Asynchronous Distributed Power and Rate Control in Ad Hoc Networks: A Game-Theoretic Approach

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***Abstract*—This paper analyzes distributed asynchronous power and rate control for wireless ad hoc networks. Importantly, all network transmitters are considered to be independent of any management infrastructure and to have the freedom to choose their *own* arbitrary control rules, using as input only information on local interference and achieved carrier signal-to-interference ratio (CIR). Such an approach respects diverse user preferences of on quality of service (QoS) and allows them to adapt to local network conditions in contrast with conventional cellular systems, whose users must follow centralized control commands from serving base stations. For this purpose, we develop a general non-cooperative game-theoretic framework and characterize the resulting power and rate allocation dynamics in terms of its convergence to network-wide acceptable equilibrium states under stochastic communication channels. Chief among the attractive features of our proposed framework is the fact that it is developed in an entirely abstract way without any particular technological or architectural assumptions, which are typically made in related works. Numerical simulations prove the potential of our approach to provide for fair, robust and comparably better CIR allocation in ad hoc networks with varying topology and user density.**

***Index Terms*—Distributed, asynchronous, power and rate control, best-response, game theory, stochastic, convergence.**

I. INTRODUCTION

THIS paper proposes a new approach to resources management in wireless networks, whereby innovative asynchronous and distributive solutions for the transmit power and rate control problem are introduced in the challenging context of ad hoc networks with stochastic channels.

Ad hoc networks are characterized by the absence of any preestablished (cellular) infrastructure, which predetermines their usage for e.g. disaster rescue or military applications [1]. However, the choice for independence from slowly deployable, vulnerable and costly base stations requires their substitution by new distributed control algorithms carried out by network users themselves, whereby asynchronous solutions to the collective decision-making are naturally preferred.

Besides the need for distributivity and asynchrony, what makes the design of such control algorithms a difficult task is also the fact that ad hoc network users are generally non-cooperative and selfish. This holds especially in terms of power/rate control - users tend to use higher power outputs to overcome experienced inband interference from others and maximize their immediate quality of service (QoS) disregarding the mutually caused interference.

One approach to cutting this vicious circle in resource management, which ultimately degrades the performance of the entire ad hoc network, consists in solving general problems of maximizing a minimum signal-to-interference and noise ratio ($SINR$) [2] or satisfying target levels of such ratios with minimum total transmit powers [3].

Inspired by works of Goodman et al., recent studies such as [4]–[11] have tried to apply game theory to directly reflect the selfishness of network users and also to solve the problem of early algorithms' possible divergence. Yet the primary orientation of these works towards distributed maximization (minimization) of QoS-based metrics leads to a strong application dependence and, thus, to low practical universality and/or mathematical generality of derived solutions.

In this paper, we use a *best-response* approach to distributed power and rate control for multihop and multiband ad hoc communications. Our system model assumes each active link (concurrent single-hop transmissions) to periodically adjust, in an individual and asynchronous manner, its transmit rate and power based on so-called "rate assignment" and "best-response" functions, using respectively the achieved carrier signal-to-interference ratio (CIR) and interference level experienced at the receiver as their only input. Unconventionally, we leave entirely up to links *themselves* how they concretely define the two aforementioned update functions. This freedom to flexibly express their time-varying QoS preferences or needs then allows for dynamic adaptation of the resources to diverse service requirements and unfavorable network conditions.

For this purpose, we develop a game-theoretic framework, which (i) defines *existence conditions* for network-wide acceptable outcomes of the power/rate allocation, defined distributively by links throughout their private update functions

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and (ii) proposes update algorithms for *asynchronous* and *distributive* search of such optimum solutions (iii) with robust performance under *stochastic* channels. In contrast to previous works, our analysis is (iv) carried out in a purely *theoretical* way and (v) focusing on analytical clarity and simplicity without limiting itself by unnecessary assumptions.

The paper is organized as follows. Section II defines the system model. Section III then characterizes Nash equilibria within the system, whereas Section IV describes its power/rate allocation dynamics. Algorithms for finding said equilibrium states under stochastic channels assuming nonlinear or linear/linearized best-response functions are specified subsequently in Sections V and VI. The latter one moreover defines a matched-up admission control for the special case of linear/linearized best-response functions. Section VII presents numerical results and comparisons upholding our theoretical analysis, followed by conclusions drawn in Section VIII.

II. System Model For Ad Hoc Networks

A wireless ad hoc network using data relaying over multihop connections on multiple frequency bands can be abstracted into a set of mutually orthogonal (non-interacting) CDMA or TDMA networks [12]. Our system model thus considers, with no loss of generality, the reduced problem of a network with N simultaneously active one-hop links sharing a single band. An active link is identified by a transmitter (TX) and a receiver (RX), the latter experiencing interference from concurrent transmissions pertaining to surrounding links in the network.

The time-varying channel gain between the TX of link j and the RX of link i is denoted by $\tilde{h}_{ij}$ and models all radio wave propagation phenomena such as e.g. path loss, shadowing or multipath fading.[1] We assume that the values of $\tilde{h}_{ij}$ are given by some *stochastic* function $\tilde{h}_{ij}\left(h_{ij},t\right)$, whose arguments are the *deterministic* mean channel gain h_{ij} and time t and which is defined such that $\mathbb{E}\left[\tilde{h}_{ij}\right]=h_{ij}$.

The mean channel gain h_{ij} is proportional to path loss due to physical energy dissipation of the transmitted signal around the transmitter antenna, large-scale shadowing and for simplicity also the spreading/processing gain of CDMA transmissions. Assume therefore that the value of h_{ij} varies only according to the *geometrical distance* between TX_j and RX_i [13]. Then the variability rate of h_{ij} in time depends exclusively on the speed, at which links change their locations within the network, i.e., on network mobility. So if links i and j are immobile, h_{ij} and h_{ji} become time-invariable constants.

In contrary, the random changes of the overall stochastic channel gain $\tilde{h}_{ij}$ around the mean channel gain h_{ij} are possibly much faster than network mobility. In the case of Rayleigh fading channels, $\tilde{h}_{ij}$ is exponentially distributed.

As for the power and rate control in the network, the TX of every link $i\in\mathscr{I}$ from the set $\mathscr{I}=\{1,\ldots,N\}$ of all network links updates its transmit power σ_i and rate ϱ_i periodically with period T in time instances t_i^k for integer $k\in\mathbb{N}$ using update data calculated and fed back by the corresponding RX. Time instances t_i^k are considered to be asynchronous with the timing of other links j, i.e., $t_i^k\neq t_j^k$ for a given k and any $i,j\in\mathscr{I}$ and $j\neq i$. The update period T is assumed to be set small enough to assure the mean channel gain h_{ij} to remain quasi-constant within T, but network transmitters have no knowledge on the instantaneous values of randomizing $\tilde{h}_{ij}$.

We think of the power/rate control process as of individual decision-making by selfish links in a non-cooperative environment. Updates of transmit power $\sigma_i\left(t_i^k\right)$ are calculated at the RX of link i based on the inband interference $\sum_{j\neq i}h_{ij}\left(t_i^k\right)\sigma_j\left(t_i^k\right)$ from other links $j\neq i$, experienced at the RX in time t_i^k, using so-called best-response (BR) function $\beta_i^{\mathrm{RX}}\left(\sum_{j\neq i}h_{ij}\sigma_j,t\right)\geqslant 0$ [14], [15], which formally describes the most *desirable* value of received power $h_{ii}\sigma_i$ at the RX, necessary to overcome a given interference $\sum_{j\neq i}h_{ij}\sigma_j$ in order to achieve an acceptable connection (e.g. a target CIR_i). A predefined initial transmit power $\sigma_i\left(t_i^0\right)$ is used in time t_i^0 for starting the data transmission.[2] Transmit data rate $\varrho_i\left(t_i^k\right)$ is updated similarly based on so-called rate assignment function $R_i^{\mathrm{TX}}\left(CIR_i,t\right)\geqslant 0$, which assigns preferred data rates to $CIR_i=h_{ii}\sigma_i/\sum_{j\neq i}h_{ij}\sigma_j$, achieved at RX_i.

Such system model requires each RX_i to send two overhead update data to TX_i (e.g., together with data acknowledgements) in order to let the TX_i update σ_i and ϱ_i to values calculated at the RX_i. Although such overhead is undesirable, note that it represents the minimum possible price for gaining adaptive and distributed power/rate control in ad hoc networks.

To individualize among links, we allow each link to define its *own* functions β_i^{RX} and R_i^{TX} independently from others and based solely on its own private CIR_i preferences or QoS needs. Said functions are therefore discussed in a general form and moreover assumed to be time-variant as defined hereafter.

To simplify the notation, we denote as $-i$ the set $\mathscr{I}\setminus\{i\}$ of links j other than link i, and introduce a vector/matrix based notation with boldface symbols by defining $\boldsymbol{\sigma}(t)$ to be a columnwise-oriented vector composed of power values $\sigma_i(t)$ at time t of all N network links $i\in\mathscr{I}$. Symbols $\boldsymbol{\sigma}_{-i}(t)$, $\boldsymbol{h}_{-i}(t)$ and $\tilde{\boldsymbol{h}}_{-i}(t)$ represent analogically vectors composed of $N-1$ elements $\sigma_j(t)$, $h_{ij}(t)$ and $\tilde{h}_{ij}\left(h_{ij},t\right)$ for all $j\in -i$, respectively.

We further define two *transmitter*-oriented BR functions - β_i^{TX} and $\tilde{\beta}_i^{\mathrm{TX}}$. The definition of β_i^{TX} by

$$\beta_i^{\mathrm{TX}}\left(\boldsymbol{\sigma}_{-i},t\right)\overset{\mathrm{def}}{=}\frac{1}{h_{ii}}\beta_i^{\mathrm{RX}}\left(\sum_{j\neq i}h_{ij}\sigma_j,t\right)=\frac{1}{h_{ii}}\beta_i^{\mathrm{RX}}\left(\boldsymbol{h}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i},t\right). \tag{1}$$

is based on the notion of the *mean* channel gain h_{ij}, whose variations can links track down by a proper choice of T and ignores the stochastic components in the overall channel gain $\tilde{h}_{ij}$, "superposed" over h_{ij}.

The notion of β_i^{TX} is sufficient for system model formulation and a general description of the power control dynamics in the network. However, when discussing the effects of fading $\tilde{h}_{ij}$ on power allocation later on, a more realistic TX-oriented BR function $\tilde{\beta}_i^{\mathrm{TX}}$ will be needed, whose definition by

$$\tilde{\beta}_i^{\mathrm{TX}}\left(\boldsymbol{\sigma}_{-i},t\right)\overset{\mathrm{def}}{=}\frac{1}{\tilde{h}_{ii}}\beta_i^{\mathrm{RX}}\left(\tilde{\boldsymbol{h}}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i},t\right) \tag{2}$$

[1] Additive noise is ignored as simultaneous transmissions in a shared band are clearly interference-limited.

[2] Conditions for global convergence to an equilibrium state are discussed later, whereby in such a case $\sigma_i\left(t_i^0\right)$ can be chosen arbitrarily.

substitutes the deterministic mean channel gains h_{ij} in β_i^{TX} by the stochastic ones $\tilde{h}_{ij}$.

