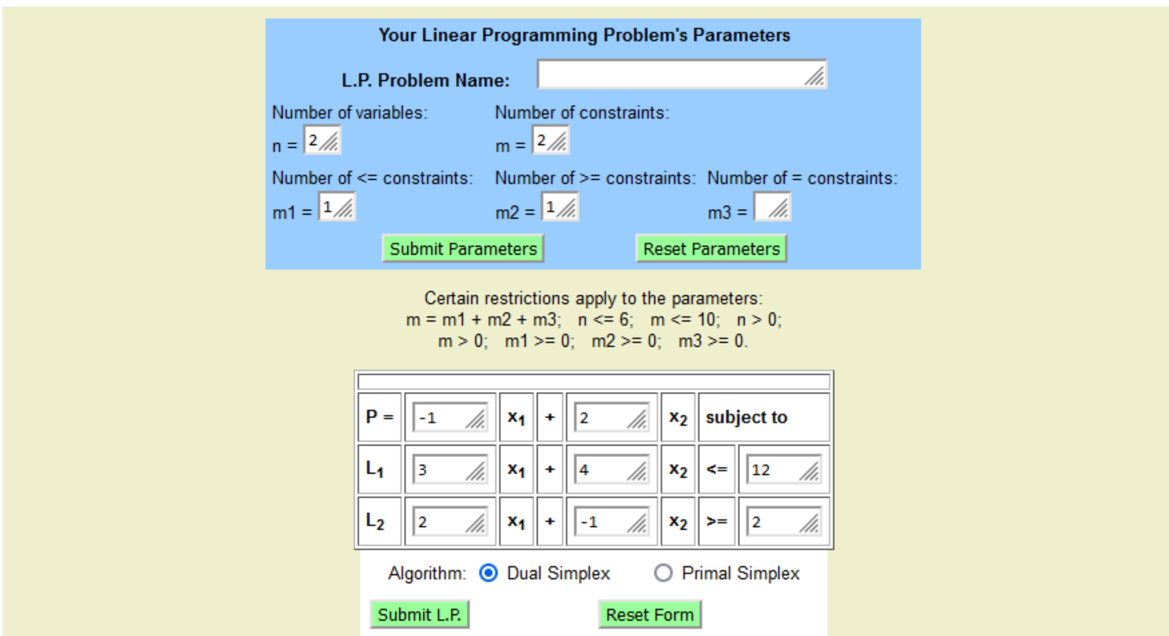
Problema 6.1

6.1 Consider the following problem.

$$\begin{aligned} \text{Maximize} \quad & -x_1 + 2x_2 \\ \text{Subject to} \quad & 3x_1 + 4x_2 \leq 12 \\ & 2x_1 - x_2 \geq 2 \\ & x_1, \quad x_2 \geq 0 \end{aligned}$$

a. Solve the problem graphically.
b. State the dual and solve it graphically. Utilize the theorems of duality to obtain the values of all the primal variables from the optimal dual solution.


Your Linear Programming Problem's Parameters

L.P. Problem Name:

Number of variables: n = 2

Number of constraints: m = 2

Number of <= constraints: m1 = 1

Number of >= constraints: m2 = 1

Number of = constraints: m3 =

Submit Parameters   Reset Parameters

Certain restrictions apply to the parameters:
m = m1 + m2 + m3; n <= 6; m <= 10; n > 0;
m > 0; m1 >= 0; m2 >= 0; m3 >= 0.

P =	-1	x_1	+	2	x_2	subject to	
L_1	3	x_1	+	4	x_2	<=	12
L_2	2	x_1	+	-1	x_2	>=	2

Algorithm: (•) Dual Simplex ( ) Primal Simplex

Submit L.P.   Reset Form

Phase II: Goal: get ß >= 0.

Tableau^1						
	b^1	x^1_1	x^1_2	s^1_1	s^1_2	row sum
$L^1_1 = L^0_1 / (4)$	3	0.75	1	0.25	0	5
$L^1_2 = L^0_2 - (1) * L^1_1$	-5	-2.75	0	-0.25	1	-7
$P^1 = P^0 - (-2) * L^1_1$	6	2.5	0	0.5	0	9
$-P^1 / L^1_2$	0	0.909	0	2	0	0

Basis for Tableau^1: [x_2, s_2,]. Value of Objective Function = 6.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Complete.

Phase II: Goal: get ß >= 0.

A. In Tableau^1:

1. Select a **pivot row, row,** with $b^1_{row} < 0$: row = 2 associated with $b^1_2 = -5$.

2. Compute the ratios $-Ø / L_2$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 1 associated with 0.909. Thus **$Û_{2,1}$ = -2.75 is the pivot**; variable s_2 will leave the basis; variable x_1 will enter the basis.

B. To create Tableau^2:

3. Compute row $L^2_2 = L^1_2 / (-2.75)$.

4. Subtract multiples of row L^2_2 from all other rows of Tableau^1 so that $x_1 = e_2$ in Tableau^2.

Tableau^2						
	b^2	x^2_1	x^2_2	s^2_1	s^2_2	row sum
$L^2_1 = L^1_1 - (0.75) * L^2_2$	1.636	0	1	0.182	0.273	3.091
$L^2_2 = L^1_2 / (-2.75)$	1.818	1	-0	0.091	-0.364	2.545
$P^2 = P^1 - (2.5) * L^2_2$	1.455	0	0	0.273	0.909	2.636
$-P^2 / L^2_{-1}$	0	0	0	0	0	0

Basis for Tableau^2: [x_2, x_1,]. Value of Objective Function = 1.45.

Phase 0: Complete.

Phase I: Complete.

Phase II: Complete.

Primal Solution: [x_2, x_1,] = [1.636, 1.818,]; P = 1.455.

(Primal x variables not in the basis have a value of 0).

Dual Solution: [y_1, y_2,] = [0.273, 0.909,]; D = 1.455.

End of the Linear Programming Dual Simplex Method

Problema 6.2

6.2 Consider the following problem.

$$\begin{array}{lrcl} \text{Minimize} & 2x_1+3x_2+5x_3+6x_4 & & \\ \text{Subject to} & x_1+2x_2+3x_3+x_4 & \geq & 2 \\ & -2x_1+x_2-x_3+3x_4 & \leq & -3 \\ & x_1,\ x_2,\ x_3,\ x_4 & \geq & 0 \end{array}$$

Phase I: Goal: get Ø >= 0.

