Lab02-Divide and Conquer

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2021.

∗ If there is any problem, please contact TA Haolin Zhou.

∗ Name:_____ Student ID:_____ Email: _____

1. *Recurrence examples.* Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for sufficiently small n. Make your bounds as tight as possible.

   (a) $T(n) = 4T(n/3) + n \log n$

   (b) $T(n) = 4T(n/2) + n^2\sqrt{n}$

   (c) $T(n) = T(n-1) + n$

   (d) $T(n) = 2T(\lfloor\sqrt{n}\rfloor) + \log n$

2. *Divide-and-conquer.* Given an integer array $A[1..n]$ and two integers $lower \leq upper$, design an algorithm using **divide-and-conquer** method to count the number of ranges (i, j) $(1 \leq i \leq j \leq n)$ satisfying

$$lower \leq \sum_{k=i}^{j} A[k] \leq upper.$$

   Example:

   Given $A = [1, -1, 2]$, $lower = 1$, $upper = 2$, return 4.

   The resulting four ranges are $(1, 1)$, $(3, 3)$, $(2, 3)$ and $(1, 3)$.

   (a) Complete the implementation in the provided C/C++ source code (The source code *Code-Range.cpp* is attached on the course webpage).

   (b) Write a recurrence for the running time of the algorithm and solve it by recurrence tree (You can modify the figure sources *Fig-RecurrenceTree.vsdx* or *Fig-RecurrenceTree.pptx* to illustrate your derivation).

   (c) Can we use the Master Theorem to solve the recurrence above? Please explain your answer.

3. *Transposition Sorting Network.* A comparison network is a **transposition network** if each comparator connects adjacent lines, as in the network in Fig. 1.

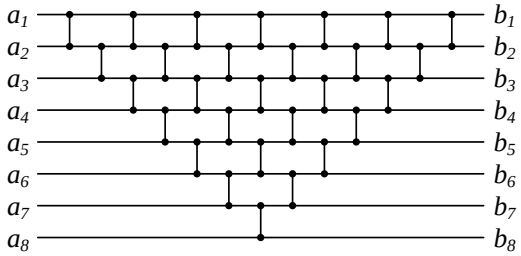

Figure 1: A Transposition Network Example

   (a) Prove that a transposition network with n inputs is a sorting network if and only if it sorts the sequence $\langle n, n-1, \cdots, 1\rangle$. (Hint: Use an induction argument analogous to the *Domain Conversion Lemma*.)

   (b) (Optional Sub-question with Bonus) Given any $n \in \mathbb{N}$, write a program using Tkinter in Python to draw a figure similar to Fig. 1 with n input wires.