Analogically to the relation of $\tilde{h}_{ij}$ and h_{ij}, the deterministic BR function β_i^{TX} represents the "mean" of the stochastic BR function $\tilde{\beta}_i^{\mathrm{TX}}$, i.e., β_i^{TX} corresponds to $\mathbb{E}\left[\tilde{\beta}_i^{\mathrm{TX}}\right]$. The symbol $\tilde{}$ in β_i^{TX} emphasizes the stochastic nature of $\tilde{\beta}_i^{\mathrm{TX}}$.

Having established β_i^{TX} and R_i^{TX}, we can from now on work only with TX-oriented system functions, and omit for clarity indexes TX (similarly, the dependency of $\sigma_i(t)$, $h_{ij}(t)$, $\tilde{h}_{ij}(h_{ij}, t)$, $\beta_i(\boldsymbol{\sigma}_{-i}, t)$ on their arguments will be emphasized only if necessary, otherwise purely symbolic notation σ_i, h_{ij}, $\tilde{h}_{ij}$, β_i will be preferred). The system model can then be formalized as a non-cooperative power allocation game [16]:

Definition 1: Denote $\mathscr{G} = \{\mathscr{I}, \sigma_i, \beta_i\}$ a strategic game defined by the following three elements:

1) set $\mathscr{I}$ of N active links i (game players), simultaneously accessing shared frequency band;
2) transmit power $\sigma_i \in \Sigma_i$ of link i (game strategy), where its power range Σ_i is a subset of positive real numbers $\mathbb{R}^{1+}$;
3) BR function β_i of link i (game playing rules), which assigns a power σ_i from the power range Σ_i to each power vector $\boldsymbol{\sigma}_{-i}$ from the power profile $\boldsymbol{\Sigma}_{-i}$, wherein $\boldsymbol{\Sigma}_{-i}$ with elements $\boldsymbol{\sigma}_{-i}$ represents the power profile of opponent links $-i$, given by the Cartesian product of Σ_j for all $j \in -i$, and $\boldsymbol{\Sigma}$ with elements $\boldsymbol{\sigma}$ denotes the entire power profile of the power control game $\mathscr{G}$, given by the Cartesian product of Σ_i for all $i \in \mathscr{I}$. $\square$

Such a system model definition reflects the fact that all network links take independently their best available actions defined by β_i and R_i ("best responses") in order to pursue their own individual objectives as expressed by the particular choice of β_i and R_i. What makes this a strategic game is that what is best for one link depends in general upon actions of other links, because links act based on the evaluation of *mutual* interference and related CIR.

Both transmit rate ϱ_i and rate assignment function R_i do not constitute a part of the game-theoretic system model formulation, because transmit rates ϱ_i can be uniquely determined herein as $\varrho_i = R_i(CIR_i) = R_i\left(h_{ii}\beta_i(\boldsymbol{\sigma}_{-i}) / \boldsymbol{h}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}\right)$. Thus, reaching a network-wide equilibrium in the β_i-driven power control game also implies stabilization of the rate adjustment process, irrespectively of the fact whether link i subordinates its choice of β_i to its primary demands on R_i (e.g., a car user with good energetic supplies) or if in contrary R_i is derived based on transmit power restrictions of link i (e.g., a power concerned user).

III. Nash Equilibrium As Power Control Game Outcome

Having formalized the system model as a dynamic power control game $\mathscr{G}$, we proceed with defining desirable game outcomes and analyzing conditions for their existence in order to examine the characteristics of β_i that would allow links to reach some mutually acceptable solution to their most likely conflicting resources allocation interests.

The outcome of game $\mathscr{G}$ is preferably a *network-wide* acceptable state, characterized by such an allocation of transmit powers from which none of the concurrently transmitting links has an incentive to unilaterally deviate, i.e., change its transmit power, while other active links keep their powers unchanged.

Such an optimum outcome is formally a set of allocated powers $\hat{\boldsymbol{\sigma}}$, characterized by an ideal interference adaptation of all links such that $\hat{\sigma}_i = \beta_i(\hat{\boldsymbol{\sigma}}_{-i}, t)\ \forall i \in \mathscr{I}$, which advantageously corresponds to the notion of Nash equilibrium [16]. Evidently, if a new link becomes active or conversely terminates its transmission, a new equilibrium state has to be found, to dynamically reflect the changed network configuration and resulting variations in local interference.

In order to allow links to dynamically adapt not only to changing interference, but also to newly emerging network conditions and corresponding variations in service preferences, we allow them to vary in time the shape of their BR functions β_i as follows:[3]

Assumption 1: [On Short Term Time-Invariance of β_i] Assume that (i) the period T of power adaptations in the game $\mathscr{G}$ is much smaller that the topological changes in the network. Furthermore, if power updates in the game $\mathscr{G}$ converge for given β_is to Nash equilibrium $\hat{\boldsymbol{\sigma}}$, then (ii) the intersection of time intervals of all links, over which β_is of all links remain invariant, is larger than the time scale for reaching $\hat{\boldsymbol{\sigma}}$. $\square$

The first requirement on power adjustments to be faster than topological changes concerns the variability of $\beta_i(\boldsymbol{h}_{-i}, t)$ due to fluctuations of h_{ij} under mobility. It in fact assures that channel gains h_{ij} remain quasi-constant on the evolution time scale of $\mathscr{G}$ and power adaptations can then easily track down the effects of link mobility.

The second requirement limits the frequency at which links can freely redefine their $\beta_i(\boldsymbol{h}_{-i}, t)$, i.e., limits the dependence of β_i on its second argument - time t. The purpose is to guarantee slow time-variance or temporary invariance of β_i and provide so links with enough time to achieve the Nash equilibrium on an appropriate time scale. Distributively verifiable conditions of power control convergence are discussed hereafter, whereby reaching $\hat{\boldsymbol{\sigma}}$ results in an easily detectable usage of a constant transmit power $\hat{\sigma}_i$, thus the assumption make sense in the light of the upcoming text.

As a whole, the assumption is necessary for characterizing conditions for existence of Nash equilibria in $\mathscr{G}$ and guarantees the predictability of the game outcome. It postulates links to be rational players in the sense that their behavior constitutes rational optimizing behavior and they do not randomize $\mathscr{G}$ e.g. by egoistic alternations of once chosen β_i.

Based on the premise, we can proceed with examining how to assure the *general* existence of Nash equilibrium/equilibria in its power profile $\boldsymbol{\Sigma}$ by making some additional assumptions on β_i. It shows up that for this purpose it is necessary but sufficient to require the continuity of β_i in its argument $\boldsymbol{\sigma}_{-i}$ on a compact and convex power profile $\boldsymbol{\Sigma}_{-i}$:

Theorem 1: Assume a non-cooperative power control game $\mathscr{G} = \{\mathscr{I}, \sigma_i, \beta_i\}$ as defined in *Definition 1*. Then if β_i is a continuous function in a non-empty compact and convex power range Σ_i for all $i \in \mathscr{I}$, the game $\mathscr{G}$ admits at least one Nash equilibrium such that $\hat{\sigma}_i = \beta_i(\hat{\boldsymbol{\sigma}}_{-i}, t)$. $\square$

[3]In contrary to β_i, R_i can vary in time *arbitrarily* as long as its technical compatibility with β_i is ensured, because data rate choice of one link does not directly affect the transmit power decisions of other links in the game $\mathscr{I}$.

Proof: If the power range Σ_i of each link i is a non-empty compact and convex set of $\mathbb{R}^{1+}$, then their Cartesian products $\boldsymbol{\Sigma}$ and $\boldsymbol{\Sigma}_{-i}$ have the same properties.

Define now a joint BR function $\boldsymbol{\beta}$ as a Cartesian product of β_i for all $i \in \mathscr{I}$ and substitute formally its argument $\boldsymbol{\sigma}_{-i} \in \boldsymbol{\Sigma}_{-i}$ by $\boldsymbol{\sigma} \in \boldsymbol{\Sigma}$, i.e., extend the domain set of $\boldsymbol{\beta}$ from $\boldsymbol{\Sigma}_{-i}$ to $\boldsymbol{\Sigma}$ by formally including also Σ_i, in order to be able to work with $\boldsymbol{\beta}$ in the context of the entire power profile $\boldsymbol{\Sigma}$.

Using Brouwer's fixed point theorem [14] as a special case of Kakutani's theorem [16], one can state that if $\boldsymbol{\beta} : \boldsymbol{\Sigma} \to \boldsymbol{\Sigma}$ is a continuous function from a non-empty, compact and convex set $\boldsymbol{\Sigma} \subset \mathbb{R}^{N+}$ into itself, there exists a fixed point $\hat{\boldsymbol{\sigma}} \in \boldsymbol{\Sigma}$ of $\boldsymbol{\beta}$ such that $\hat{\boldsymbol{\sigma}} = \boldsymbol{\beta}(\hat{\boldsymbol{\sigma}}, t)$, i.e., $\mathscr{G}$ admits Nash equilibrium. ■

Note from the above proof the theoretical necessity of the continuity (*Assumption 3*). If β_i was only piecewise continuous, then there might not be a point such that $\hat{\sigma}_i = \beta_i(\hat{\boldsymbol{\sigma}}_{-i}, t)$, i.e., no intersection of $\beta_i(\boldsymbol{\sigma}_{-i}, t)$ with the line σ_i. This means that is not possible to prove the general existence of Nash equilibria for discontinuous or only piecewise continuous BR functions. Consequently, the next two assumptions are incorporated into the system model to assure existence of at least one equilibrium $\hat{\boldsymbol{\sigma}}$:

Assumption 2: [On Power Ranges Σ_i] Let the power range Σ_i of each link i be a non-empty, compact and convex set of positive real numbers $\mathbb{R}^{1+}$ for all $i \in \mathscr{I}$. □

Assumption 3: [On Continuity of BR Functions β_i] Let β_i be a continuous function in Σ_i for all $i \in \mathscr{I}$. □[4]

Knowing conditions for existence of Nash equilibria in an ad hoc network with distributively defined power/rate control, one may be tempted to proceed with studying the system of discrete time difference equations $\sigma_i\left(t_i^k + T\right) = \beta_i\left(\boldsymbol{\sigma}_{-i}\left(t_i^k\right), t_i^k\right)$ with the perspective of determining conditions, under which the periodical power updates converge to optimum power vector $\hat{\boldsymbol{\sigma}}$.

However, fading effects characteristic for mobile wireless environment can cause significant variations of the channel gains $\tilde{h}_{ij}$ in time and, consequently, high variability of local inband interference $\tilde{\boldsymbol{h}}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}$ within the period T.

Calculating power updates based on β_i with fluctuating interference values as input then in fact corresponds to the usage of stochastic BR function $\tilde{\beta}_i$ as defined in (2) - instead of the originally envisaged deterministic "mean" BR function β_i defined by (1). The occurrence of large fluctuations of σ_i then degrades the performance of the network [17].

In ergodic channels (e.g., fast fading channels), the randomness of channel gains $\tilde{h}_{ij}$, that is to say of interference $\tilde{\boldsymbol{h}}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}$, can be removed by simple averaging of interference measurements collected during each update period T, because $\mathbb{E}\left[\tilde{\boldsymbol{h}}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}\right] = \mathbb{E}\left[\tilde{\boldsymbol{h}}_{-i}^{\mathrm{T}}\right]\boldsymbol{\sigma}_{-i} = \boldsymbol{h}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}$.

This way of estimating the mean interference value $\boldsymbol{h}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}$ is however not possible in non-ergodic channels (e.g., slow fading channels), whose short time scale average of $\tilde{h}_{ij}$ is not equal to the overall average h_{ij}. Thus, a more advanced practical implementation of link transmit power updates is needed for the non-ergodic case.

