Homework 4 Iterative Methods

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a)

The answer is this problem is well posed. The reason is compatibility condition in Neumannn conditions is required focing f and the boundary condition need to fuffill specific compatibility conditions. Physically, it expresses that flux through the boundary needs to be compensated with a source term inside the domain. For this question, the dp/dn=0, so the integral of force should also be 0. For this homework, I calculated sum of f, the ans is 1.7e-15, which is nearly equal to 0, so it fulfill the compatibility condition.

b)

For this question, the steps are below:

1: Discretized the Laplace equation in all boundaries.

2: Insert the discretization equation in Gauss-Seidel method

3: Implemented to Successive Over-Relaxation method.

Then got the answer.

for i=1

$$p_{1,1}^{(m+1)} = (1-\omega)p_{1,1}^{(m)} + \frac{\omega}{1+\beta^2}\left[p_{2,1}^{(m)} + \beta^2 p_{1,2}^{(m)} - h_x^2 f_{1,1}\right]$$

$$p_{1,j}^{(m+1)} = (1-\omega)p_{1,j}^{(m)} + \frac{\omega}{2(1+\beta^2)-1}\left[p_{2,j}^{(m)} + \beta^2 p_{1,j+1}^{(m)} + \beta^2 p_{1,j-1}^{(m+1)} - h_x^2 f_{1,j}\right](j = 2, N_y - 1)$$

$$p_{1,N_y}^{(m+1)} = (1-\omega)p_{1,N_y}^{(m)} + \frac{\omega}{1+\beta^2}\left[p_{2,N_y}^{(m)} + \beta^2 p_{1,N_y-1}^{(m+1)} - h_x^2 f_{1,N_y}\right]$$

For i=N_x

$$p_{Nx,1}^{(m+1)} = (1-\omega)p_{Nx,1}^{(m)} + \frac{\omega}{1+\beta^2}\left[p_{Nx-1,1}^{(m+1)} + \beta^2 p_{Nx,2}^{(m)} - h_x^2 f_{Nx,1}\right]$$

$$p_{Nx,j}^{(m+1)} = (1-\omega)p_{Nx,j}^{(m)} + \frac{\omega}{2(1+\beta^2)-1}\left[p_{Nx-1,j}^{(m+1)} + \beta^2 p_{Nx,j+1}^{(m)} + \beta^2 p_{Nx,j-1}^{(m+1)} - h_x^2 f_{1,j}\right](j = 2, N_y - 1)$$

$$p_{Nx,N_y}^{(m+1)} = (1-\omega)p_{Nx,N_y}^{(m)} + \frac{\omega}{1+\beta^2}\left[p_{Nx-1,N_y}^{(m+1)} + \beta^2 p_{Nx,N_y-1}^{(m+1)} - h_x^2 f_{Nx,N_y}\right]$$

For j=1

$$p_{1,1}^{(m+1)} = (1-\omega)p_{1,1}^{(m)} + \frac{\omega}{1+\beta^2}\left[p_{2,1}^{(m)} + \beta^2 p_{1,2}^{(m)} - h_y^2 f_{1,1}\right]$$

$$p_{i,1}^{(m+1)} = (1-\omega)p_{i,1}^{(m)} + \frac{\omega}{2(1+\beta^2)-\beta^2}\left[p_{i-1,1}^{(m+1)} + p_{i+1,1}^{(m)} + \beta^2 p_{i,2}^{(m)} - h_y^2 f_{i,1}\right](i = 2, N_x - 1)$$

$$p_{N_x,1}^{(m+1)} = (1-\omega)p_{N_x,1}^{(m)} + \frac{\omega}{1+\beta^2}\left[p_{Nx-1,1}^{(m+1)} + \beta^2 p_{Nx,2}^{(m)} - h_y^2 f_{Nx,1}\right]$$