Tableau^0

	b^0	x^0_1	x^0_2	x^0_3	x^0_4	s^0_1	s^0_2	row sum
L^0_1	-3	-2	1	-1	3	1	0	-1
L^0_2	-2	-1	-2	-3	-1	0	1	-8
P^0	0	-2	-3	0	0	0	0	-5
$-P^0 / L^0_1$	0	0	3	0	0	0	0	0

Basis for Tableau^0: [s_1, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^0:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^0_2 = -3$, tcol = 2.

2. Select any row, r, with a positive entry in tcol = 2 as the **pivot row:** row = 1 associated with $Û_{1,2} = 1$ and constraint L_1.

3. Compute the ratios $-Ø / L_1$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 3 associated with 0. Thus **$Û_{1,3} = -1$ is the pivot**; variable s_1 will leave the basis; variable x_3 will enter the basis.

B. To create Tableau^1:

4. Compute row $L^1_1 = L^0_1 / (-1)$.

5. Subtract multiples of row L^1_1 from all other rows of Tableau^0 so that $x_3 = e_1$ in Tableau^1.

Tableau^1								
	b^1	x^1_1	x^1_2	x^1_3	x^1_4	s^1_1	s^1_2	row sum
$L^1_1 = L^0_1 / (-1)$	3	2	-1	1	-3	-1	0	1
$L^1_2 = L^0_2 - (-3) * L^1_1$	7	5	-5	0	-10	-3	1	-5
$P^1 = P^0 - (0) * L^1_1$	0	-2	-3	0	0	0	0	-5
$-P^1 / L^1_2$	0	0.4	0	0	0	0	0	0

Basis for Tableau^1: [x_3, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^1:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^1_1 = -2$, tcol = 1.

2. Select any row, r, with a positive entry in tcol = 1 as the **pivot row:** row = 2 associated with $Û_{2,1} = 5$ and constraint L_2.

3. Compute the ratios $-Ø / L_2$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 4 associated with 0. Thus **$Û_{2,4}$ = -10 is the pivot**; variable s_2 will leave the basis; variable x_4 will enter the basis.

B. To create Tableau^2:

4. Compute row $L^2_2 = L^1_2 / (-10)$.

5. Subtract multiples of row L^2_2 from all other rows of Tableau^1 so that $x_4 = e_2$ in Tableau^2.

Tableau^2								
	b^2	x^2_1	x^2_2	x^2_3	x^2_4	s^2_1	s^2_2	row sum
$L^2_1 = L^1_1 - (-3) * L^2_2$	0.9	0.5	0.5	1	0	-0.1	-0.3	2.5
$L^2_2 = L^1_2 / (-10)$	-0.7	-0.5	0.5	0	1	0.3	-0.1	0.5
$P^2 = P^1 - (0) * L^2_2$	0	-2	-3	0	0	0	0	-5
$-P^2 / L^2_1$	0	4	6	0	0	0.001	0	0

Basis for Tableau^2: [x_3, x_4,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^2:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^2_1 = -2$, tcol = 1.

2. Select any row, r, with a positive entry in tcol = 1 as the **pivot row:** row = 1 associated with $Û_{1,1} = 0.5$ and constraint L_1.

3. Compute the ratios $-Ø / L_1$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 6 associated with 0. Thus **$Û_{1,6}$ = -0.3 is the pivot**; variable x_3 will leave the basis; variable s_2 will enter the basis.

B. To create Tableau^3:

4. Compute row $L^3_1 = L^2_1 / (-0.3)$.

5. Subtract multiples of row L^3_1 from all other rows of Tableau^2 so that $s_2 = e_1$ in Tableau^3.

Tableau^3								
	b^3	x^3_1	x^3_2	x^3_3	x^3_4	s^3_1	s^3_2	row sum
$L^3_1 = L^2_1$ / (-0.3)	-3	-1.667	-1.667	-3.333	-0	0.333	1	-8.333
$L^3_2 = L^2_2$ - (-0.1) * L^3_1	-1	-0.667	0.333	-0.333	1	0.333	0	-0.333
$P^3 = P^2$ - (0) * L^3_1	0	-2	-3	0	0	0	0	-5
$-P^3 / L^3_2$	0	0	9	0	0	0	0	0

Basis for Tableau^3: [s_2, x_4,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^3:

1. Select a **target column, tcol,** with $Ø_{tcol}$ < 0: $Ø^3_2$ = -3, tcol = 2.

2. Select any row, r, with a positive entry in tcol = 2 as the **pivot row:** row = 2 associated with $Û_{2,2}$ = 0.333 and constraint L_2.

3. Compute the ratios -Ø / L_2 as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 3 associated with 0. Thus **$Û_{2,3}$ = -0.333 is the pivot**; variable x_4 will leave the basis; variable x_3 will enter the basis.

B. To create Tableau^4:

4. Compute row $L^4_2 = L^3_2$ / (-0.333).

5. Subtract multiples of row L^4_2 from all other rows of Tableau^3 so that $x_3 = e_2$ in Tableau^4.

Tableau^4								
	b^4	x^4_1	x^4_2	x^4_3	x^4_4	s^4_1	s^4_2	row sum
$L^4_1 = L^3_1$ - (-3.333) * L^4_2	7	5	-5	0	-10	-3	1	-5
$L^4_2 = L^3_2$ / (-0.333)	3	2	-1	1	-3	-1	-0	1
$P^4 = P^3$ - (0) * L^4_2	0	-2	-3	0	0	0	0	-5
$-P^4 / L^4_1$	0	0.4	0	0	0	0	0	0

Basis for Tableau^4: [s_2, x_3,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^4:

1. Select a **target column, tcol,** with $Ø_{tcol}$ < 0: $Ø^4_1$ = -2, tcol = 1.

2. Select any row, r, with a positive entry in tcol = 1 as the **pivot row:** row = 1 associated with $Û_{1,1}$ = 5 and constraint L_1.

3. Compute the ratios -Ø / L_1 as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 4 associated with 0. Thus **$Û_{1,4}$ = -10 is the pivot**; variable s_2 will leave the basis; variable x_4 will enter the basis.