[4]As for practical discrete systems, our results for continuous BR functions are still applicable with a near optimal performance by operating on discrete levels, which are the nearest to optimal continuous solutions.

IV. Best-Response Power Control Dynamics

Observe from the proof of *Theorem 1* that the search for Nash equilibrium $\hat{\boldsymbol{\sigma}} = \boldsymbol{\beta}(\hat{\boldsymbol{\sigma}}, t)$ is equivalent to finding a zero root of an auxiliary function $\boldsymbol{f}(\boldsymbol{\sigma}, t) = \boldsymbol{\beta}(\boldsymbol{\sigma}, t) - \boldsymbol{\sigma}$ with $\boldsymbol{\beta}$ given by the Cartesian product of β_i on $\boldsymbol{\Sigma}$. Knowing that the usage of $\boldsymbol{\beta}$ under stochastic channel gains $\tilde{h}_{ij}$ corresponds to the usage of $\tilde{\boldsymbol{\beta}}$, we can model power/rate control in fading channels as a zero root search problem for a "noisy" function $\tilde{\boldsymbol{f}} = \tilde{\boldsymbol{\beta}} - \boldsymbol{\sigma}$ with $\tilde{\boldsymbol{\beta}}$ composed of components as defined in (2).

To distributively implement such stochastic search, one can employ an iterative algorithm of SA class. These algorithms are known to be the most suitable for this kind of task [18], [19]. Their general form has been proposed in [20] and importantly allows every link i to distributively calculate its updates of σ_i based on

$$\sigma_i(t+T) = \sigma_i(t) + a_i^k \tilde{f}_i(\boldsymbol{\sigma}_{-i}(t), t), \tag{3}$$

having as input only consecutive observations of the interference $\tilde{\boldsymbol{h}}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}$ as required by the system model, whereby $a_i^k > 0$ is the algorithmic step size and $\tilde{f}_i(\sigma_{-i}, t) = \tilde{\beta}_i(\sigma_{-i}, t) - \sigma_i$ with noisy $\tilde{\beta}_i$ defined in (2) denotes the i-th component of $\tilde{\boldsymbol{f}}$. Yet the algorithm undesirably requires all power updates to be performed *simultaneously*, i.e., a network-wide synchronization, whereby it is typically assumed that $a_i^k = a_j^k$ for any $i, j \in \mathscr{I}$.

We propose more elaborate algorithms for power updates at *asynchronous* time instants t_i^k in Sections V and VI. To derive their convergence conditions, we first need to characterize the driving force of SA class algorithms, that is to say of the evolutionary dynamics of $\boldsymbol{\sigma}$, during the search for the Nash equilibrium $\hat{\boldsymbol{\sigma}}$ of the game $\mathscr{G}$ based on (3) with given BR functions. It shows up that the dynamics is, for large k, directly related by the Arzelà-Ascoli theorem [19] to the asymptotical properties of a standard law of motion [15], governed by N coupled first-order scalar differential equations:

$$\begin{aligned} \frac{d\boldsymbol{\sigma}(t)}{dt} &= \boldsymbol{\beta}(\boldsymbol{\sigma}(t), t) \\ \boldsymbol{\sigma}\left(t^0\right) &= \boldsymbol{\sigma}_0. \end{aligned} \tag{4}$$

The continuous model (4) is advantageously a deterministic one thanks to the inherent property of SA algorithms to average out channel fluctuations. Moreover, its parameters such as importantly the channel gains h_{ij} are constant as a consequence of *Assumption 1* and the assumption of link mobility to be much larger than that of the power/rate adaptation.

It is clear that in order to be able to predict the future states $\boldsymbol{\sigma}$ of the algorithm (3) (and thus its convergence) for some initial $\boldsymbol{\sigma}_0$, there must exist a unique solution to (4), i.e., a continuous function $\boldsymbol{\sigma} : \left[t^0, t^1\right] \to \mathbb{R}^{N+}$ such that $\frac{d\boldsymbol{\sigma}(t)}{dt}$ is defined and $\frac{d\boldsymbol{\sigma}(t)}{dt} = \boldsymbol{\beta}(\boldsymbol{\sigma}(t), t)$ for all $t \in \left[t^0, t^1\right]$.

The continuity on β_i (*Assumption 3*) must be therefore tightened by requiring them to also have limited first derivative in $\boldsymbol{\sigma}$. Then the following theorem holds.

Theorem 2: Assume a short-term variant $\boldsymbol{\beta}(\boldsymbol{\sigma}, t)$ as defined in *Assumption 1* such that it is piecewise continuous in t and satisfies the Lipschitz condition $\|\boldsymbol{\beta}(\boldsymbol{\sigma}, t) - \boldsymbol{\beta}(\boldsymbol{\tau}, t)\| \leqslant L\|\boldsymbol{\sigma} - \boldsymbol{\tau}\|$ for some $L > 0, \forall \boldsymbol{\sigma}, \boldsymbol{\tau} \in \boldsymbol{\Sigma}$ and $\forall t \in \left[t^0, t^1\right]$. Then the power control dynamics $\frac{d\boldsymbol{\sigma}(t)}{dt} = \boldsymbol{\beta}(\boldsymbol{\sigma}, t)$ with $\boldsymbol{\sigma}(t^0) = \boldsymbol{\sigma}^0$ has a unique solution over the time interval $[t^0, t^1]$. □

Proof is available in books on ordinary differential equations.

Importantly, it holds that the unique solution depends continuously on the initial state $\boldsymbol{\sigma}(t^0)$ and parameters of $\boldsymbol{\beta}$ under the above conditions, which gives us the possibility to arbitrarily select $\sigma_i(t_i^0)$ for all links $i \in \mathscr{I}$, and tolerate continuous changes of mean channel gains h_{ij} in $\boldsymbol{\beta}$ as an effect of link mobility.

Having obtained a power control game with analytically predictable dynamics, we proceed with defining convergence of the dynamics (4) to an isolated Nash equilibrium $\hat{\boldsymbol{\sigma}}$ from its neighborhood and consequently analyze the convergence conditions.

As power ranges are subsets of Euclidean space, consider, without loss of generality, a shift of $\hat{\boldsymbol{\sigma}}$ to the origin of $\mathbb{R}^N$ so that $\hat{\boldsymbol{\sigma}} = \mathbf{0}$. Then the notion of dynamics (4) *converging* to an equilibrium point $\hat{\boldsymbol{\sigma}}$ from its neighborhood can be equivalently referred to as the *stability* of $\hat{\boldsymbol{\sigma}}$ and conveniently defined as:

Definition 2: An equilibrium $\hat{\boldsymbol{\sigma}}$ is

- stable, if for each $\xi > 0$ there exists $\delta = \delta(\xi) > 0$ such that $\|\boldsymbol{\sigma}(0)\| < \delta$ implies $\|\boldsymbol{\sigma}(t)\| < \xi$ for all $t \geqslant 0$;
- asymptotically stable, if it is stable and δ can be chosen such that $\|\boldsymbol{\sigma}(0)\| < \delta$ implies $\lim_{t\to\infty} \boldsymbol{\sigma}(t) = \mathbf{0}$;
- unstable, if it is not stable. □

According to this definition, the stability of $\hat{\boldsymbol{\sigma}}$ implies that after some time $\boldsymbol{\sigma}$ remains only in a close neighborhood of $\hat{\boldsymbol{\sigma}}$, called the region of stability whose size is arbitrarily given by some function of ξ. In the case of asymptotical stability, the close neighborhood is shrinking in time and reduces to the equilibrium point itself as time goes to infinity. Analogically, instability of the equilibrium is equivalent to divergence of the power control process.

When finding ways for determining the stability of $\hat{\boldsymbol{\sigma}}$, we avoid the intuitive search for specific analytical solutions of (4) as this approach limits itself by assuming a particular shape of $\boldsymbol{\beta}$, which makes it application-dependent, and rather take advantage of Lyapunov indirect analysis [21]. Its main result gives *general* sufficient (but not necessary) stability conditions:

Theorem 3 (Lyapunov stability): Consider power control dynamics $\frac{d\boldsymbol{\sigma}(t)}{dt} = \boldsymbol{\beta}(\boldsymbol{\sigma})$ with an equilibrium point $\hat{\boldsymbol{\sigma}} = \mathbf{0}$ in the origin. Let $D \subset \mathbb{R}^N$ be a domain containing the equilibrium $\hat{\boldsymbol{\sigma}} = \mathbf{0}$. Then, if there exists a scalar continuously differentiable function $v(\boldsymbol{\sigma}) : D \to \mathbb{R}^1$ such that $v(\mathbf{0}) = 0$ and $v(\boldsymbol{\sigma}) > 0$ in $D\backslash\mathbf{0}$, and satisfying also (i) $\dot{v}(\boldsymbol{\sigma}) \leqslant 0$ in D or (ii) $\dot{v}(\boldsymbol{\sigma}) < 0$ in $D\backslash\mathbf{0}$; then the equilibrium point $\hat{\boldsymbol{\sigma}}$ in the origin is (i) stable or (ii) asymptotically stable, respectively, whereby $\dot{v}$ denotes the derivative of v along the trajectories $\boldsymbol{\beta}$ of (4), i.e., $\dot{v} = \sum_i \frac{\partial v}{\partial \sigma_i}\frac{d\sigma_i}{dt} = \sum_i \frac{\partial v}{\partial \sigma_i}\beta_i = \operatorname{grad} v^{\mathrm{T}}\boldsymbol{\beta}$. □

In other words, the equilibrium is (asymptotically) stable, if there exists a continuously differentiable positive definite function v so that $\dot{v}$ is negative semidefinite (definite). The Barbashin-Krasovskii theorem [21] extends this results by stating *global* asymptotical stability if v is positive definite, $\dot{v}$ is negative definite $\forall\boldsymbol{\sigma} \neq \mathbf{0}$ and $\|\boldsymbol{\sigma}\| \to \infty \Rightarrow v \to \infty$.

Thus, if v approaches infinity when $\|\boldsymbol{\sigma}\| \to \infty$, global convergence to a unique equilibrium follows without assuming further restrictions on $\boldsymbol{\beta}$. This is an important observation as *Theorem 1* does not guarantee the uniqueness of $\hat{\boldsymbol{\sigma}}$, but only its existence. Hence, if the BR power control converges *globally* to some isolated equilibrium, i.e., the domain D is the whole power profile $\boldsymbol{\Sigma}$, then by contradiction there cannot exist another isolated equilibrium. As such, unwanted oscillations between several equilibria can be avoided.

A general method for generating Lyapunov functions for checking the power and rate control stability through *Theorem 3* consists in Schultz-Gibson variable gradient method [22].

V. Asynchronous Convergence Under Stochastic Channels

A. Proposed Algorithm for Solving Power/Rate Control Game

This section introduces an SA algorithm, which is more suitable for solving the power control game $\mathscr{G}$ than the one presented in (3) and builds up on the previously established facts. It implements *asynchronous* iterative search for one of the previously established, but potentially *multiple* Nash equilibria in $\mathscr{G}$ under the influence of stochastic communication channels $\tilde{h}_{ij}$, which moreover allows for link-level transmit power *resets* if the estimate σ_i of the equilibrium component $\hat{\sigma}_i$ departs at some iteration out of the predefined range Σ_i.