$j=N_y$

$$p_{1,N_y}^{(m+1)} = (1-\omega)p_{1,N_y}^{(m)} + \frac{\omega}{1+\beta^2}\left[p_{2,N_y}^{(m)} + \beta^2 p_{1,N_y-1}^{(m+1)} - h_x^2 f_{1,N_y} \right]$$

$$p_{i,N_y}^{(m+1)} = (1-\omega)p_{i,N_y}^{(m)} + \frac{\omega}{2(1+\beta^2)-\beta^2}\left[p_{i+1,N_y}^{(m)} + p_{i-1,N_y}^{(m+1)} + \beta^2 p_{i,N_y-1}^{(m+1)} - h_x^2 f_{i,N_y} \right](i=2,N_x-1)$$

$$p_{Nx,N_y}^{(m+1)} = (1-\omega)p_{Nx,N_y}^{(m)} + \frac{\omega}{1+\beta^2}\left[p_{Nx-1,N_y}^{(m+1)} + \beta^2 p_{Nx,N_y-1}^{(m+1)} - h_x^2 f_{Nx,N_y} \right]$$

c)

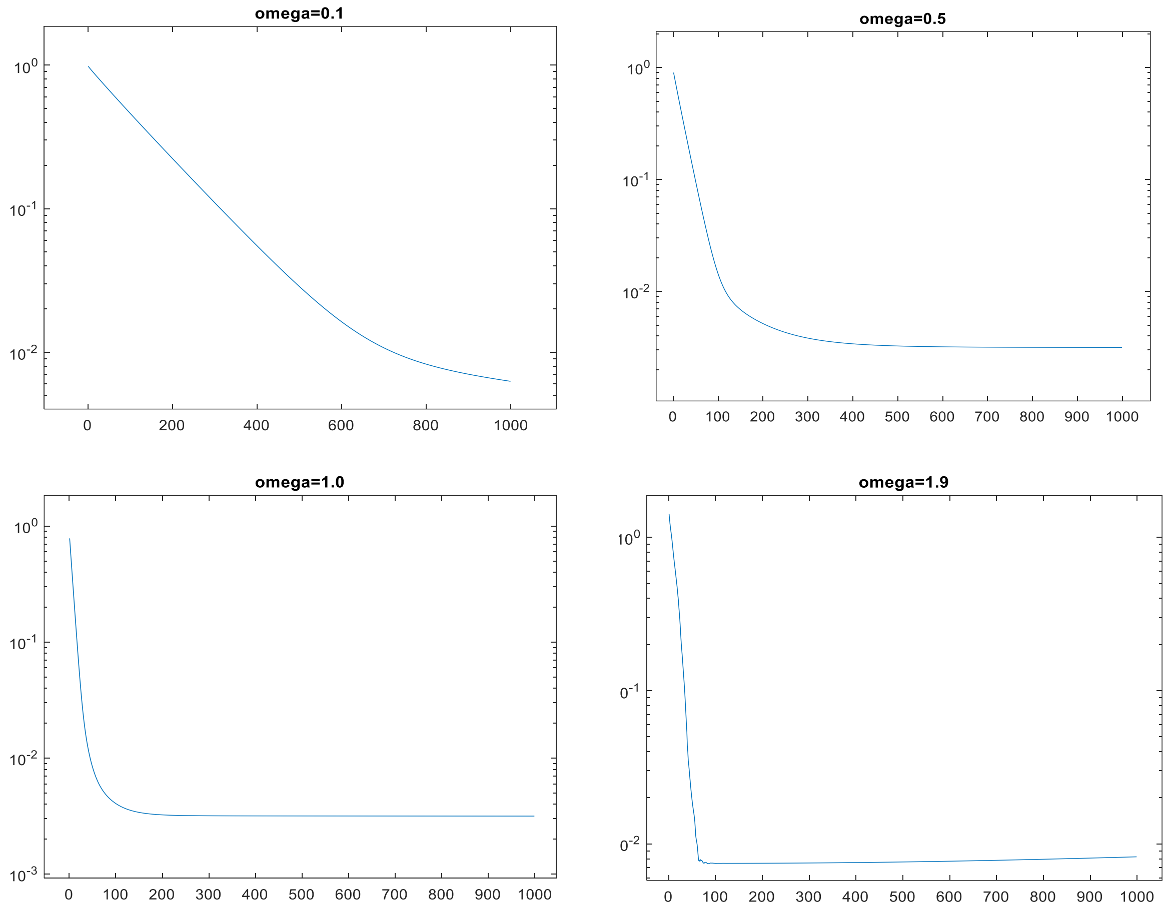

Figure 1 Relative residual with interaction number m

As shown in figure 1, the relative residual are all decreased. While for the omega=1.9, the convergence is the fastest and omega=0.1 is the slowest to convergence. When omega is 1.0, it is the GS method, which is slower than SOR method when omega larger than 1. So, the SOR could accelerate convergence.