B. To create Tableau^5:

4. Compute row $L^5_1 = L^4_1$ / (-10).

5. Subtract multiples of row L^5_1 from all other rows of Tableau^4 so that $x_4 = e_1$ in Tableau^5.

Tableau^5								
	b^5	x^5_1	x^5_2	x^5_3	x^5_4	s^5_1	s^5_2	row sum
$L^5_1 = L^4_1 / (-10)$	-0.7	-0.5	0.5	-0	1	0.3	-0.1	0.5
$L^5_2 = L^4_2 - (-3) * L^5_1$	0.9	0.5	0.5	1	0	-0.1	-0.3	2.5
$P^5 = P^4 - (0) * L^5_1$	0	-2	-3	0	0	0	0	-5
$-P^5 / L^5_2$	0	4	6	0	0	0.001	0	0

Basis for Tableau^5: [x_4, x_3,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^5:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^5_1 = -2$, tcol = 1.

2. Select any row, r, with a positive entry in tcol = 1 as the **pivot row:** row = 2 associated with $Û_{2,1} = 0.5$ and constraint L_2.

3. Compute the ratios $-Ø / L_2$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 6 associated with 0. Thus **$Û_{2,6}$ = -0.3 is the pivot**; variable x_3 will leave the basis; variable s_2 will enter the basis.

B. To create Tableau^6:

4. Compute row $L^6_2 = L^5_2 / (-0.3)$.

5. Subtract multiples of row L^6_2 from all other rows of Tableau^5 so that $s_2 = e_2$ in Tableau^6.

Tableau^6								
	b^6	x^6_1	x^6_2	x^6_3	x^6_4	s^6_1	s^6_2	row sum
$L^6_1 = L^5_1 - (-0.1) * L^6_2$	-1	-0.667	0.333	-0.333	1	0.333	0	-0.333
$L^6_2 = L^5_2 / (-0.3)$	-3	-1.667	-1.667	-3.333	-0	0.333	1	-8.333
$P^6 = P^5 - (0) * L^6_2$	0	-2	-3	0	0	0	0	-5
$-P^6 / L^6_1$	0	0	9	0	0	0	0	0

Basis for Tableau^6: [x_4, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^6:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^6_2 = -3$, tcol = 2.

2. Select any row, r, with a positive entry in tcol = 2 as the **pivot row:** row = 1 associated with $Û_{1,2} = 0.333$ and constraint L_1.

3. Compute the ratios $-Ø / L_1$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 3 associated with 0. Thus **$Û_{1,3}$ = -0.333 is the pivot**; variable x_4 will leave the basis; variable x_3 will enter the basis.

B. To create Tableau^7:

4. Compute row $L^7_1 = L^6_1 / (-0.333)$.

5. Subtract multiples of row L^7_1 from all other rows of Tableau^6 so that $x_3 = e_1$ in Tableau^7.

Tableau^7								
	b^7	x^7_1	x^7_2	x^7_3	x^7_4	s^7_1	s^7_2	row sum
$L^7_1 = L^6_1$ / (-0.333)	3	2	-1	1	-3	-1	-0	1
$L^7_2 = L^6_2$ - (-3.333) * L^7_1	7	5	-5	0	-10	-3	1	-5
$P^7 = P^6$ - (0) * L^7_1	0	-2	-3	0	0	0	0	-5
$-P^7 / L^7_2$	0	0.4	0	0	0	0	0	0

Basis for Tableau^7: [x_3, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^7:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^7_1 = -2$, tcol = 1.

2. Select any row, r, with a positive entry in tcol = 1 as the **pivot row:** row = 2 associated with $Û_{2,1} = 5$ and constraint L_2.

3. Compute the ratios -Ø / L_2 as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 4 associated with 0. Thus **$Û_{2,4}$ = -10 is the pivot**; variable s_2 will leave the basis; variable x_4 will enter the basis.

B. To create Tableau^8:

4. Compute row $L^8_2 = L^7_2$ / (-10).

5. Subtract multiples of row L^8_2 from all other rows of Tableau^7 so that $x_4 = e_2$ in Tableau^8.

Tableau^8								
	b^8	x^8_1	x^8_2	x^8_3	x^8_4	s^8_1	s^8_2	row sum
$L^8_1 = L^7_1$ - (-3) * L^8_2	0.9	0.5	0.5	1	0	-0.1	-0.3	2.5
$L^8_2 = L^7_2$ / (-10)	-0.7	-0.5	0.5	-0	1	0.3	-0.1	0.5
$P^8 = P^7$ - (0) * L^8_2	0	-2	-3	0	0	0	0	-5
$-P^8 / L^8_1$	0	4	6	0	0	0.001	0	0

Basis for Tableau^8: [x_3, x_4,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^8:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^8_1 = -2$, tcol = 1.

2. Select any row, r, with a positive entry in tcol = 1 as the **pivot row:** row = 1 associated with $Û_{1,1} = 0.5$ and constraint L_1.

3. Compute the ratios -Ø / L_1 as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 6 associated with 0. Thus **$Û_{1,6}$ = -0.3 is the pivot**; variable x_3 will leave the basis; variable s_2 will enter the basis.

B. To create Tableau^9:

4. Compute row $L^9_1 = L^8_1$ / (-0.3).

5. Subtract multiples of row L^9_1 from all other rows of Tableau^8 so that $s_2 = e_1$ in Tableau^9.

Tableau^9								
	b^9	x^9_1	x^9_2	x^9_3	x^9_4	s^9_1	s^9_2	row sum
$L^9_1 = L^8_1 / (-0.3)$	-3	-1.667	-1.667	-3.333	-0	0.333	1	-8.333
$L^9_2 = L^8_2 - (-0.1) * L^9_1$	-1	-0.667	0.333	-0.333	1	0.333	0	-0.333
$P^9 = P^8 - (0) * L^9_1$	0	-2	-3	0	0	0	0	-5
$-P^9 / L^9_2$	0	0	9	0	0	0	0	0

Basis for Tableau^9: [s_2, x_4,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^9:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^9_2 = -3$, tcol = 2.

2. Select any row, r, with a positive entry in tcol = 2 as the **pivot row:** row = 2 associated with $Û_{2,2} = 0.333$ and constraint L_2.