Importantly, the requirements for assuring convergence of the proposed algorithm are defined (except of standard noise and stepsize conditions) based solely on the customized definition of link (noisy) BR functions or, more precisely, based on the *existence* of a Lyapunov function for the corresponding dynamics (4) as stated in *Theorem 3*. The redundancy of knowing the *exact* form of the Lyapunov function gives our approach a new potential for deriving *distributed* admission control schemes for assuring the network power and rate control stability with respect to the distributively defined and time-varying BR functions. An exemplary scheme is developed in Section VI for the linear/linearized case.

The proposed algorithm is defined in accordance with the system model by periodical (period T) and asynchronous transmit power updates in times t_i^k, whereby all active links start with some initial transmit power $\sigma_i(t_i^0)$ and use the *fluctuating* interference information $\tilde{\boldsymbol{h}}_{-i}(t_i^k)^{\mathrm{T}}\boldsymbol{\sigma}_{-i}(t_i^k)$ given by the current value of the asynchronously updated transmit powers $\boldsymbol{\sigma}_{-i}(t_i^k)$ to update their powers with respect to their BR functions using the formula

$$\sigma_i(t_i^k + T) = \mathbb{L}\left[\sigma_i(t_i^k) + a_i^k \tilde{f}_i(\boldsymbol{\sigma}_{-i}(t_i^k), t_i^k)\right] + (1 - \mathbb{L})\sigma_i^*, \tag{5}$$

where a_i^k denotes the algorithmic step size of link i and $\tilde{f}_i(\sigma_{-i}, t) = \tilde{\beta}_i(\sigma_{-i}, t) - \sigma_i(t)$ with noisy $\tilde{\beta}_i$ defined in (2) denotes the i-th component of the auxiliary function $\tilde{\boldsymbol{f}}(\boldsymbol{\sigma}, t) = \tilde{\boldsymbol{\beta}}(\boldsymbol{\sigma}, t) - \boldsymbol{\sigma}$ with Nash equilibrium (equilibria) $\hat{\boldsymbol{\sigma}}$ as its zero root(s).

The logical indicator function $\mathbb{L}$ in (5) is equal to 1 if the candidate value $\sigma_i(t_i^k + T) = \sigma_i(t_i^k) + a_i^k \tilde{f}_i(\boldsymbol{\sigma}_{-i}(t_i^k), t_i^k)$ of the estimated equilibrium component $\hat{\sigma}_i$ at time $t_i^k + T$ exits from the predefined transmit power range Σ_i, containing by *Assumption 2, 3* at least one Nash equilibrium, and becomes 0 otherwise. In other words, if $\left(\sigma_i(t_i^k) + a_i^k \tilde{f}_i(\boldsymbol{\sigma}_{-i}(t_i^k))\right) \in \Sigma_i$, the value of $\sigma_i(t_i^k + T)$ is pulled back to some predetermined initial power $\sigma_i^* \in \Sigma_i$, from which the iterative search is restarted. For the moment, we assume that if some link i resets its power to σ_i^* in t_i^k, other links $j \in -i$ join it by switching to σ_j^* in t_j^k too.

B. Convergence Statement

In order to state the convergence conditions of the algorithm (5) in the next theorem, we make the following assumption on the fluctuating nature of $\tilde{\beta}_i$ (respective on $\tilde{f}_i$), using the notion of canonical additive noise ϵ_i defined by $\epsilon_i(t) = \tilde{\beta}_i(\boldsymbol{\sigma}_{-i}, t) - \beta_i(\boldsymbol{\sigma}_{-i}, t)$, i.e., as the difference between the BR function, comprising channel gains $\tilde{\boldsymbol{h}}_{-i}$ with randomizing effects, namely $\tilde{\beta}_i$, and the "mean" β_i, considering only the deterministic propagation model based on mean gains $\boldsymbol{h}_{-i}$.

Assumption 4: Assume for any $i \in \mathscr{I}$, any convergent subsequence $\{\sigma_i(t_i^n)\}$ of $\{\sigma_i(t_i^k)\}$ and any $\theta_k \in [0, \theta]$ that

$$\limsup_{\theta \to 0} \limsup_{k \to \infty} \frac{1}{\theta} \left| \sum_{s=n}^{y(n,\theta_k) \wedge r(i, g_i^n + 1)} a_i^s \epsilon_i(t_i^s) \right| = 0, \quad (6)$$

where "$\wedge$" denotes the minimum between $y(n, \theta_k) = \inf\{l \geqslant n, \sum_{s=n}^{l} a_i^s > \theta_k\}$ and $r(i, l) = \inf\{k > 0, g_i^k = l\}$, whereby g_i^k is defined by $g_i^{k+1} = g_i^k + (1 - \mathbb{L})$ with $g_i^0 = 0\ \forall i$ and $\mathbb{L}$ defined as in (5). □

This represents a standard condition on noise ϵ_i for SA algorithms (in fact the minimum possible one) and naturally assumes ϵ_i to be bounded to allow its averaging out by a suitably small (decreasing) step size a_i^k. The above formulation of the noise condition is not the only possible one. For instance, see [23] for other equivalent formulations or [18] for therefrom resulting conditions on stochastic processes $\tilde{h}_{ij}$ for given h_{ij}.

Furthermore, standard restrictions on the stepsize in SA algorithms are assumed, whereby network links are allowed to use their *own* stepsize to respect their independent status in an ad hoc network.

Assumption 5: Assume that each link $i \in \mathscr{I}$ iterates the recursions (5) with its own step size a_i^k such that $a_i^k > 0$; $a_i^k \xrightarrow{k \to \infty} 0$; $\sum_{k=0}^{\infty} a_i^k = \infty$ and

$$0 < c_i^{\min} \leqslant \liminf_{k \to \infty} \frac{a_i^k}{a_{j \neq i}^k} \leqslant \limsup_{k \to \infty} \frac{a_i^k}{a_{j \neq i}^k} \leqslant c_i^{\max}\ a.s. \quad (7)$$

for some $c_i^{\min}$ and $c_i^{\max}$ and $\forall i$. □

Then the convergence of (5) can be stated as follows:

Theorem 4: Assume a non-cooperative power control game $\mathscr{G} = \{\mathscr{I}, \sigma_i, \beta_i\}$ as defined in *Definition 1* and suppose $\boldsymbol{\beta}$ to be given by a finite[5] sequence $\{\boldsymbol{\beta}^K\}$ of totally $K^{\max}$ functions $\boldsymbol{\beta}^K : \boldsymbol{\Sigma} \to \boldsymbol{\Sigma}$, being for all integers $K \in \{1, \ldots, K^{\max}\}$ short-term invariant, as stated in *Assumption 1*, and globally Lipschitz, as required in *Theorem 2*, on a non-empty compact and convex power profile $\boldsymbol{\Sigma} \subset \mathbb{R}^{N+}$ (*Assumption 2*).

Let the transmit power $\sigma_i(t_i^k)$ of link i be periodically updated with period T at time instances t_i^k for integer $k \in \mathbb{N}$ based on (5) assuming some initial power $\sigma_i(t_i^0) < \max[\Sigma_i]$ and reset power $\sigma_i^* < \max[\Sigma_i]$, whereby each link i defines its own step size a_i^k according to *Assumption 5*. Furthermore, assume that the set $J^K = \{\hat{\boldsymbol{\sigma}} \in \boldsymbol{\Sigma} : \boldsymbol{f}^K(\hat{\boldsymbol{\sigma}}, t) = 0\}$ of Nash equilibria $\hat{\boldsymbol{\sigma}}$ in $\mathscr{G}$ established by *Theorem 1* for each $\boldsymbol{\beta}^K$, i.e., the set of zero root(s) of $\boldsymbol{f}^K(\boldsymbol{\sigma}, t) = \boldsymbol{\beta}^K(\boldsymbol{\sigma}, t) - \boldsymbol{\sigma}$, is a finite set of isolated points and there exists a set $\{v^K\}$ of $K^{\max}$ scalar twice continuously differentiable functions $v^K : \boldsymbol{\Sigma} \to \mathbb{R}^1$ such that for all respective $\boldsymbol{\sigma} \in \boldsymbol{\Sigma} \setminus J^K$

$$\sup_{c_i \in [c_i^{\min}, c_i^{\max}]} \left[(\boldsymbol{f}^K)^{\mathrm{T}} \operatorname{diag}(1, c_2, \ldots, c_N) \operatorname{grad} v^K \right] < 0 \quad (8)$$

(diag stands for diagonal matrix). Then, if it is true that

$$v^K(\boldsymbol{\sigma}^*) < \inf_{\boldsymbol{\sigma}: \sigma_i = \max[\Sigma_i]; \sigma_{j \neq i} \leqslant \max[\Sigma_{j \neq i}]} v^K(\boldsymbol{\sigma})\ \forall K \quad (9)$$

together with the *Assumption 4* on noise ϵ_i, the iterations $\boldsymbol{\sigma}(t^k)$ of algorithm (5) tend to J^K for respective $\tilde{\boldsymbol{\beta}}^K$, whereby

$$\operatorname{dist}\left[\boldsymbol{\sigma}(t^k), J^{K^{\max}}\right] \xrightarrow{k \to \infty} 0\ a.s. \quad (10)$$

for $\operatorname{dist}\left[\boldsymbol{\sigma}(t^k), J^K\right] = \inf\{\|\boldsymbol{\sigma}(t^k) - \hat{\boldsymbol{\sigma}}\|\ \forall \hat{\boldsymbol{\sigma}} \in J^K\}$. □

Proof: The theorem formulation guarantees by *Theorem 1* the existence of at least one Nash equilibrium $\hat{\boldsymbol{\sigma}}$ in the power control game $\mathscr{G}$, whereby in order to assure the convergence to $\hat{\boldsymbol{\sigma}}$ it requires for each $\boldsymbol{\beta}^K$ the predictability of the corresponding dynamics (4) in the sense of *Theorem 2* and also its stability throughout the existence of respective Lyapunov functions v^K as in *Theorem 3*.

Knowing that $J^K \subset \boldsymbol{\Sigma}$, it is meaningful to restrict the search for $\hat{\boldsymbol{\sigma}}$ only to power profile $\boldsymbol{\Sigma}$ of $\mathscr{G}$ by using the function $\mathbb{L}$ in (5) for truncating equilibrium transmit power estimates $\sigma_i(t_i^k)$ to some $\sigma_i^* < \max[\Sigma_i]$ if their values exceed the corresponding power range Σ_i. Truncating the sequence $\{\boldsymbol{\sigma}(t^k)\}$ by $\mathbb{L}$ on fixed truncation bounds $\max\{\Sigma_i\}$ then result in boundedness of the sequence, which implies by Bolzano-Weierstrass theorem the existence of a convergent subsequence in the sequence $\boldsymbol{\sigma}(t^k)$.

This fact allows us to use Lemma 5.6.3 from [18] stating that if condition (9) holds, then the values $v^K(\boldsymbol{\sigma}(t^k))$ of Lyapunov functions v^K, calculated for the iterations $\boldsymbol{\sigma}(t^k)$ of (5) with given $\tilde{\boldsymbol{\beta}}^K$ in time instances $t_i^k\ \forall i \in \mathscr{I}$, cannot cross infinitely many times some interval $[\gamma_1, \gamma_2]$ such that $\operatorname{dist}\left[[\gamma_1, \gamma_2], v(J^K)\right] > 0$ for some $\gamma_1 < \gamma_2$, or converge to γ_1 if $\gamma_1 = \gamma_2$.

