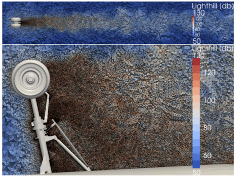
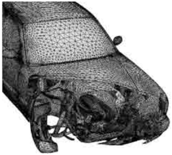
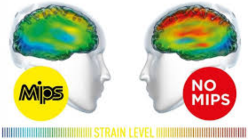
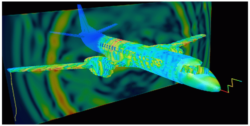
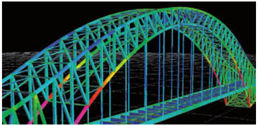
Example of applications using FEM











Finite Element Method

- A numerical method for computing approximate solutions to differential equations.
- Strengths. It can handle:
  - Complex geometries
  - General boundary conditions
  - Variable, rough and non-linear coefficients
  - Strong theoretical foundation

About the course

- The Finite Element Method (SF2561 and FSF3561),
- Teachers
  - Lectures:
  - Exercise:
- 7.5 hp, which splits into

  4.5 hp:
  - Theoretical homework sets - Problem Set A&B;
  - Practical homework sets - Lab A (for everyone) and Lab B (only compulsory for FSF3561-students!).

  3.0 hp:
  - Final examination -

About the course

- Finite element method in 1D, 2D, and 3D.
  - Poisson problems with Dirichlet and Neumann boundary conditions.
  - Time-dependent parabolic problems.

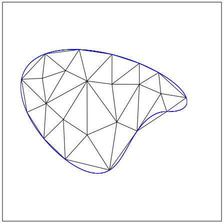
- Both theoretical and practical aspects will be covered:

  Theoretical:
  - Well-posedness, existence & uniqueness, convergence, etc.

  Practical:
  - Implementation, mesh generation, etc.

Important concepts

- Geometry
  - Computational domain
  - Mesh: a discrete representation of the domain consisting of geometric objects such as, e.g interval (1D), triangle (2D), tetrahedron (3D), quadrilateral (2D), brick (3D).



- Approximation
  - Finite dimensional function space - Piecewise polynomials
  - Basis functions

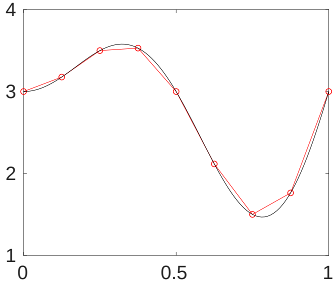
Approximation of functions - Interpolation

Interval: $I = [0, 1]$.

Mesh: uniform mesh with 9 nodes $x_j = \frac{j}{8}, \quad j = 0, 1, \cdots, 8$.

Function: $f(x) = 2x\sin(2\pi x) + 3$.

Interpolation: continuous piecewise linear interpolation of $f(x)$ on the mesh.



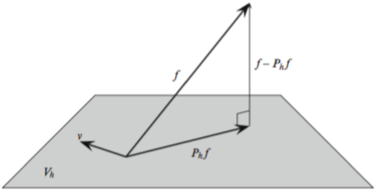
Approximation of functions - Projection

Interval: $I = [0, 1]$.

V_h: space of continuous piecewise linear functions on the mesh.

$P_h f$: the L^2-projection of f onto V_h.

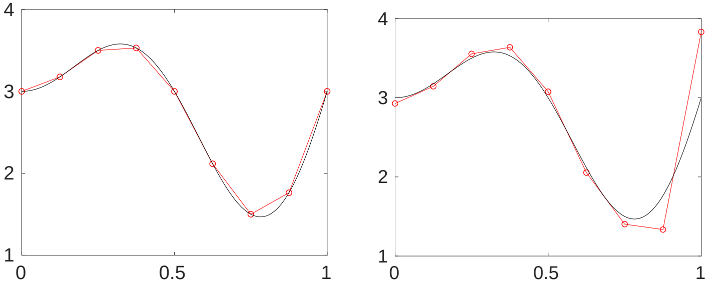
$$\int_I (f - P_h f)\, v\, dx = 0 \quad \forall v \in V_h$$



Approximation of a function

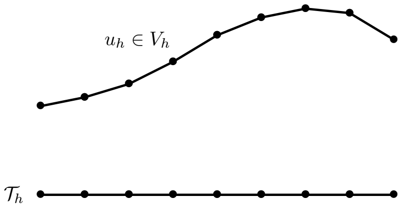
Read chapter 1 in the course book

- Interpolation: the interpolant approximates f exactly at the nodes.
- Projection: the L^2-projection approximates f "on average".



FEM in 1D

The basics.



- Divide domain into nodes and elements to create a mesh.
- Seek approximate solution to given differential equation in finite dimensional function space.
- Use appropriate basis for the finite dimensional space.
- Rewrite the differential equation as simple algebraic equation.
- The algebraic equation is solved in a computer program and gives an approximate solution to the differential equation.