3. Compute the ratios $-Ø / L_2$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 3 associated with 0. Thus **$Û_{2,3}$ = -0.333 is the pivot**; variable x_4 will leave the basis; variable x_3 will enter the basis.

B. To create Tableau^{10}:

4. Compute row $L^{10}_2 = L^9_2 / (-0.333)$.

5. Subtract multiples of row L^{10}_2 from all other rows of Tableau^9 so that $x_3 = e_2$ in Tableau^{10}.

Tableau^{10}								
	b^{10}	x^{10}_1	x^{10}_2	x^{10}_3	x^{10}_4	s^{10}_1	s^{10}_2	row sum
$L^{10}_1 = L^9_1 - (-3.333) * L^{10}_2$	7	5	-5	0	-10	-3	1	-5
$L^{10}_2 = L^9_2 / (-0.333)$	3	2	-1	1	-3	-1	-0	1
$P^{10} = P^9 - (0) * L^{10}_2$	0	-2	-3	0	0	0	0	-5
$-P^{10} / L^{10}_1$	0	0.4	0	0	0	0	0	0

Basis for Tableau^{10}: [s_2, x_3,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^{10}:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^{10}_1 = -2$, tcol = 1.

2. Select any row, r, with a positive entry in tcol = 1 as the **pivot row:** row = 1 associated with $Û_{1,1} = 5$ and constraint L_1.

3. Compute the ratios $-Ø / L_1$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 4 associated with 0. Thus **$Û_{1,4}$ = -10 is the pivot**; variable s_2 will leave the basis; variable x_4 will enter the basis.

B. To create Tableau^{11}:

4. Compute row $L^{11}_1 = L^{10}_1 / (-10)$.

5. Subtract multiples of row L^{11}_1 from all other rows of Tableau^{10} so that $x_4 = e_1$ in Tableau^{11}.

Possible cycling in Phase 1. Results may be in error!

Possible cycling in Phase 1. Results may be in error!

Phase II: Goal: get ß >= 0.

Tableau^{11}								
	b^{11}	x^{11}_1	x^{11}_2	x^{11}_3	x^{11}_4	s^{11}_1	s^{11}_2	row sum
$L^{11}_1 = L^{10}_1 / (-10)$	-0.7	-0.5	0.5	-0	1	0.3	-0.1	0.5
$L^{11}_2 = L^{10}_2 - (-3) * L^{11}_1$	0.9	0.5	0.5	1	0	-0.1	-0.3	2.5
$P^{11} = P^{10} - (0) * L^{11}_1$	0	-2	-3	0	0	0	0	-5
$-P^{11} / L^{11}_1$	0	0	0	0	0	0	0.001	0

Basis for Tableau^{11}: [x_4, x_3,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Complete.

Phase II: Goal: get ß >= 0.

A. In Tableau^{11}:

1. Select a **pivot row, row,** with $b^{11}_{row} < 0$: row = 1 associated with $b^{11}_1 = -0.7$.

2. Compute the ratios $-Ø / L_1$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 6 associated with 0.001. Thus **$Û_{1,6}$ = -0.1 is the pivot**; variable x_4 will leave the basis; variable s_2 will enter the basis.

B. To create Tableau^{12}:

3. Compute row $L^{12}_1 = L^{11}_1 / (-0.1)$.

4. Subtract multiples of row L^{12}_1 from all other rows of Tableau^{11} so that $s_2 = e_1$ in Tableau^{12}.

Tableau^{12}								
	b^{12}	x^{12}_1	x^{12}_2	x^{12}_3	x^{12}_4	s^{12}_1	s^{12}_2	row sum
$L^{12}_1 = L^{11}_1 / (-0.1)$	7	5	-5	0	-10	-3	1	-5
$L^{12}_2 = L^{11}_2 - (-0.3) * L^{12}_1$	3	2	-1	1	-3	-1	0	1
$P^{12} = P^{11} - (0) * L^{12}_1$	0	-2	-3	0	0	0	0	-5
$-P^{12} / L^{12}_{-1}$	0	0	0	0	0	0	0	0

Basis for Tableau^{12}: [s_2, x_3,]. Value of Objective Function = 0.

Phase 0: Complete.

Phase I: Complete.

Phase II: Complete.

Primal Solution: [s_2, x_3,] = [7, 3,]; P = 0.

(Primal x variables not in the basis have a value of 0).

Dual Solution: [y_1, y_2,] = [0, 0,]; D = 0.

End of the Linear Programming Dual Simplex Method

6.3

6.3 Solve the following linear program by a graphical method.

$$\begin{aligned} \text{Maximize} \quad & 3x_1 + x_2 + 4x_3 \\ \text{Subject to} \quad & 6x_1 + 3x_2 + 5x_3 \leq 25 \\ & 3x_1 + 4x_2 + 5x_3 \leq 20 \\ & x_1, \quad x_2, \quad x_3 \geq 0 \end{aligned}$$

(*Hint*. Utilize the dual problem.)

Phase I: Goal: get Ø >= 0.

Tableau^0							
	b^0	x^0_1	x^0_2	x^0_3	s^0_1	s^0_2	row sum
L^0_1	25	6	3	5	1	0	40
L^0_2	20	3	4	5	0	1	33
P^0	0	-3	-1	0	0	0	-4
$-P^0 / L^0_1$	0	0.5	0.333	0	0	0	0

Basis for Tableau^0: [s_1, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^0:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^0_1 = -3$, tcol = 1.

2. Select any row, r, with a positive entry in tcol = 1 as the **pivot row:** row = 1 associated with $Û_{1,1} = 6$ and constraint L_1.

3. Compute the ratios $-Ø / L_1$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 2 associated with 0.333. Thus **$Û_{1,2} = 3$ is the pivot**; variable s_1 will leave the basis; variable x_2 will enter the basis.

B. To create Tableau^1:

4. Compute row $L^1_1 = L^0_1 / (3)$.

5. Subtract multiples of row L^1_1 from all other rows of Tableau^0 so that $x_2 = e_1$ in Tableau^1.

Tableau^1							
	b^1	x^1_1	x^1_2	x^1_3	s^1_1	s^1_2	row sum
$L^1_1 = L^0_1 / (3)$	8.333	2	1	1.667	0.333	0	13.333
$L^1_2 = L^0_2 - (4) * L^1_1$	-13.333	-5	0	-1.667	-1.333	1	-20.333
$P^1 = P^0 - (-1) * L^1_1$	8.333	-1	0	1.667	0.333	0	9.333
$-P^1 / L^1_1$	0	0.5	0	0	0	0	0

Basis for Tableau^1: [x_2, s_2,]. Value of Objective Function = 8.33.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^1:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^1_1 = -1$, tcol = 1.