Hence, the algorithm (5) performs only a finite number of truncations during its iterations and evolves, for a sufficiently large k, as the Robbins-Monro algorithm (3). But then assuming the standard noise condition from *Assumption 4* and stepsize restriction from *Assumption 5*, it follows by [18] that $\operatorname{dist}\left[\boldsymbol{\sigma}(t^k), cls\{J^K\}\right] \xrightarrow{k \to \infty} 0\ a.s.$. The symbol $cls\{J^K\}$ denotes the closure of J^K on $\boldsymbol{\Sigma}$, i.e., J^K plus its limit points.

Since the Euclidean space $\mathbb{R}^N$ is a topological space of T1 kind having its topology induced by the Euclidean metric, any two distinct points in $\boldsymbol{\Sigma}$ can be separated. Thus the finite set J^k has no limit points and its closure $cls\{J^K\}$ is equal to J^K itself. The iterations of (5) thus tend to J^K and $\operatorname{dist}\left[\boldsymbol{\sigma}(t^k), J^{K^{\max}}\right] \xrightarrow{k \to \infty} 0\ a.s.$.

It should be mentioned as a concluding remark that the usage of said Lemma requires three conditions to be fulfilled: (i) the measurability and uniform local boundedness of $\boldsymbol{f}^K$, which is evidently true; (ii) nowhere dense functions v^K in J^K, which holds as J^K are finite sets and v^K are continuous

[5] Infinite sequences can also be assumed after some theorem modifications.

thanks to their continuous differentiability; (iii) satisfaction of $\lim_{t_i^k \to\infty} \sum \epsilon_i \left(t_i^k\right) a_i^k = 0$ a.s., which follows as a consequence of $a_i^k \xrightarrow{k\to\infty} 0$, because the delays between power updates of links j and i at time k are bounded by T.

Lastly, although Lemma 5.6.3 uses spherical truncation limits, our usage of rectangular bounds $\max\{\Sigma_i\}$ has no effect on the proof. The proof remains unchanged if continuous time-variance of $\boldsymbol{f}^K$ is assumed too. ■

C. Design Issues

We can clearly see that the above convergence conditions for algorithm (5) do not exceed restrictions on $\mathscr{G}$ from previous sections - apart of verifying *Assumption 4* on noise ϵ_i or $\tilde{h}_{ij}$ for some particular channel type of interest. Moreover, it is apparent from condition (9) that Lyapunov functions v^K in *Theorem 4* do *not* need to be non-negative as in the case of v in *Theorem 3*.

Note, however, that assuring the existence of *positive definite* functions v^K not only automatically implies satisfaction of (9), but importantly allows to implement *partial* iteration resets of (5). To be more specific, *full* network-wide resets were assumed until now, i.e., if some links i exceeds Σ_i with its estimate of $\hat{\sigma}_i$ in time t_i^k and resets $\sigma_i\left(t_i^k\right)$ to σ_i^*, other links $-i$ join it by switching to $\boldsymbol{\sigma}_{-i}^*$. Yet if $v^K > 0$, the condition (9) holds for *any* $\boldsymbol{\sigma}^* \in \boldsymbol{\Sigma}$. So also, for $\boldsymbol{\sigma}^*$ defined by components σ_i^* and $\sigma_j\left(t_i^k\right)\ \forall j \in -i$ whose usage corresponds, on the network level, to a situation where link i resets its iterations of (5), but other links ignore this fact and do not alter their own iteration by in any way, i.e., to what we called a partial iteration reset. Finding $v^K > 0$ thus removes the undesirable overhead for implementing coordinated full network-wide resets in the search of $\hat{\boldsymbol{\sigma}}$.

Another important observation is that even if *temporary* instabilities arise during the usage of the proposed SA algorithm, the algorithm (5) converges *on the whole* to J^K, that is to say to $J^{K^{\max}}$. Understandably, such instabilities have no benefit for the network power control performance and should be avoided to promote energetical efficiency of ongoing transmissions.

Note also that the algorithmic step size a_i^k attains progressively decreasing values (e.g. $a_i^k = 1/k$), disregarding the dynamic changes in the network, so the speed of convergence to time-variant equilibria can be different in different stages of the power control process. This is due to the fact that the magnitude of a_i^k plays a key role in a well-known tradeoff between the algorithmic convergence rate and error distance of the allocated power vector from the optimum equilibrium vector. In the early stages of the equilibrium search, a bigger step size implies generally speaking faster convergence with lower precision of the equilibrium estimate, whereas a smaller step size allows reaching the equilibrium without e.g. oscillations around it, but with slower convergence rate (detailed numerical simulations, related to said stepsize choice trade-off, can be found e.g. in [24]).

In this context, it was proposed e.g. by [17], [19] to use a constant step size rather than a decreasing time-variant one to provide SA algorithms with a balanced long-term performance in terms of tracking down time-varying equilibria or responding to network admission events, which also cause equilibrium time-variance. Such an approach, however, sacrifices the above shown almost sure convergence to equilibrium sets J^K and offers only convergence in distribution [19]. It is therefore more reasonable to implement flexible network-wide resets of a_i^k, using e.g. a binary "reset" tone on a broadcasting signaling frequency, to handle significant network changes.

An interesting design alternative of algorithm (5) consists in averaging of the estimate sequence $\sigma_i\left(t_i^k\right)$ at each link i [25] and using the averaged output values for determining actual transmit powers [26]. It has been shown [25] that the sequence of simple arithmetic averages of powers allocated by (5) converges to the desired equilibrium $\hat{\boldsymbol{\sigma}}$ with optimal rate, whereby under some appropriate assumptions, the choice of a_i^k does not affect this rate. Importantly, averaging allows the use of a step size that decays on a slower pace than the classical choice of $O\left(1/k\right)$. Implementation details are studied in [26] (see references therein for advanced averaging techniques).

VI. Distributed Medium Access Control For Networks With Linear/Linearized BR Functions

A. Proposed Algorithm for Linear Power/Rate Control

This section proposes a *distributed* and *asynchronous* admission control scheme, whose operation on the network data link layer automatically implies convergence to the previously presented SA algorithm, running on the underlying physical layer.

For the purpose of its derivation, we assume a system model with linear or linearized BR functions in the form $\beta_i^{\mathrm{RX}}\left(\boldsymbol{\sigma}_{-i}, t\right) = B_i + A_i \boldsymbol{h}_{-i}^{\mathrm{T}} \boldsymbol{\sigma}_{-i}$ for some possibly time-variant $B_i \in \mathbb{R}^{1+}$ and $A_i \in \mathbb{R}^1$. Then $\beta_i^{\mathrm{TX}} = \left(B_i + A_i \boldsymbol{h}_{-i}^{\mathrm{T}} \boldsymbol{\sigma}_{-i}\right)/h_{ii}$ and $\tilde{\beta}_i^{\mathrm{TX}} = \left(B_i + A_i \tilde{\boldsymbol{h}}_{-i}^{\mathrm{T}} \boldsymbol{\sigma}_{-i}\right)/\tilde{h}_{ii}$. In the case of approximating a nonlinear BR function $\boldsymbol{\beta}^{\mathrm{TX}}$ by a linear one, we assume continuous differentiability of the nonlinear $\boldsymbol{\beta}^{\mathrm{TX}}$. We will again work with $\tilde{\beta}_i^{\mathrm{TX}}$ only and thus drop the index TX for simplicity as before.

Two significant power/rate control schemes can be successfully modeled in such a way. Firstly, if $B_i > 0$ and $A_i < 0$, then the preferences of each link on the desirable CIR_i decrease inverse proportionally with increasing interference, representing typically the desire of high-power high-speed transmissions for low inband interference and rather slow (but energetically not demanding) transmissions for higher interference.

If the interference level exceeds the threshold B_i/A_i, for which $\beta_i\left(B_i/A_i, t\right) = 0$, link i becomes passive and waits for better interference conditions while saving its power budget. This implements a receiver-based admission control of CSMA/CA kind (Carrier Sensing Multiple Access/Collision Avoidance), which can be combined with the standard transmitter-based CSMA/CA, using the evaluation of a maximum interference threshold.

Secondly, if $B_i = 0$ and $A_i > 0$, the system model represents links with fixed target CIRs similarly to [3].

The next theorem extends the results of the previous section and states admission control conditions for assuring convergent asynchronous power/rate control with linear or linearized

BR functions to a unique Nash equilibrium under the effects of channel fluctuations:

Theorem 5: Assume a non-cooperative power control game $\mathscr{G} = \{\mathscr{I}, \sigma_i, \beta_i\}$ as defined in *Definition 1* and suppose $\boldsymbol{\beta}$ to be given by a finite sequence $\{\boldsymbol{\beta}^K\}$ of totally $K^{\max}$ functions $\boldsymbol{\beta}^K$, whose i-th component β_i^K is a short-term invariant (*Assumption 1*) and linear BR function $\beta_i^K = \left(B_i + A_i \sum_{j \neq i} h_{ij} \sigma_j\right) / h_{ii}$ in a non-empty compact and convex power range $\Sigma_i \subset \mathbb{R}^{1+}$ (*Assumption 2*) for all $i \in \mathscr{I}$ and integer $K \in \{1, \ldots, K^{\max}\}$.

Let the transmit power $\sigma_i\left(t_i^k\right)$ of link i be periodically updated with period T at time instances t_i^k for integer $k \in \mathbb{N}$ based on (5) assuming some initial power $\sigma_i\left(t_i^0\right) < \max\left[\Sigma_i\right]$ and reset power $\sigma_i^* < \max\left[\Sigma_i\right]$, whereby each link i defines its own step size a_i^k according to *Assumption 5*.

Then if all links $i \in \mathscr{I}$ satisfy for all $K \geqslant K^{\max}$

$$h_{ii} > |A_i| \sum_{j \neq i} h_{ij}, \tag{11}$$

there exists a unique Nash equilibrium $\hat{\boldsymbol{\sigma}}^K$ for each $\boldsymbol{\beta}^K$ in the power profile $\mathbb{R}^N$. If it moreover holds that $\hat{\boldsymbol{\sigma}}^K \in \boldsymbol{\Sigma} \subset \mathbb{R}^{N+}\ \forall K$ and *Assumptions 4* and *5* on noise ϵ_i and step size a_i^k are true,[6] the iteration resets of (5) at link i based on function $\mathbb{L}$ can be performed independently from other links $-i$, whereby $\boldsymbol{\sigma}\left(t^k\right)$ tend to J^K for respective $\tilde{\boldsymbol{\beta}}^K$ with $\left\|\boldsymbol{\sigma}\left(t^k\right) - \hat{\boldsymbol{\sigma}}^{K^{\max}}\right\| \xrightarrow{k\to\infty} 0\, a.s.$. □

Proof: The above theorem states in comparison with *Theorem 4* that the linear nature of $\boldsymbol{\beta}^K$ and condition (11) assure (i) unique Nash equilibrium $\hat{\boldsymbol{\sigma}}^K$ for each $\boldsymbol{\beta}^K$ in $\mathbb{R}^N$ and, if $\hat{\boldsymbol{\sigma}}^K \in \boldsymbol{\Sigma} \forall K$, imply together with the standard *Assumptions 4* and *5* on noise ϵ_i and step size a_i^k the satisfaction of all main convergence conditions of *Theorem 4*: (ii) $\boldsymbol{\beta}^K$ being globally Lipschitz functions; (iii) J^K being sets of isolated points; (iv) existence of Lyapunov functions v^K for assuring algorithmic stability; and (v) condition (9) on $\boldsymbol{\sigma}^*$ while allowing partial resets. So the proof must validate all these five statements.

