Apuntes Procesos

Yanina De Luna Ocampo, Kevin Ramírez Méndez

October 2022

1 Introduction

1.1 Example 1.8

According to the Howard Hughes Medical Institute, about 7% of men and 0.4% of women are color blind-either cannot distinguish red from green or see red and green differently from the most people. In the United States, about 49% of the population is male and 51% female. A person is selected at random. What is the probability they are colorblind.

Gender	ColorBlind	Not ColorBlind
Male	7%	93%
Female	.4%	99.6%

Thus the probability of being colorblind in the United States is given by the probability of being colorblind given is Male multiplied the probability of being male, plus the probability of being colorblind given is female multiplied by the probability of being female.

$P(A) = P(C|M)P(M) + P(C|F)P(F)$
$= (0.07)(0.49) + (0.004)(0.51) = 0.03634$

1.2 Example 1.9

In a standard deck of card, the probability that the suit of a random card is hearts is $13/52 = 1/4$. Assume that a standard deck has one card missing. A card is picked from the deck. Find the probability that it is a Heart.

Solution Assume that the missing card can be any of the 52 cards picked uniformly at random. Let M denote the event that the missing card is a heart, with the complement M^C the event that the missing card is not a heart. Let H denote the event that the card that is picked from the deck is a heart. By the law of total probability,

$$P(H) = P(H|M)P(M) + P(H|M^C)P(M^C)$$
$$= (\tfrac{12}{51})\tfrac{1}{4} + (\tfrac{13}{51})\tfrac{3}{4} = \tfrac{1}{4}$$

1.3 Bayes' Rule

For events A and B

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|B^C)P(B^C)}$$

For any event B and B^C partition the sample space. Given a countable sequence of events $B_1, B_1, ...,$ which partition the sample space, a more general form of Bayes´rule is

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_j P(A|B_j)P(B_j)}$$

1.4 Example 1.11

The use of polygraphs (lie detectors) is controversial, and many scientists feel that they should be banned. On the contrary, some polygraph advocates claim that they are mostly accurate. In 1998, the Supreme Court supported the right of state and federal governments to bar polygraph evidence in court.

Assume that one person in a company of 100 employees is a thief. To find the thief the company will administer a polygraph test to all its employees. The lie detector has the property that if a subject is a liar, there is a 95% probability that the polygraph will detect that they are lying. However, if the subject is telling the truth, there is a 10% chance the polygraph will report a false positive and assert that the subject is lying.

Assume that a random employee is given the polygraph test and asked whether they are the thief. The employee says "no," and the lie detector reports that they are lying. Find the probability that the employee is in fact lying.

Solution Let L denote the event that the employee is a liar. Let D denote the event that the lie detector reports that the employee is a liar, The desired probability is $P(L|D)$.

$$P(L|D) = \frac{P(D|L)P(L)}{P(D|L)P(L)+P(D|L^C)P(L^C)}$$
$$= \frac{(0.95)(0.01)}{(0.95)(0.01)+(0.10)(0.99))} = 0.088$$

1.5 Example 1.12

Max chooses an integer X uniformly at random between 1 and 100. If $X = x$, Mary then chooses an integer Y uniformly at random between 1 and x. Find the conditional pmf of Y given $X = x$.

Solution By the structure of this two-stage random experiment, the conditional distribucion of Y given $X = x$ is uniform on $1, ..., x$. Thus, the conditional pmf is

$$P(Y = y|X = x) = \frac{1}{x}, for y = 1, ..., x$$

1.6 Example 1.13

The joint pmf of X and Y is

$$P(X = x, Y = y) = \frac{x+y}{18}$$
$$\text{for } x, y = 0, 1, 2$$

Find the conditional pmf of Y given $X = x$

Solution The marginal distribution of X is

$$P(X = x) = \sum_{y=0}^{2} P(X = x, Y = y) = \frac{x}{18} + \frac{x+1}{18} + \frac{x+2}{18} = \frac{x+1}{6}$$
$$\text{for } x = 0, 1, 2. \text{ The conditional pmf is}$$

$$P(Y = y|X = x) = \frac{P(X=x,Y=y)}{P(X=x)} = \frac{\frac{x+y}{18}}{\frac{x+1}{6}} = \frac{x+y}{3(x+1)}$$

1.7 Example 1.14

A bag contains 2 red, 3 blue, and 4 white balls. Three balls are picked from the bag (sampling without replacement). Let B be the number of blue balls picked. Let R be the number of red balls picked. Find the conditional pmf of B given $R = 1$.

Solution We have

$$P(B = b|R = 1) = \frac{P(B=b,R=1)}{P(R=1)}$$

$$= \frac{\frac{\binom{3}{b}\binom{2}{1}\binom{4}{3-b-1}}{\binom{9}{3}}}{\frac{\binom{2}{1}\binom{7}{2}}{93}} = \frac{2\binom{3}{b}\binom{4}{2-b}}{42} = \frac{\binom{3}{b}\binom{4}{2-b}}{21}$$
$$\text{if } b = 0, 2/7$$
$$\text{if } b = 1, 4/7$$
$$\text{if } b = 2, 1/7$$