2. Select any row, r, with a positive entry in tcol = 1 as the **pivot row:** row = 1 associated with $Û_{1,1} = 2$ and constraint L_1.

3. Compute the ratios $-Ø / L_1$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 1 associated with 0.5. Thus **$Û_{1,1} = 2$ is the pivot**; variable x_2 will leave the basis; variable x_1 will enter the basis.

B. To create Tableau^2:

4. Compute row $L^2_1 = L^1_1 / (2)$.

5. Subtract multiples of row L^2_1 from all other rows of Tableau^1 so that $x_1 = e_1$ in Tableau^2.

Tableau^2							
	b^2	x^2_1	x^2_2	x^2_3	s^2_1	s^2_2	row sum
$L^2_1 = L^1_1 / (2)$	4.167	1	0.5	0.833	0.167	0	6.667
$L^2_2 = L^1_2 - (-5) * L^2_1$	7.5	0	2.5	2.5	-0.5	1	13
$P^2 = P^1 - (-1) * L^2_1$	12.5	0	0.5	2.5	0.5	0	16
$-P^2 / L^2_{-1}$	0	0	0	0	0	0	0

Basis for Tableau^2: [x_1, s_2,]. Value of Objective Function = 12.5.

Phase 0: Complete.

Phase I: Complete.

Phase II: Complete.

Primal Solution: [x_1, s_2,] = [4.167, 7.5,]; P = 12.5.

(Primal x variables not in the basis have a value of 0).

Dual Solution: [y_1, y_2,] = [0.5, 0,]; D = 12.5.

End of the Linear Programming Dual Simplex Method

Tableau^0									
	b^0	x^0_1	x^0_2	x^0_3	x^0_4	x^0_5	s^0_1	s^0_2	row sum
L^0_1	19	1	1	2	3	5	1	0	32
L^0_2	57	2	4	3	2	1	0	1	70
P^0	0	-10	-24	0	0	0	0	0	-34
$-P^0 / L^0_2$	0	5	6	0	0	0	0	0	0

Basis for Tableau^0: [s_1, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^0:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^0_1 = -10$, tcol = 1.

2. Select any row, r, with a positive entry in tcol = 1 as the **pivot row:** row = 2 associated with $\hat{U}_{2,1} = 2$ and constraint L_2.

3. Compute the ratios $-Ø / L_2$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 1 associated with 5. Thus **$\hat{U}_{2,1} = 2$ is the pivot**; variable s_2 will leave the basis; variable x_1 will enter the basis.

B. To create Tableau^1:

4. Compute row $L^1_2 = L^0_2 / (2)$.

5. Subtract multiples of row L^1_2 from all other rows of Tableau^0 so that $x_1 = e_2$ in Tableau^1.

Tableau^1									
	b^1	x^1_1	x^1_2	x^1_3	x^1_4	x^1_5	s^1_1	s^1_2	row sum
$L^1_1 = L^0_1 - (1) * L^1_2$	-9.5	0	-1	0.5	2	4.5	1	-0.5	-3
$L^1_2 = L^0_2 / (2)$	28.5	1	2	1.5	1	0.5	0	0.5	35
$P^1 = P^0 - (-10) * L^1_2$	285	0	-4	15	10	5	0	5	316
$-P^1 / L^1_2$	0	0	2	0	0	0	0	0	0

Basis for Tableau^1: [s_1, x_1,]. Value of Objective Function = 285.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^1:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^1_2 = -4$, tcol = 2.

2. Select any row, r, with a positive entry in tcol = 2 as the **pivot row:** row = 2 associated with $Û_{2,2} = 2$ and constraint L_2.

3. Compute the ratios $-Ø / L_2$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 2 associated with 2. Thus **$Û_{2,2} = 2$ is the pivot**; variable x_1 will leave the basis; variable x_2 will enter the basis.

B. To create Tableau^2:

4. Compute row $L^2_2 = L^1_2 / (2)$.

5. Subtract multiples of row L^2_2 from all other rows of Tableau^1 so that $x_2 = e_2$ in Tableau^2.

Tableau^2									
	b^2	x^2_1	x^2_2	x^2_3	x^2_4	x^2_5	s^2_1	s^2_2	row sum
$L^2_1 = L^1_1 - (-1) * L^2_2$	4.75	0.5	0	1.25	2.5	4.75	1	-0.25	14.5
$L^2_2 = L^1_2 / (2)$	14.25	0.5	1	0.75	0.5	0.25	0	0.25	17.5
$P^2 = P^1 - (-4) * L^2_2$	342	2	0	18	12	6	0	6	386
$-P^2 / L^2_{-1}$	0	0	0	0	0	0	0	0	0

Basis for Tableau^2: [s_1, x_2,]. Value of Objective Function = 342.

Phase 0: Complete.

Phase I: Complete.

Phase II: Complete.

Primal Solution: [s_1, x_2,] = [4.75, 14.25,]; P = 342.

(Primal x variables not in the basis have a value of 0).

Dual Solution: [y_1, y_2,] = [0, 6,]; D = 342.

End of the Linear Programming Dual Simplex Method

6.7

6.7 Consider the following linear program.

$$\text{P: Minimize} \quad 6x_1 + 2x_2$$

$$\text{Subject to} \quad x_1 + 2x_2 \geqslant 3$$

$$x_2 \geqslant 0$$

$$x_1 \quad \text{unrestricted}$$

The **"three-phase method"** of the **dual simplex algorithm**:

Phase 0 - **drive all artificial variables (associated with = constraints) to zero, i.e. eliminate them from the basis;**

Phase I - **find a tableau with Ø >= 0, i.e. a feasible dual program;**

Phase II - **generate tableaux that decrease the value of μ turning ß >= 0, without dropping back into Phase 0 or I, i.e. find a feasible basic dual program that minimizes the objective function D.**

Warning: A non-numeric value encountered in **/services/webpages/e/g/egwald.ca/public/operationsresearch/functions/dualsimplex.php** on line **171**

Phase I: Goal: get Ø >= 0.