To simplify the representation of β^K, we define $\boldsymbol{B}^K$ to be a vector with components $B_i^K / h_{ii}\ \forall i \in \mathscr{I}$, and $\boldsymbol{A}^K$ to be a matrix defined by $\boldsymbol{A}_{ii}^K = 0$ and $\boldsymbol{A}_{ij}^K = A_i^K h_{ij} / h_{ii}$ for $j \neq i$ and $i \in \mathscr{I}$. Then $\beta^K = \boldsymbol{B}^K + \boldsymbol{A}^K \boldsymbol{\sigma}$. Define also $\boldsymbol{E}$ to be N-by-N unit matrix.

The elements of both $\boldsymbol{B}^K$ and $\boldsymbol{A}^K$ can be practically assumed to be bounded in time, so $\boldsymbol{\beta}^K$ fulfills the global Lipschitz condition on $\mathbb{R}^N$, given that $\left\|\boldsymbol{\beta}^K(\boldsymbol{\sigma}, t) - \boldsymbol{\beta}^K(\boldsymbol{\tau}, t)\right\| = \left\|\boldsymbol{A}^K \boldsymbol{\sigma} - \boldsymbol{A}^K \boldsymbol{\tau}\right\| = \left\|\boldsymbol{A}^K\right\| \cdot \|\boldsymbol{\sigma} - \boldsymbol{\tau}\|$, which proves point (ii).

As for point (i) and (iii), assume for the moment that they hold by supposing that each J^K contains only one unique Nash equilibrium $\hat{\boldsymbol{\sigma}}^K$, satisfying $\hat{\boldsymbol{\sigma}}^K = \boldsymbol{B}^K + \boldsymbol{A}^K \hat{\boldsymbol{\sigma}}^K$. Then $\hat{\boldsymbol{\sigma}}^K = -\left(\boldsymbol{A}^K - \boldsymbol{E}\right)^{-1} \boldsymbol{B}^K$, which implies invertibility of the matrix $\left(\boldsymbol{A}^K - \boldsymbol{E}\right)$.

This premise allows us to proceed with showing that the theorem formulation assures the existence of Lyapunov functions v^K for each $\boldsymbol{f}^K = \boldsymbol{\beta}^K - \boldsymbol{\sigma} = \boldsymbol{B}^K + \left(\boldsymbol{A}^K - \boldsymbol{E}\right)\boldsymbol{\sigma}$ such that condition (8) is satisfied. Perform first with no loss of generality a substitution of $\boldsymbol{\sigma}$ in $\mathscr{G}$ by $\boldsymbol{s} = \boldsymbol{\sigma} - \hat{\boldsymbol{\sigma}}^K$, which results in a coordinate shift of $\mathscr{G}$ into the origin of $\mathbb{R}^N$ and yields $\hat{\boldsymbol{s}}^K = \boldsymbol{0}$ and $\boldsymbol{f}^K(\boldsymbol{s}_{-i}, t) = \boldsymbol{B}^K + \left(\boldsymbol{A}^K - \boldsymbol{E}\right)(\boldsymbol{s} + \hat{\boldsymbol{\sigma}}) = \left(\boldsymbol{A}^K - \boldsymbol{E}\right)\boldsymbol{s}$.

Suppose now a candidate Lyapunov function v^K in a polynomial form $v^K = \boldsymbol{\sigma}^{\mathrm{T}} \boldsymbol{V}^K \boldsymbol{\sigma}$ for some N-by-N matrix $\boldsymbol{V}^K$. Then the derivative of v^K along the trajectories $\boldsymbol{f}^K(\boldsymbol{s}_{-i}, t)$ in the shifted system, scaled component-wise by $\{1, c_2, \ldots, c_N\}$ as defined in *Theorem 4*, is given in the present case of linear BR functions by $\dot{v} = \left(\boldsymbol{f}^K\right)^{\mathrm{T}} \boldsymbol{C}\, \mathrm{grad}\, v^K = \boldsymbol{s}^{\mathrm{T}} \left[\boldsymbol{V}^K \left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right) + \left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)^{\mathrm{T}} \boldsymbol{V}^K\right] \boldsymbol{s} \stackrel{\mathrm{def}}{=} -\boldsymbol{s}^{\mathrm{T}} \boldsymbol{W}^K \boldsymbol{s}$, where $\boldsymbol{C}$ is diagonal matrix given by $\boldsymbol{C} = \mathrm{diag}(1, c_2, \ldots, c_N)$.

Based on this derivation, one can see that each matrix $\boldsymbol{V}^K$ must satisfy for some chosen $\boldsymbol{W}^K$ the equation

$$\boldsymbol{V}^K \left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right) + \left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)^{\mathrm{T}} \boldsymbol{V}^K = -\boldsymbol{W}^K, \tag{12}$$

to meet the requirements of (8). It is a known result (e.g. [28]) that there exists a positive definite symmetric matrix $\boldsymbol{V}^K$ that satisfies (12) for some *symmetric positive definite* matrix $\boldsymbol{W}^K$ if and only if the matrix $\left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)$ is Hurwitz (its eigenvalues have negative real parts).

To see if the above theorem implies the matrix $\left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)$ to be Hurwitz, we examine Gerŝgorin's disks [29] of $\left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)$, whose union identifies the region in the complex plane $\boldsymbol{z}$ that contains all N eigenvalues λ_i of $\left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)$. The disk of the i-th eigenvalue λ_i is given by

$$\left|z - \left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)_{ii}\right| \leqslant \sum_{j \neq i} \left|\left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)_{ij}\right|, \tag{13}$$

which given that $\left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)_{ii} = -c_i$ and $\left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)_{ij} = c_i A_i h_{ij} / h_{ii}$ can be rewritten as $|z + c_i| \leqslant c_i |A_i| \sum_{j \neq i} h_{ij} / h_{ii}$. The Hurwitz nature of $\left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)$ is then satisfied if and only if $\forall i$ it holds that $c_i - c_i |A_i| \sum_{j \neq i} h_{ij} / h_{ii} > 0$, i.e., $h_{ii} \geqslant |A_i| \sum_{j \neq i} h_{ij}$. But this requirement is evidently fulfilled by condition (11), so the existence of Lyapunov functions v^K for given $\boldsymbol{f}^K$ is established as required in *Theorem 4* and point (iv) proved.

The condition (11) however requires a stricter *sharp* inequality $h_{ii} > |A_i| \sum_{j \neq i} h_{ij}$, which means that *none* of the disks (13) contains the point $z = 0$. As it holds that the determinant of a square matrix is equal to the product of its eigenvalues, the determinant of matrix $\left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)$ cannot be equal to zero. Hence, there can exist only a unique solution $\hat{\boldsymbol{\sigma}}^K$ to $\boldsymbol{\sigma} = \boldsymbol{B}^K + \boldsymbol{A}^K \boldsymbol{\sigma}$ and the assumption on each J^K to contain a unique Nash equilibrium $\hat{\boldsymbol{\sigma}}^K$ in $\mathbb{R}^N$ was therefore correct, being in fact implied as a consequence of condition (11). This proves both points (i) and (iii).

Lastly, if $\boldsymbol{W}^K > 0$ is used in (12), then also $\boldsymbol{V}^K > 0$ and the polynomial function $v^K > 0$ is strictly increasing because all its coefficients are positive. But then condition (9) from *Theorem 4* is true for any $\boldsymbol{\sigma}^*$ smaller than $\boldsymbol{\sigma}$ such that $\sigma_i = \max\left[\Sigma_i\right]$ and $\sigma_j \leqslant \max\left[\Sigma_j\right] \forall j \neq i$, which proves point (v) and allows partial resets of (5) as discussed previously. ■

B. Physical Interpretation

The theorem states that (i) if condition (11) is fulfilled, the linear power/rate control game $\mathscr{G}$ admits unique Nash equilibrium $\hat{\boldsymbol{\sigma}}^K$ in $\mathbb{R}^N$ for given $\boldsymbol{\beta}^K$, and (ii) if power ranges

[6]See [27] for a similar noise condition applicable to linear $\mathscr{G}$ too.

$\boldsymbol{\Sigma}$, that is to say the truncation bounds $\max|\boldsymbol{\Sigma}|$, are set such that all $\hat{\boldsymbol{\sigma}}^K \in \boldsymbol{\Sigma}$, the algorithm (5) converges to $\hat{\boldsymbol{\sigma}}^K$, that is to say to $\hat{\boldsymbol{\sigma}}^{K^{\max}}$. This renders redundant the requirements of *Theorem 1*, predicting solely the equilibrium existence, which were necessary to be assumed in the previous general *Theorem 4*. Furthermore, the well-known Foshini-Miljevic algorithm can be incorporated under such circumstances into our framework and its key, but vague requirement for "algorithmic feasibility" can be substituted by requiring validity of the proposed condition (11).

In fact, condition (11) can be regarded as an admission control scheme on the data link layer, which is cross-layer optimized with the underlying physical layer in terms of assuring the convergence of its power and rate control and moreover supplementing the admission control, implicit to our system model and based on defining link BR functions so as to assign zero transmit powers to some chosen interference values. Note also the cross-layer relation to the network layer through its parameter B_i, which is adaptively adjusted in response to routing purposes so that always enough transmit power is allocated for a given transmission distance of link i.

Using only said condition (11), each link can determine *independently* from others, whether its transmission would affect the stability of the overall network under given channel gains h_{ii} and h_{ij} and with a BR function characterized by B_i and A_i. All of these parameters are *locally* available information for link i and, thus, there is no need for some collective decision-making causing undesirable overhead.

To be more specific, link i with an intention to start transmitting data can proceed only if $h_{ii} > |A_i| \sum_{j\neq i} h_{ij}$. Following this admission rule by all links assures that the power/rate control converges globally and asymptotically with an exponential motion to a unique Nash equilibrium for any initial condition. Note that the *absolute* value of local interference is not important as e.g. in the case of CSMA/CA. However, transmission by a link violating the admission control condition (11) results in an undesirable exponential divergence of all transmit powers in the network towards minimum or maximum limits of the transmit power profile $\boldsymbol{\Sigma}$ (note that small violations may be tolerable due to the approximate nature of Gerŝgorin's disks used in proof of *Theorem 5*). Instantaneous violations due to randomization of h_{ij} by $\tilde{h}_{ij}$ are negligible as their speed is much faster than the network reaction abilities or more precisely the update frequency $1/T$.

It is apparent from the theorem proof that divergence surely follows only if at least one eigenvalue λ_i of the matrix $\left(\boldsymbol{C}\boldsymbol{A}^K - \boldsymbol{C}\right)$ has a positive real part or if at least one eigenvalue λ_i such that $\left|\lambda_i\left(\boldsymbol{A}^K\right)\right| = 1$ corresponds to a Jordan cell with dimension more than 1, because then there exists no v^K to satisfy (12).

As for the interpretation of the admission control condition (11), it can be understood in the following way. Suppose that the path loss channel gain h_{ij} decreases proportionally to $1/l_{ij}^{\alpha}$, where l_{ij} is the length of the physical distance between $\mathrm{TX_i}$ and $\mathrm{TX_j}$ and $\alpha \in [2,5]$. Then (11) can be rewritten as

$$\frac{|A_i|}{h_{ii}} \sum_{\forall j\neq i} h_{ij} = |A_i|\, l_{ii}^{\alpha} \sum_{\forall j\neq i} \frac{1}{l_{ij}^{\alpha}} < 1. \tag{14}$$

So the length of link i must be such that

$$l_{ii} < \left(|A_i| \sum_{\forall j\neq i} l_{ij}^{-\alpha}\right)^{-1/\alpha}. \tag{15}$$

Supposing for simplicity only one strongly interfering link j and neglecting the contribution of others, the last inequality reduces to

$$l_{ij} > l_{ii}\, \sqrt[\alpha]{|A_i|}. \tag{16}$$

Then the condition (11) means that in order to assure the convergence of SA power/rate allocation, the interfering transmitter j must be located *outside* of a disk area around the receiver of link i, whose diameter is given by the distance to its corresponding transmitter i and scaled by factor $\sqrt[\alpha]{|A_i|}$. Hence, the condition is in fact equivalent to position-based control for given propagation/channel model.

