Lab05-DynamicProgramming

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2021.

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1. *Optimal Binary Search Tree.* Given a sorted sequence $K = \langle k_1, k_2, \ldots, k_n \rangle$ of n distinct keys, and we wish to build a binary search tree from these keys. For each key k_i, we have a probability p_i that a search will be for k_i. Some searches may be for values not in K, and so we also have $n+1$ *dummy keys* $d_0, d_1, d_2, \ldots, d_n$ representing values not in K. In particular, d_0 represents all values less than k_1, and d_n represents all values greater than k_n. For $i = 1, 2, \ldots, n-1$, the dummy key d_i represents all values between k_i and k_{i+1}. For each dummy key d_i, we have a probability q_i that a search will correspond to d_i. Each key k_i is an internal node, and each dummy key d_i is a leaf. Every search is either successful (finding some key k_i) or unsuccessful (finding some dummy key d_i), and so we have $\sum_{i=1}^{n} p_i + \sum_{i=0}^{n} q_i = 1$.

   (a) Prove that if an optimal binary search tree T (T has the smallest expected search cost) has a subtree T' containing keys $k_i, \ldots, k_j$, then this subtree T' must be optimal as well for the subproblem with keys $k_i, \ldots, k_j$ and dummy keys $d_{i-1}, \ldots, d_j$.

   (b) We define $e[i,j]$ as the expected cost of searching an optimal binary search tree containing the keys $k_i, \ldots, k_j$. Our goal is to compute $e[1,n]$. Write the state transition equation and pseudocode using **dynamic programming** to find the minimum expected cost of a search in a given binary tree. (**Remark**: You may use $w(i,j) = \sum_{l=i}^{j} p_l + \sum_{l=i-1}^{j} q_l$).

   (c) Implement your proposed algorithm in C/C++ and analyze the time complexity. (The framework Code-OBST.cpp is attached on the course webpage). Give the minimum search cost calculated by your algorithm. The test case is given as following:

i	0	1	2	3	4	5	6	7
p_i		0.04	0.06	0.08	0.02	0.10	0.12	0.14
q_i	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05

   (d) Please draw the structure of the optimal binary search tree in the test case, and explain the drawing process.

2. *Dynamic Time Warping Distance.* **DTW** stretches the series along the time axis in a dynamic way over different portions to enable more effective matching. Let $DTW(i,j)$ be the optimal distance between the first i and first j elements of two time series $\bar{X} = (x_1 \ldots x_n)$ and $\bar{Y} = (y_1 \ldots y_m)$, respectively. Note that the two time series are of lengths n and m, which may not be the same. Then, the value of $DTW(i,j)$ is defined recursively as follows:

$$DTW(i,j) = |x_i - y_j| + \min(DTW(i,j-1), DTW(i-1,j), DTW(i-1,j-1))$$

   (a) Implement the proposed DTW algorithm in C/C++ and analyze the time complexity of your implementation. (The framework Code-DTW.cpp is attached on the course webpage). Two test cases have been given in the source code.

   (b) The window constraint imposes a minimum level w of positional alignment between matched elements. The window constraint requires that $DTW(i,j)$ be computed only when $|i-j| \le w$. Modify your code to add a window constraint and give the results of $w = 0$ and $w = 1$ on the two test cases.

Remark: You need to include your .pdf and .tex and 2 source code files in your uploaded .rar or .zip file. Screenshots of test case results are acceptable.