Tableau^0						
	b^0	x^0_1	x^0_2	s^0_1	s^0_2	row sum
L^0_1	-3	-1	-2	1	0	-5
L^0_2	0	0	-1	0	1	0
P^0	0	-6	-2	0	0	-8
$-P^0 / L^0_{-1}$	0	0	0	0	0	0

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get Ø >= 0.

A. In Tableau^0:

1. Select a **target column, tcol,** with $Ø_{tcol} < 0$: $Ø^0_2 = -2$, tcol = 2.

2. Select any row, r, with a positive entry in tcol = 2 as the **pivot row: No positive entry exists in the target column = 2, so no pivot exists.**

No feasible pivot in Phase I - Dual problem is unfeasible!

End of the Linear Programming Dual Simplex Method

6.27

6.27 Solve the following problem by the dual simplex method.

$$\begin{array}{ll} \text{Maximize} & -4x_1 - 6x_2 - 18x_3 \\ \text{Subject to} & x_1 \quad\quad + 3x_3 \geqslant 3 \\ & \quad\quad x_2 + 2x_3 \geqslant 5 \\ & x_1, \quad x_2, \quad x_3 \geqslant 0 \end{array}$$

Give the optimal values of all the primal and dual variables. Demonstrate that complementary slackness holds.

Phase II: Goal: get ß >= 0.

Tableau^0

	b^0	x^0_1	x^0_2	x^0_3	s^0_1	s^0_2	row sum
L^0_1	-3	-1	0	-3	1	0	-6
L^0_2	-5	0	-1	-2	0	1	-7
P^0	0	4	6	0	0	0	10
$-P^0 / L^0_2$	0	0	6	0	0	0	0

Basis for Tableau^0: [s_1, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Complete.

Phase II: Goal: get ß >= 0.

A. In Tableau^0:

1. Select a **pivot row, row,** with $b^0_{row} < 0$: row = 2 associated with $b^0_2 = -5$.

2. Compute the ratios -Ø / L_2 as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 3 associated with 0. Thus **$Û_{2,3}$ = -2 is the pivot**; variable s_2 will leave the basis; variable x_3 will enter the basis.

B. To create Tableau^1:

3. Compute row $L^1_2 = L^0_2 / (-2)$.

4. Subtract multiples of row L^1_2 from all other rows of Tableau^0 so that $x_3 = e_2$ in Tableau^1.

Tableau^1							
	b^1	x^1_1	x^1_2	x^1_3	s^1_1	s^1_2	row sum
$L^1_1 = L^0_1 - (-3) * L^1_2$	4.5	-1	1.5	0	1	-1.5	4.5
$L^1_2 = L^0_2 / (-2)$	2.5	0	0.5	1	0	-0.5	3.5
$P^1 = P^0 - (0) * L^1_2$	0	4	6	0	0	0	10
$-P^1 / L^1_{-1}$	0	0	0	0	0	0	0

Basis for Tableau^1: [s_1, x_3,]. Value of Objective Function = 0.

Phase 0: Complete.

Phase I: Complete.

Phase II: Complete.

Primal Solution: [s_1, x_3,] = [4.5, 2.5,]; P = 0.

(Primal x variables not in the basis have a value of 0).

Dual Solution: [y_1, y_2,] = [0, 0,]; D = 0.

End of the Linear Programming Dual Simplex Method

6.29

6.29 Solve the following linear program by the dual simplex method.

$$\begin{aligned}
\text{Minimize} \quad & 2x_1 + 3x_2 + 5x_3 + 6x_4 \\
\text{Subject to} \quad & x_1 + 2x_2 + 3x_3 + x_4 \geqslant 2 \\
& -2x_1 + x_2 - x_3 + 3x_4 \leqslant -3 \\
& x_1, \quad x_2, \quad x_3, \quad x_4 \geqslant 0
\end{aligned}$$

Phase II: Goal: get ß >= 0.

Tableau^0								
	b^0	x^0_1	x^0_2	x^0_3	x^0_4	s^0_1	s^0_2	row sum
L^0_1	-2	-1	-2	-3	-1	1	0	-8
L^0_2	-3	-2	1	-1	3	0	1	-1
P^0	0	2	3	0	0	0	0	5
$-P^0 / L^0_2$	0	1	0	0	0	0	0	0

Basis for Tableau^0: [s_1, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Complete.

Phase II: Goal: get ß >= 0.

A. In Tableau^0:

1. Select a **pivot row, row,** with $b^0_{row} < 0$: row = 2 associated with $b^0_2 = -3$.

2. Compute the ratios $-Ø / L_2$ as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 3 associated with 0. Thus **$Û_{2,3}$ = -1 is the pivot**; variable s_2 will leave the basis; variable x_3 will enter the basis.

B. To create Tableau^1:

3. Compute row $L^1_2 = L^0_2 / (-1)$.

4. Subtract multiples of row L^1_2 from all other rows of Tableau^0 so that $x_3 = e_2$ in Tableau^1.

Tableau^1								
	b^1	x^1_1	x^1_2	x^1_3	x^1_4	s^1_1	s^1_2	row sum
$L^1_1 = L^0_1 - (-3) * L^1_2$	7	5	-5	0	-10	1	-3	-5
$L^1_2 = L^0_2 / (-1)$	3	2	-1	1	-3	0	-1	1
$P^1 = P^0 - (0) * L^1_2$	0	2	3	0	0	0	0	5
$-P^1 / L^1_{-1}$	0	0	0	0	0	0	0	0

Basis for Tableau^1: [s_1, x_3,]. Value of Objective Function = 0.

Phase 0: Complete.

Phase I: Complete.

Phase II: Complete.

Primal Solution: [s_1, x_3,] = [7, 3,]; P = 0.

(Primal x variables not in the basis have a value of 0).

Dual Solution: [y_1, y_2,] = [0, 0,]; D = 0.