VII. Numerical Results

Our simulations test whether the proposed approach to power and rate control has the potential to allocate satisfactorily high CIRs for links in densely populated ad hoc networks. This kind of evaluation is needed, because the above work analyzes power/rate control updates based solely on experienced local inband interference, whereas many fundamental formulae from information theory depend on the *relative* CIR [13].[7]

The results show that given the *same* amount of total network power, a representative algorithm from our framework allocates *higher* link CIRs than a hereafter introduced comparison algorithm under changing network density (Subsection A) and topological randomness (Subsection B). This is an expectable result due to the direct connection of such network parameters to local interference, which is in turn the main algorithmic input in our framework. However, the proposed approach to CIR allocation is comparatively also more robust against malicious attacks (Subsection C) and fairer in its distribution (Subsection D).

We assume in all our scenarios a square area with 10 km edge and $N = 100$ stationary links (or N changing from 1 to 100), whereby their length is set to 100 meters in order to simplify the interpretation of the following simulation results (our conclusions can be readily extended to the general case).

The location of individual links depends on the scenario setup, but in each scenario, all links update their transmit powers asynchronously and distributively using algorithm (5) (partially reset from $\sigma_i^* = 0.1$ W if needed and using $a_i^k = 1/k$) and the same linear BR function

$$\beta_i\left(\boldsymbol{\sigma}_{-i}\right) = h_{ii}\sigma_i^{\max} - \boldsymbol{h}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}, \tag{17}$$

whereby the network maximum transmit power $\sigma_i^{\max} = \max\left[\Sigma_i\right] = 1$ W. This corresponds to the usage of "bursty" high-speed high-power transmissions for low interference and vice versa as discussed in Section VI. Channel gains h_{ij} exhibit a path loss with exponent $\alpha = 3.5$ and are subject to Rayleigh fading $\tilde{h}_{ij}$ with maximum Doppler frequency shift

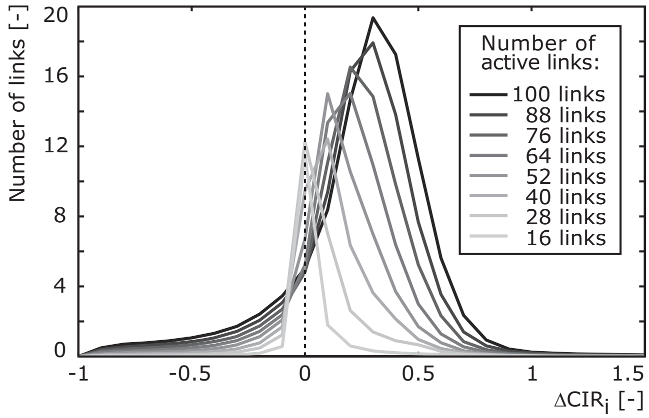

Fig. 1. Histogram of differences $\Delta CIR_i = CIR_i^{\mathrm{prop}} - CIR_i^{\mathrm{comp}}$ between CIR_is allocated by network links i using the proposed algorithm (CIR_i^{prop}) and the comparison algorithm (CIR_i^{comp}). Eight cases of different network density are shown, each obtained by averaging over 10^4 topologies with uniformly randomly distributed links.

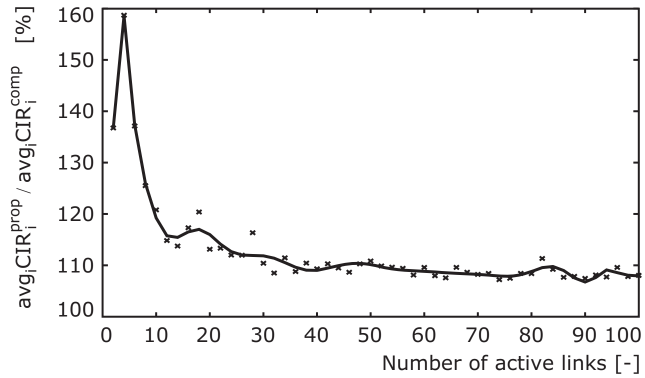

Fig. 2. Maximum value of the ratio of average network CIRs ($\mathrm{avg}_i \mathrm{CIR}_i = \sum_{\forall i} \mathrm{CIR}_i / N$), allocated by network links i in 10^4 different topologies using the proposed algorithm ($\mathrm{avg}_i \mathrm{CIR}_i^{\mathrm{prop}}$) and the comparison algorithm ($\mathrm{avg}_i \mathrm{CIR}_i^{\mathrm{comp}}$), shown as a function of increasing number of active links, distributed uniformly randomly in the network.

250 Hz. Naturally, only stable topologies in terms of condition (11) are considered.

For a precisely-defined and fair comparison, we compare *equilibrium* CIRs allocated based on the usage of BR functions (17) ("proposed algorithm") with CIRs obtained by a comparison algorithm, which (i) allocates the transmit power of every link such that every network RX receives the *same* power (recall that all $\mathrm{TX}-\mathrm{RX}$ pairs are equidistant); whereby (ii) the level of said received power is such that the total sum of transmit powers used by such "constant received power" algorithm is equal to the total of our linear BR algorithm.

A. Scenario with Varying Network Density

In the first scenario, we slowly increase from 1 to 100 the number of active links, distributed uniformly randomly in the network, and compare the performance of both algorithms in terms of allocated equilibrium CIRs.

Fig. 1 depicts a histogram of CIR differences $\Delta CIR_i = CIR_i^{\mathrm{prop}} - CIR_i^{\mathrm{comp}}$ between CIR_is allocated by network links i using the proposed algorithm (CIR_i^{prop}) and the comparison one (CIR_i^{comp}), whereby each curve represents a different total number N of active links. We observe that growing network link density (and thus worse interference conditions) implies a shift of the histogram curves in the direction of *positive* values, which means a progressive *improvement* of CIR_is allocated by the proposed algorithm with respect to the comparison one under worsening network density conditions.

Fig. 2 then concerns average network CIRs, denoted as $\mathrm{avg}_i CIR_i = \sum_{\forall i} CIR_i / N$, allocated by the proposed algorithm ($\mathrm{avg}_i CIR_i^{\mathrm{prop}}$) and the comparison algorithm ($\mathrm{avg}_i CIR_i^{\mathrm{comp}}$). For comparison purposes, it depicts the ratio $\mathrm{avg}_i CIR_i^{\mathrm{prop}} / \mathrm{avg}_i CIR_i^{\mathrm{comp}}$ as a function of increasing density, whereby it plots *maximum* values of this ratio achieved in 10^4 different topologies, i.e., a numerical upperbound $\max_{10^4 topologies} [\mathrm{avg}_i CIR_i^{\mathrm{prop}} / \mathrm{avg}_i CIR_i^{\mathrm{comp}}]$. We can see that our proposed algorithm outperforms the constant received power one by allocating up to 60 % higher average CIRs in sparse networks and is more efficient by about 10 % even in dense networks.

The large improvement difference between sparse and dense networks lies in the fact that the distribution of local interference is less homogenous in rather sparse networks, and power allocation with the *same* BR functions thus results in mutually more different powers and consequently CIRs throughout the network. This enables significant local CIR improvements by the proposed algorithm in sparser networks, whereas in denser networks, the interference is more homogenous yielding an allocation of comparable powers and hence only lower and more balanced CIR improvement possibilities.

B. Scenario with Varying Topological Randomness

The second scenario evaluates, how the proposed algorithm can improve link CIRs if the randomness of simulated network topologies varies having a constant number of links. We simulate the network randomness variation as a continuous transition from regular lattice topologies (0 % randomness) to uniformly randomly distributed topologies (100 % randomness). Specifically, we superpose a regular lattice with 10 rows and 10 columns over the network and define a virtual square area around each of the 100 intersections of the lattice. The geometrical centers of 100 network links are then placed uniformly randomly *anywhere* into these deviation areas, whereby their size is initially set to zero (i.e., only the intersection point itself is contained) and then progressively increased such that finally links can be placed anywhere in the network. The value of the network randomness can be then calculated basically as the ratio of the virtual deviation area edge size and network area edge size.

Fig. 3 shows analogically to Fig. 1 four histograms of CIR differences ΔCIR_i between CIR_is allocated by network links i using the proposed algorithm and the comparison one for four representative levels of topological randomness - 0%, 70%, 80% and 100%. We can again observe a progressive shift of CIR differences to *positive* values in response to increasing topological randomness, which proves better

[7]As the performance improvement of power/rate-controlled systems has been already demonstrated, e.g., in [30] or [31], it is not being repeated here.

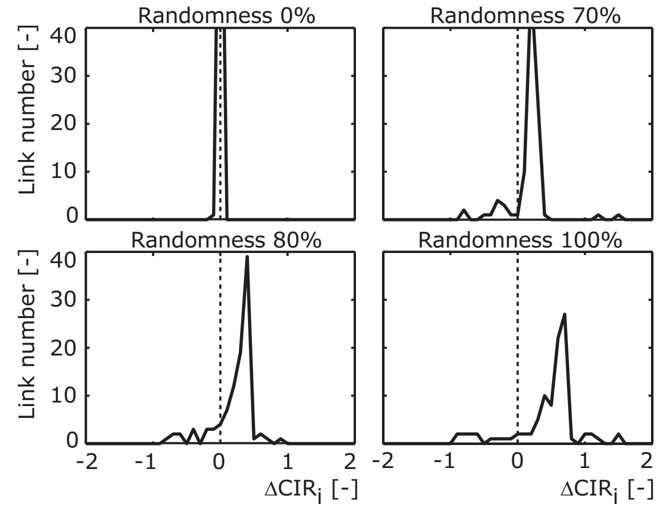

Fig. 3. Histogram of differences $\Delta\mathrm{CIR}_i = \mathrm{CIR}_i^{\mathrm{prop}} - \mathrm{CIR}_i^{\mathrm{comp}}$ between CIR_is allocated by network links i using the proposed algorithm ($\mathrm{CIR}_i^{\mathrm{prop}}$) and the comparison algorithm ($\mathrm{CIR}_i^{\mathrm{comp}}$). Four cases of increasing topological randomness in a network with 100 active links are illustrated, each obtained by averaging over 10^4 topologies.

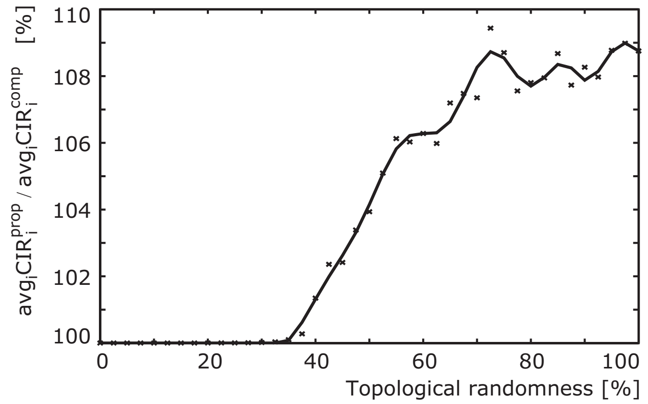

Fig. 4. Maximum value of the ratio of average network CIRs ($\mathrm{avg}_i\mathrm{CIR}_i = \sum_{\forall i}\mathrm{CIR}_i/N$), allocated by network links i in 10^4 different topologies using the proposed algorithm ($\mathrm{avg}_i\mathrm{CIR}_i^{\mathrm{prop}}$) and the comparison one ($\mathrm{avg}_i\mathrm{CIR}_i^{\mathrm{comp}}$), shown as a function of increasing topological randomness in a network with 100 active links.

