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Procesos Estocásticos

(1)

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix} ; \quad \pi = (\pi_1, \pi_2, \pi_3, \pi_4)P$$

$$\begin{aligned} \left(\tfrac{1}{2}\right)\pi_1 + (0)\pi_2 + \left(\tfrac{1}{4}\right)\pi_3 + (0)\pi_4 &= \pi_1 \\ \left(\tfrac{1}{4}\right)\pi_1 + \left(\tfrac{1}{2}\right)\pi_2 + \left(\tfrac{1}{4}\right)\pi_3 + \left(\tfrac{1}{4}\right)\pi_4 &= \pi_2 \\ (0)\pi_1 + \left(\tfrac{1}{2}\right)\pi_2 + \left(\tfrac{1}{2}\right)\pi_3 + \left(\tfrac{1}{2}\right)\pi_4 &= \pi_3 \\ \left(\tfrac{1}{4}\right)\pi_1 + (0)\pi_2 + (0)\pi_3 + \left(\tfrac{1}{4}\right)\pi_4 &= \pi_4 \\ \pi_1 + \pi_2 + \pi_3 + T_4 &= 1 \end{aligned}$$

$\Rightarrow$

$$\frac{\pi_3}{4} - \frac{\pi_1}{2} = 0 \quad \text{I}$$

$$\frac{\pi_4}{4} - \frac{\pi_1}{4} - \frac{\pi_2}{2} + \frac{\pi_3}{4} = 0 \quad \text{II}$$

$$\frac{\pi_4}{2} + \frac{\pi_2}{2} - \frac{\pi_3}{2} = 0 \quad \text{III}$$

$$\frac{\pi_1}{4} - \frac{3\pi_4}{4} = 0 \quad \text{IV}$$

de I :

$$\frac{\pi_1}{2} = \frac{\pi_3}{4} \Rightarrow \pi_1 = \frac{\pi_3}{2} \quad ; \quad \pi_3 = 2\pi_1$$

de III :

$$\frac{\pi_4}{2} + \frac{\pi_2}{2} = \frac{\pi_3}{2}$$

$$\frac{\pi_4}{2} + \frac{\pi_2}{2} = \pi_1$$

$$\left(\frac{\pi_1}{6}\right) + \frac{\pi_2}{2} = \pi_1$$

$$\frac{\pi_2}{2} = \frac{5\pi_1}{6}$$

$$\pi_2 = \frac{5\pi_1}{3}$$

de IV:

$$\frac{3\pi_4}{4} = \frac{\pi_1}{4} \Rightarrow \pi_4 = \frac{\pi_1}{3}$$

Luego :

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

$$\pi_1 + \left(\frac{5}{3}\right)\pi_1 + (2)\pi_1 + \left(\frac{1}{3}\right)\pi_1 = 1$$

$$5\pi_1 = 1 \Rightarrow \underline{\pi_1 = \frac{1}{5}}$$

$$\pi_2 = \left(\frac{5}{3}\right)\pi_1 = \left(\frac{5}{3}\right)\left(\frac{1}{5}\right) \Rightarrow \underline{\pi_2 = \frac{1}{3}}$$

$$\pi_3 = 2\pi_1 = 2\left(\frac{1}{5}\right) \Rightarrow \underline{\pi_3 = \frac{2}{5}}$$

$$\pi_4 = \left(\frac{1}{3}\right)\pi_1 = \left(\frac{1}{3}\right)\left(\frac{1}{5}\right) \Rightarrow \underline{\pi_4 = \frac{1}{15}}$$

$\therefore$ La distribución estacionaria está dada por

$$\underline{\pi = \left(\frac{1}{5}, \frac{1}{3}, \frac{2}{5}, \frac{1}{15}\right)}$$

2 Recordamos propiedades de Markov

$$\sum_i \pi_i p_{ij} = \pi_j \qquad y \qquad \pi P = \pi$$

Una matriz doblemente estocástica quiere decir que:

$$\sum_{i=1}^{k} p_{ij} = 1 \quad \text{(A)}$$

Ahora, una matriz estacionaria: $\pi = (\pi_1, \pi_2, \ldots \pi_n)$

Ahora, estacionaria uniforme: $\pi_i = \frac{1}{k} \Rightarrow \pi = \left(\frac{1}{k}, \ldots, \frac{1}{k}\right)$

Entonces:
$$\pi_i P = \sum_{i=1}^{k} \pi_i p_{ij}$$
$$= \sum_{i=1}^{k} \left(\frac{1}{k}\right) p_{ij}$$
$$= \left(\frac{1}{k}\right)\sum_{i=1}^{k} p_{ij} \rightarrow \text{(A)}$$
$$= \left(\frac{1}{k}\right)(1) = \frac{1}{k}$$

$$\underline{\pi_i P = \pi_i} \quad \forall i \Rightarrow \underline{\pi P = \pi}$$

$\therefore$ La cadena de Markov con matriz doblemente estacionaria posee una distribución estacionaria uniforme