End of the Linear Programming Dual Simplex Method

6.30

6.30 Consider the following problem.

$$\text{Minimize} \quad 3x_1+5x_2-x_3+2x_4-4x_5$$

$$\begin{aligned}\text{Subject to} \quad x_1+x_2+x_3+3x_4+x_5 &\leq 6\\ -x_1-x_2+2x_3+x_4-x_5 &\geq 3\\ x_1,\ x_2,\ x_3,\ x_4,\ x_5 &\geq 0\end{aligned}$$

Phase II: Goal: get ß >= 0.

Tableau^0									
	b^0	x^0_1	x^0_2	x^0_3	x^0_4	x^0_5	s^0_1	s^0_2	row sum
L^0_1	-3	1	1	-2	-1	1	1	0	-2
L^0_2	6	1	1	1	3	1	0	1	14
P^0	0	3	5	0	0	0	0	0	8
$-P^0 / L^0_1$	0	0	0	0	0	0	0	0	0

Basis for Tableau^0: [s_1, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Complete.

Phase II: Goal: get ß >= 0.

A. In Tableau^0:

1. Select a **pivot row, row,** with $b^0_{row} < 0$: row = 1 associated with $b^0_1 = -3$.

2. Compute the ratios -Ø / L_1 as per the last row. Discard ratios which are not positive and ratios associated with artificial variables. Select the column with the least positive ratio as the **pivot column:** col = 3 associated with 0. Thus **$\hat{U}_{1,3}$ = -2 is the pivot**; variable s_1 will leave the basis; variable x_3 will enter the basis.

B. To create Tableau^1:

3. Compute row $L^1_1 = L^0_1 / (-2)$.

4. Subtract multiples of row L^1_1 from all other rows of Tableau^0 so that $x_3 = e_1$ in Tableau^1.

Tableau^1									
	b^1	x^1_1	x^1_2	x^1_3	x^1_4	x^1_5	s^1_1	s^1_2	row sum
$L^1_1 = L^0_1 / (-2)$	1.5	-0.5	-0.5	1	0.5	-0.5	-0.5	0	1
$L^1_2 = L^0_2 - (1) * L^1_1$	4.5	1.5	1.5	0	2.5	1.5	0.5	1	13
$P^1 = P^0 - (0) * L^1_1$	0	3	5	0	0	0	0	0	8
$-P^1 / L^1_{-1}$	0	0	0	0	0	0	0	0	0

Basis for Tableau^1: [x_3, s_2,]. Value of Objective Function = 0.

Phase 0: Complete.

Phase I: Complete.

Phase II: Complete.

Primal Solution: [x_3, s_2,] = [1.5, 4.5,]; P = 0.

(Primal x variables not in the basis have a value of 0).

Dual Solution: [y_1, y_2,] = [0, 0,]; D = 0.

End of the Linear Programming Dual Simplex Method

6.44

6.44 Solve the following problem by the primal-dual algorithm.

$$
\begin{aligned}
\text{Minimize} \quad & x_1 + 2x_3 - x_4 \\
\text{Subject to} \quad & x_1 + x_2 + x_3 + x_4 \leqslant 6 \\
& 2x_1 - x_2 + 3x_3 - 3x_4 \geqslant 5 \\
& x_1, \ x_2, \ x_3, \ x_4 \geqslant 0
\end{aligned}
$$

Phase I: Goal: get ß >= 0.

Tableau^0									
	b^0	x^0_1	x^0_2	x^0_3	x^0_4	s^0_1	s^0_2	row sum	$b / \hat{U}_k$
L^0_1	6	1	1	1	1	1	0	11	6
L^0_2	-5	-2	1	-3	3	0	1	-5	1.667
P^0	0	-1	0	0	0	0	0	-1	

Basis for Tableau^0: [s_1, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Goal: get ß >= 0.

A. In Tableau^0:

1. Select a **target row, r,** with $b_r < 0$: $b^0_2 = -5$, r = 2.

2. Select any column, col, with a negative entry in row = 2 as the **pivot column:** col = 3 associated with $\hat{U}_{2,3} = -3$ and constraint L_2.

3. Compute the ratios $b_i / \hat{U}_{i,3}$ as per the last column. Select the row with the least positive ratio as the **pivot row:** row = 2 associated with constraint L_2. Thus **$\hat{U}_{2,3}$ = -3 is the pivot**; variable s_2 will leave the basis; variable x^1_3 will enter the basis.

B. To create Tableau^1:

4. Compute row $L^1_2 = L^0_2 / (-3)$.

5. Subtract multiples of row L^1_2 from all other rows of Tableau^0 so that $x^1_3 = e_2$ in Tableau^1.

Phase II: Goal: get Ø >= 0.

Tableau^1									
	b^1	x^1_1	x^1_2	x^1_3	x^1_4	s^1_1	s^1_2	row sum	$b / \hat{U}_k$
$L^1_1 = L^0_1 - (1) * L^1_2$	4.333	0.333	1.333	0	2	1	0.333	9.333	13
$L^1_2 = L^0_2 / (-3)$	1.667	0.667	-0.333	1	-1	0	-0.333	1.667	2.5
$P^1 = P^0 - (0) * L^1_2$	0	-1	0	0	0	0	0	-1	

Basis for Tableau^1: [s_1, x_3,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Complete.

Phase II: Goal: get Ø >= 0.

A. In Tableau^1:

1. Select the **pivot column, col,** with the most negative value in Ø: col = 1, $Ø_1 = -1$: x^2_1 will enter the basis.

2. Compute the ratios $b_i / \hat{U}_{i,1}$ as per the last column. Select the row with the least positive ratio as the **pivot row:** row = 2 associated with constraint L_2. Thus **$\hat{U}_{2,1}$ = 0.667 is the pivot**; variable x_3 will leave the basis; variable x^2_1 will enter the basis.

B. To create Tableau^2:

3. Compute row $L^2_2 = L^1_2 / (0.667)$.

4. Subtract multiples of row L^2_2 from all other rows of Tableau^1 so that $x^2_1 = e_2$ in Tableau^2.

Tableau^2									
	b^2	x^2_1	x^2_2	x^2_3	x^2_4	s^2_1	s^2_2	row sum	$b / Û_k$
$L^2_1 = L^1_1 - (0.333) * L^2_2$	3.5	0	1.5	-0.5	2.5	1	0.5	8.5	1.4
$L^2_2 = L^1_2 / (0.667)$	2.5	1	-0.5	1.5	-1.5	0	-0.5	2.5	0
$P^2 = P^1 - (-1) * L^2_2$	2.5	0	-0.5	1.5	-1.5	0	-0.5	1.5	

Basis for Tableau^2: [s_1, x_1,]. Value of Objective Function = 2.5.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Complete.

