Homework 0

Due Aug 29, 2014
in lecture and SVN
Instructions for submission into your
class SVN repository are on the webpage.

The purpose of this assignment is to give you a chance to refresh the math skills we expect you to have learned in prior classes. These particular skills will be essential to mastery of CS225, and we are unlikely to take much class time reminding you how to solve similar problems. Though you are not required to work independently on this assignment, we encourage you to do so because we think it may help you diagnose and remedy some things you might otherwise find difficult later on in the course. If this homework is difficult, please consider completing the discrete math requirement (CS173 or MATH 213) before taking CS225.

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Section (circle one): AYB Wed 7-9, AYC Thurs 9-11, AYD Thurs 11-1, AYE Thurs 1-3, AYF Thurs 3-5, AYG Thurs 5-7, AYH Thurs 7-9, AYI Fri 9-11, AYJ Fri 11-1, AYK Fri 1-3, AYL Fri 3-5, AYM Fri 5-7

Laptop Sections: AYN Thur 11-1, AYO Thur 2-4, AYP Thur 4-6, AYQ Fri 1-3, AYR Fri 3-5.

Grade		Out of 60
Grader		

1. (3 points) Using 140 characters or less, post a synopsis of your favorite movie to the course piazza space under the "HW0 tell me something!" notice, so that your post is visible to everyone in the class, and tagged by #HW0num1. Also, use Piazza's code-formatting tools to write a *private* post to course staff that includes at least 5 lines of code. It can be code of your own or from a favorite project–it doesn't even have to be syntactically correct–but it must be formatted as a code block in your post, and also include the tag #HW0num1. (Hint: Check https://piazza.com/product/features). Finally, please write the 2 post numbers corresponding to your posts here:

Favorite Movie Post (Public) number:	@28
Formatted Code Post (Private) number:	@32

2. (12 points) Simplify the following expressions as much as possible, **without using an calculator (either hardware or software)**. Do not approximate. Express all rational numbers as improper fractions. Show your work in the space provided, and write your answer in the box provided.

(a) $\prod_{k=2}^{n}(1-\frac{1}{k^2})$

$$= (1-\frac{1}{4})(1-\frac{1}{9})(1-\frac{1}{16}) \cdots (1-\frac{1}{n^2})$$

$$= \frac{3}{4} \cdot \frac{8}{9} \cdot \frac{15}{16} \cdots \frac{n^2}{n^2}$$

$$= \frac{1\times\cancel{3}}{2\times\cancel{2}} \cdot \frac{\cancel{2}\times\cancel{4}}{3\times\cancel{3}} \cdot \frac{\cancel{3}\times 5}{4\times\cancel{4}} \cdots \frac{(\cancel{n-1})(n+1)}{n\times n}$$

$$= \frac{1}{2} \cdot \frac{n+1}{n} = \frac{n+1}{2n}$$

Answer for (a):	$\frac{n+1}{2n}$

(b) 3^{1000} mod 7

$$3^{1000} \bmod 7 = (3^2)^{500} \bmod 7 = 9^{500} \bmod 7 = (9 \bmod 7)^{500} \bmod 7$$

$$= 2^{500} \bmod 7 = (2^5)^{100} \bmod 7 = (32 \bmod 7)^{100} \bmod 7$$

$$= 4^{100} \bmod 7 = (16 \bmod 7)^{50} \bmod 7 = 2^{50} \bmod 7 = \cancel{4}(32 \bmod 7)^{10} \bmod 7$$

$$= 4^{10} \bmod 7 = (16 \bmod 7)^5 \bmod 7 = 32 \bmod 7 = 4$$

Answer for (b):	4

(c) $\sum_{r=1}^{\infty}(\frac{1}{2})^r$

$= \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^\infty}$

$= \frac{\frac{1}{2}(1-\frac{1}{2^\infty})}{1-\frac{1}{2}} = 1-\frac{1}{2^\infty} \approx 1$

Answer for (c):	$1-\frac{1}{2^\infty} \approx 1$

(d) $\frac{\log_7 81}{\log_7 9}$

$= \frac{\log_7 9^2}{\log_7 9} = \frac{2\log_7 9}{\log_7 9} = 2$

Answer for (d):	2

(e) $\log_2 4^{2n}$

$= \log_2 (2)^{2 \cdot 2n}$

$= \log_2 2^{4n}$

$= 4n \log_2 2$

$= 4n$

Answer for (e):	$4n$

(f) $\log_{17} 221 - \log_{17} 13$

$\log_{17} 221 - \log_{17} 13$

$= \log_{17} \frac{221}{13}$

$= \log_{17} 17 = 1$

Answer for (f):	1

3. (8 points) Find the formula for $1 + \sum_{j=1}^{n} j!j$, and show work proving the formula is correct using induction.

