$$\exp\left[-\frac{1}{2}\left(\left(\sum_{i=1}^{n}\phi(x_i-\theta)^2\right)+\tau(\theta-\theta_0)^2\right)\right]$$
$$\propto \exp\left(-\frac{1}{2}(\tau+n\phi)\left(\theta-\frac{1}{\tau+n\phi}\left(\tau\theta_0+\phi\sum_{i=1}^{n}x_i\right)\right)^2\right)$$

Q1 :

$$\exp\left[-\tfrac{1}{2}\left(\left(\sum_{i=1}^{n}\phi(x_i-\theta)^2\right)+\tau(\theta-\theta_0)^2\right)\right]$$

$$= \exp\left[-\tfrac{1}{2}\left(\phi\sum_{i=1}^{n}(x_i^2-2x_i\theta+\theta^2)+\tau(\theta^2-2\theta\theta_0+\theta_0^2)\right)\right]$$

only focus on θ

$$\propto \exp\left[-\tfrac{1}{2}\left(\phi n\theta^2-2\phi\theta\sum_{i=1}^{n}x_i+\tau\theta^2-2\theta\tau\theta_0\right)\right]$$

$$= \exp\left[-\tfrac{1}{2}\left(\theta^2(\tau+\phi n)-2\theta\left(\phi\sum_{i=1}^{n}x_i+\tau\theta_0\right)\right)\right]$$

$$= \exp\left[-\tfrac{1}{2}(\tau+\phi n)\left(\theta^2-2\theta\,\frac{\phi\sum_{i=1}^{n}x_i+\tau\theta_0}{\tau+\phi n}\right)\right]$$

$$\propto \exp\left[-\tfrac{1}{2}(\tau+\phi n)\left(\theta^2-2\theta\,\frac{\phi\sum_{i=1}^{n}x_i+\tau\theta_0}{\tau+\phi n}+\left(\frac{\phi\sum_{i=1}^{n}x_i+\tau\theta_0}{\tau+\phi n}\right)^2\right)\right]$$

Due to $\dfrac{\phi\sum_{i=1}^{n}x_i+\tau\theta_0}{\tau+\phi n}$ has no θ which is constant.

$$= \exp\left[-\tfrac{1}{2}(\tau+\phi n)\left(\theta-\frac{\phi\sum_{i=1}^{n}x_i+\tau\theta_0}{\tau+\phi n}\right)^2\right]$$