Phase II: Goal: get Ø >= 0.

A. In Tableau^2:

1. Select the **pivot column, col,** with the most negative value in Ø: col = 4, $Ø_4$ = -1.5: x^3_4 will enter the basis.

2. Compute the ratios $b_i / Û_{i,4}$ as per the last column. Select the row with the least positive ratio as the **pivot row:** row = 1 associated with constraint L_1. Thus **$Û_{1,4}$ = 2.5 is the pivot**; variable s_1 will leave the basis; variable x^3_4 will enter the basis.

B. To create Tableau^3:

3. Compute row $L^3_1 = L^2_1 / (2.5)$.

4. Subtract multiples of row L^3_1 from all other rows of Tableau^2 so that $x^3_4 = e_1$ in Tableau^3.

Tableau^3									
	b^3	x^3_1	x^3_2	x^3_3	x^3_4	s^3_1	s^3_2	row sum	$b / Û_k$
$L^3_1 = L^2_1 / (2.5)$	1.4	0	0.6	-0.2	1	0.4	0.2	3.4	7
$L^3_2 = L^2_2 - (-1.5) * L^3_1$	4.6	1	0.4	1.2	0	0.6	-0.2	7.6	0
$P^3 = P^2 - (-1.5) * L^3_1$	4.6	0	0.4	1.2	0	0.6	-0.2	6.6	

Basis for Tableau^3: [x_4, x_1,]. Value of Objective Function = 4.6.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Complete.

Phase II: Goal: get Ø >= 0.

A. In Tableau^3:

1. Select the **pivot column, col,** with the most negative value in Ø: col = 6, $Ø_6$ = -0.2: s^4_2 will enter the basis.

2. Compute the ratios $b_i / Û_{i,6}$ as per the last column. Select the row with the least positive ratio as the **pivot row:** row = 1 associated with constraint L_1. Thus **$Û_{1,6}$ = 0.2 is the pivot**; variable x_4 will leave the basis; variable s^4_2 will enter the basis.

B. To create Tableau^4:

3. Compute row $L^4_1 = L^3_1 / (0.2)$.

4. Subtract multiples of row L^4_1 from all other rows of Tableau^3 so that $s^4_2 = e_1$ in Tableau^4.

Tableau^4										
	b^4	x^4_1	x^4_2	x^4_3	x^4_4	s^4_1	s^4_2	row sum	b / $\hat{U}_k$
$L^4_1 = L^3_1 / (0.2)$	7	0	3	-1	5	2	1	17	0
$L^4_2 = L^3_2 - (-0.2) * L^4_1$	6	1	1	1	1	1	0	11	0
$P^4 = P^3 - (-0.2) * L^4_1$	6	0	1	1	1	1	0	10	

Basis for Tableau^4: [s_2, x_1,]. Value of Objective Function = 6.

Phase 0: Complete.

Phase I: Complete.

Phase II: Complete.

Primal Solution: [s_2, x_1,] = [7, 6,]; P = 6.

(Primal x variables not in the basis have a value of 0).

Dual Solution: [y_1, y_2,] = [1, 0,]; D = 6.

End of the Linear Programming Primal Simplex Method

6.49 Consider the following linear programming problem and its optimal final tableau shown below.

$$\begin{aligned} \text{Maximize} \quad & 2x_1 + x_2 - x_3 \\ \text{Subject to} \quad & x_1 + 2x_2 + x_3 \leqslant 8 \\ & -x_1 + x_2 - 2x_3 \leqslant 4 \\ & x_1, \quad x_2, \quad x_3 \geqslant 0 \end{aligned}$$

Final Tableau

	z	x_1	x_2	x_3	x_4	x_5	RHS
z	1	0	3	3	2	0	16
x_1	0	1	2	1	1	0	8
x_5	0	0	3	-1	1	1	12

Phase II: Goal: get Ø >= 0.

Tableau^0								
	b^0	x^0_1	x^0_2	x^0_3	s^0_1	s^0_2	row sum	$b / \hat{U}_k$
L^0_1	8	1	2	1	1	0	13	8
L^0_2	4	-1	1	-2	0	1	3	0
P^0	0	-2	-1	0	0	0	-3	

Basis for Tableau^0: [s_1, s_2,]. Value of Objective Function = 0.

Proceed to the next tableau as follows:

Phase 0: Complete.

Phase I: Complete.

Phase II: Goal: get Ø >= 0.

A. In Tableau^0:

1. Select the **pivot column, col,** with the most negative value in Ø: col = 1, $Ø_1$ = -2: x^1_1 will enter the basis.

2. Compute the ratios $b_i / \hat{U}_{i,1}$ as per the last column. Select the row with the least positive ratio as the **pivot row:** row = 1 associated with constraint L_1. Thus **$\hat{U}_{1,1}$ = 1 is the pivot**; variable s_1 will leave the basis; variable x^1_1 will enter the basis.

B. To create Tableau^1:

3. Compute row $L^1_1 = L^0_1 / (1)$.

4. Subtract multiples of row L^1_1 from all other rows of Tableau^0 so that $x^1_1 = e_1$ in Tableau^1.

Tableau^1								
	b^1	x^1_1	x^1_2	x^1_3	s^1_1	s^1_2	row sum	$b / \hat{U}_k$
$L^1_1 = L^0_1 / (1)$	8	1	2	1	1	0	13	0
$L^1_2 = L^0_2 - (-1) * L^1_1$	12	0	3	-1	1	1	16	0
$P^1 = P^0 - (-2) * L^1_1$	16	0	3	2	2	0	23	

Basis for Tableau^1: [x_1, s_2,]. Value of Objective Function = 16.

Phase 0: Complete.

Phase I: Complete.

Phase II: Complete.

Primal Solution: [x_1, s_2,] = [8, 12,]; P = 16.

(Primal x variables not in the basis have a value of 0).

Dual Solution: [y_1, y_2,] = [2, 0,]; D = 16.

End of the Linear Programming Primal Simplex Method