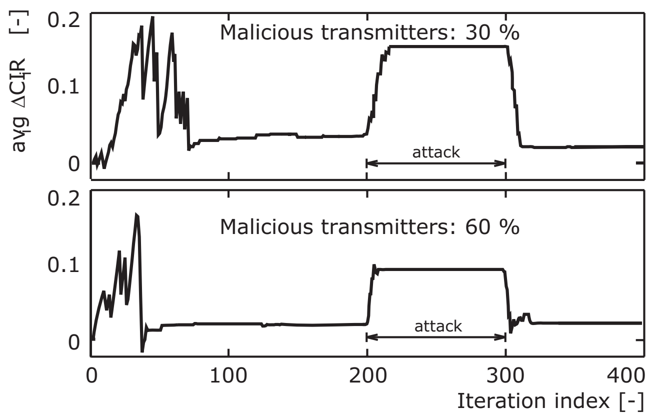

Fig. 5. Two evolution examples of the difference between the average network CIRs ($\mathrm{avg}_i\mathrm{CIR}_i$) allocated by network links i using the proposed algorithm ($\mathrm{avg}_i\mathrm{CIR}_i^{\mathrm{prop}}$) and the comparison algorithm ($\mathrm{avg}_i\mathrm{CIR}_i^{\mathrm{comp}}$), showing the effects of a malicious interference attack between time 200 and 300 iterations by randomly chosen 30 % or 60 % of the network transmitters.

comparative CIR allocation performance of the proposed algorithm in random networks. As observed, both algorithms allocate practically the same CIRs in regular topologies with grid-like layout (see the zero centered CIR differences for 0 % randomness), since the interference among equidistant links using moreover the same BR functions is rather homogenous (the influence of link angular orientations averages out). Improvement by the proposed algorithm comes analogically to our previous result with interference conditions being more heterogeneous, i.e., in more random network topologies.

In order to highlight the performance improvement trend of the proposed algorithm in progressively randomized topologies, Fig. 4 plots similarly to Fig. 2 the maximum value of the ratio $\mathrm{avg}_i CIR_i^{\mathrm{prop}}/\mathrm{avg}_i CIR_i^{\mathrm{comp}}$ of average network CIRs ($\mathrm{avg}_i CIR_i$), allocated by network links in 10^4 different topologies using the proposed algorithm ($\mathrm{avg}_i CIR_i^{\mathrm{prop}}$) and the comparison algorithm ($\mathrm{avg}_i CIR_i^{\mathrm{comp}}$) and shows it as a function of increasing topological randomness in a network with 100 links. Note as a validity cross-check the correspondence of the herein shown approximately 10% improvement, achieved in full random networks, with the corresponding result of the same 10 % improvement shown in Fig. 2 for a network with 100 uniformly randomly distributed links.

C. Scenario with Malicious Links

Assume now a network with 100 uniformly randomly distributed links as in the first scenario and suppose that some portion of the network transmitters does not use its available energy for communication purposes, but in contrary only maliciously attacks the surrounding links by causing unnecessary interference. Setting the output level of attacking transmitters to the network maximum transmit power limit $\sigma_i^{\max} = 1$ W, we investigate the immunity of our two algorithms to such attacks in terms of CIR allocation.

The malicious interference attack is simulated to occur from time 200 to 300 iterations and to be carried out by randomly chosen 30 % or 60 %, of the network transmitters. Fig. 5 then shows for both cases the evolution of the difference $\mathrm{avg}_i CIR_i^{\mathrm{prop}} - \mathrm{avg}_i CIR_i^{\mathrm{comp}}$ between the average network CIRs ($\mathrm{avg}_i CIR_i$) allocated by network links using the proposed algorithm ($\mathrm{avg}_i CIR_i^{\mathrm{prop}}$) and the comparison algorithm ($\mathrm{avg}_i CIR_i^{\mathrm{comp}}$). Note that $\mathrm{avg}_i\mathrm{CIR}_i^{\mathrm{prop}} - \mathrm{avg}_i\mathrm{CIR}_i^{\mathrm{comp}} = \mathrm{avg}_i\left[\mathrm{CIR}_i^{\mathrm{prop}} - \mathrm{CIR}_i^{\mathrm{comp}}\right] = \mathrm{avg}_i\Delta\mathrm{CIR}_i$.

Both curves represent in the first 200 iterations the process of convergence to Nash equilibrium, whose reaching corresponds to the flat parts of the plots. At time 200 iterations, when the malicious links suddenly start to disturb others, we observe in both cases that the proposed algorithm quickly allocates a new Nash equilibrium with transmit powers better adapted to the new interference conditions, thus yielding higher CIRs compared to the constant received power one. Consequently, we observe in Fig. 5 sudden improvements of the average network CIR (curves' step-ups), coinciding with the attack time frame. Such a response to increased interference proves a comparatively better robustness against interference attacks. After the attack finishes at time 300, the network returns to its original condition.

This conclusion is further supported by the evidence

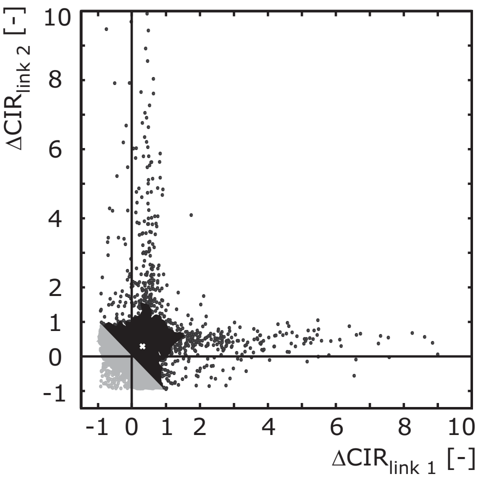

Fig. 6. Distribution of CIR differences $\Delta\mathrm{CIR}_i = \mathrm{CIR}_i^{\mathrm{prop}} - \mathrm{CIR}_i^{\mathrm{comp}}$ between CIRs allocated by two randomly chosen network links (labeled as link "1" and "2") using the proposed algorithm ($\mathrm{CIR}_i^{\mathrm{prop}}$) and the comparison algorithm ($\mathrm{CIR}_i^{\mathrm{comp}}$) in 10^4 network topologies with 100 uniformly randomly distributed active links. The coordinates of a white cross indicates the overall average values of $\Delta\mathrm{CIR}_{\mathrm{link\ 1}}$ and $\Delta\mathrm{CIR}_{\mathrm{link\ 2}}$.

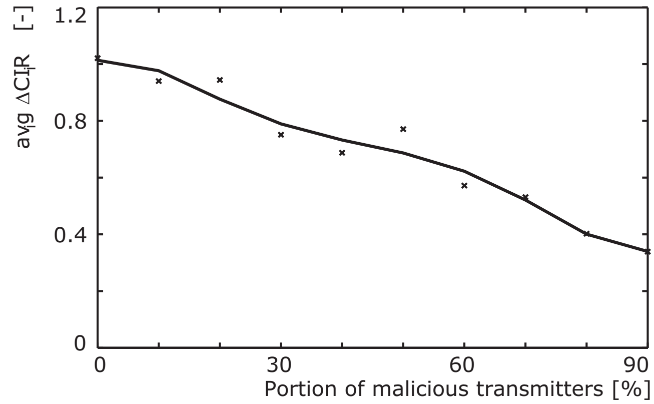

Fig. 7. Maximum value of the difference between the average network CIRs ($\mathrm{avg}_i\mathrm{CIR}_i$), allocated by network links in 10^4 different topologies using the proposed algorithm ($\mathrm{avg}_i\mathrm{CIR}_i^{\mathrm{prop}}$) and the comparison one ($\mathrm{avg}_i\mathrm{CIR}_i^{\mathrm{comp}}$), shown as a function of increasing ratio of malicious transmitters in a network with 100 uniformly randomly distributed active links.

of Fig. 7, plotting the maximum value of the difference $\mathrm{avg}_i CIR_i^{\mathrm{prop}} - \mathrm{avg}_i CIR_i^{\mathrm{comp}}$ between the average network CIRs ($\mathrm{avg}_i CIR_i$), allocated by network links in 10^4 different topologies using the proposed algorithm and the comparison one as a function of increasing ratio of malicious transmitters in the network. Note also from the figure that the shown average network CIR *difference* decreases with increasing number of malicious links, and follows hereby the clearly understandable trend of decreasing values of $\mathrm{avg}_i CIR_i^{\mathrm{prop}}$ and $\mathrm{avg}_i CIR_i^{\mathrm{comp}}$ due to the decreasing number of usefully communicating links and worsening interference conditions.

D. Scenario for Evaluating Fairness

In the last scenario, we measure CIR differences between the proposed and comparison algorithms for two arbitrarily chosen links (labeled as links "1" and "2") in 10^4 network topologies with 100 uniformly randomly distributed links. Fig. 6 represents both CIR differences $\Delta CIR_{\mathrm{link1}}$ and $\Delta CIR_{\mathrm{link2}}$ measured in each topology by a point with coordinates $[\Delta CIR_{\mathrm{link1}}; \Delta CIR_{\mathrm{link2}}]$. The coordinates of a white cross in Fig. 6 are given by the overall average values of $\Delta CIR_{\mathrm{link1}}$ and $\Delta CIR_{\mathrm{link2}}$ for all 10^4 topologies.

We can see from the distribution of measured differences $\Delta CIR_{\mathrm{link1}}$ and $\Delta CIR_{\mathrm{link2}}$ that the proposed algorithm achieves more overall CIR improvements (black points) than impairments (gray points). This argument is supported by positive coordinates of the white cross, i.e., the average values of CIR differences. The results of the last simulation can be thus understood as a demonstration of the fact that the proposed algorithm improves CIR in a generally fair way.

Observe also that in some topologies the proposed algorithm significantly improves the CIR of one of the two surveyed links by up to $\Delta CIR = 10$. We consider this as a substantial achievement, because all our simulation scenarios consider (at least at some stage) very dense topologies at the stability limit (i.e., $h_{ii} \approx |A_i| \sum_{j \neq i} h_{ij}$ in (11)), which consequently results in relatively high interference with absolute CIR values hardly exceeding 3 dB in the case of both simulated algorithms.

VIII. Conclusion

We studied distributed asynchronous power and rate control for ad hoc networks using general best-response and rate assignment functions from a game-theoretical point of view. Restricting our model by only minimal necessary mathematical assumptions, we showed conditions for global convergence of the power/rate dynamics to Nash equilibria, whereby our analysis is entirely general and application independent. As we assumed network links to have the freedom to choose their own power and rate control rules, the knowledge of the herein derived results can be used for a practical choice of these rules by individual links to guarantee network-wide existence of and convergence to optimally allocated powers and rates. The potential of our approach to provide for satisfactory CIR allocation was also shown, together with a discussion of convergence issues in stochastic channels.

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