d)

As shown in figure 2, is the different Nx=Ny=N, from 20 to 100. As can be seen, the coarse mesh, N=20 could accelerate convergence. While the fine mesh, N=100 induce slow convergence, the reason is the error reduction is related to h^2, large N cause small h so the convergence is slow.

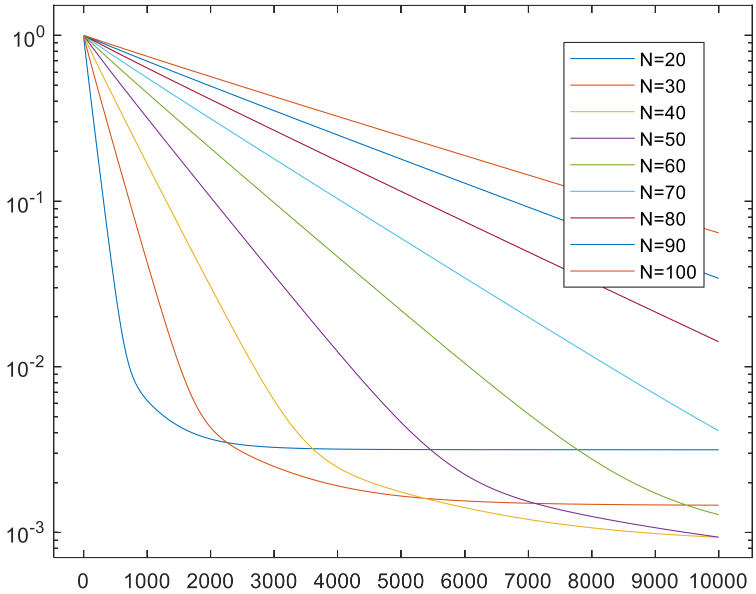

Figure 2 Different Nx and Ny induced relative residual

e)

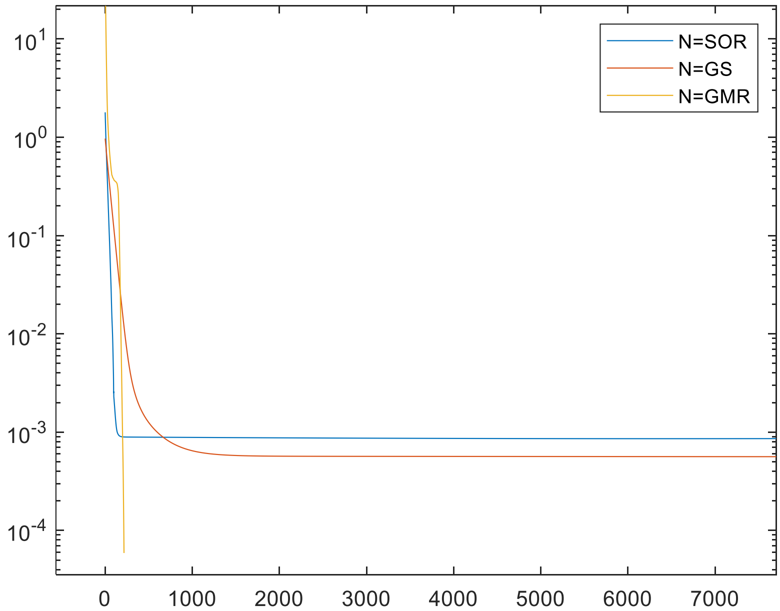

Figure 3 Comparisons of three different interation methods

As shown in figure 3, the comparisons of GS, SOR and GMR. It could be seen that when omega is 1.9, the SOR is the fastest method to convergence while the relative residual is higher than GMR. The GMR is also fast, nearly the same with SOR, but it has a very high accuracy, the relative residual is smallest. The GS is the slowest one to convergence, also has a high relative residual.