Formula:	$1 + \sum_{j=1}^{n} j!j = (n+1)!$

$S(n) = 1 + \sum_{j=1}^{n} j!j$

$S(1) = 1 + 1! \cdot 1 = 2 = 2!$

$S(2) = 1 + 2! \cdot 2 + 1! \cdot 1 = 6 = 3!$

$S(3) = 1 + 3! \cdot 3 + 2! \cdot 2 + 1! \cdot 1 = 24 = 4!$

$\therefore$ guess $S(n) = \cancel{\#}(n+1)!$

for $n=1$. $S(1) = 2$.

suppose $S(n) = (n+1)!$ is true.

$S(n+1) = 1 + \sum_{j=1}^{n} (n+1)! \cdot (n+1)$

$= \cancel{1+}\ S(n) + (n+1)!(n+1)$

$= (n+1)! + (n+1)!(n+1)$

$= (n+1)! \cdot (n+2) = (n+2)!$ $\therefore$ it is true for $n+1$. $\therefore$ it is true for all n.

4. (8 points) Indicate for each of the following pairs of expressions $(f(n), g(n))$, whether $f(n)$ is O, Ω, or Θ of $g(n)$. Prove your answers to the first two items, but just GIVE an answer to the last two.

(a) $f(n) = 4^{\log_4 n}$ and $g(n) = 2n + 1$.

$f(n) = 4^{\log_4 n} = n$

~~$O(g(n)) = n$~~

~~$\therefore f(n) = O(g(n))$~~

$\frac{f(n)}{g(n)} = \frac{n}{2n+1} = \frac{n + \frac{1}{2} - \frac{1}{2}}{2n+1}$

$= \frac{1}{2} - \frac{1}{4n+2} \leq \frac{1}{2}$

As $\frac{f(n)}{g(n)} \leq \frac{1}{2}$

As $f(n) \geq C_2 g(n)$.

~~Assume~~ $C = \frac{1}{4}$

$f(n) \geq \frac{1}{4} g(n)$.

$\therefore f(n)$ is $\Theta(g(n))$

Answer for (a):	$f(n) = \cancel{\#}\ \Theta(g(n))$

(b) $f(n) = n^2$ and $g(n) = (\sqrt{2})^{\log_2 n}$.

$g(n) = (\sqrt{2})^{\log_2 n}$

$= (2^{\frac{1}{2}})^{\log_2 n}$

$= (2^{\log_2 n})^{\frac{1}{2}}$

$= n^{\frac{1}{2}}$

$\frac{f(n)}{g(n)} = \frac{n^2}{n^{\frac{1}{2}}} = n^{\frac{3}{2}}$

As $n > 0$

$\therefore \frac{f(n)}{g(n)} > 0$

$\therefore f(n)$ is $\Omega(g(n))$

Answer for (b):	$f(n) = \Omega(g(n))$

(c) $f(n) = \log_2 n!$ and $g(n) = n \log_2 n$.

Answer for (c):	$f(n)$	$\Theta(g(n))$

(d) $f(n) = n^k$ and $g(n) = c^n$ where k, c are constants and c is >1.

Answer for (d):	$f(n)$	$O(g(n))$

5. (9 points) Solve the following recurrence relations for integer n. If no solution exists, please explain the result.

(a) $T(n) = T(\frac{n}{2}) + 5$, $T(1) = 1$, assume n is a power of 2.

$T(1) = 1$
$T(2) = T(1) + 5 = 6$
$T(4) = T(2) + 5 = 11$
$T(8) = T(4) + 5 = 16$
$\vdots$

$T(n) = T(\frac{n}{2}) + 5$
$= T(\frac{n}{4}) + 5 + 5$
$= T(\frac{n}{8}) + 5 + 5 + 5$
$= T(\frac{n}{2^k}) + k \cdot 5$

As $n \to 2n$
$T(n) \to T(n) + 5$.
$\therefore$ ~~$T(n) = 1 + 5 \cdot \log_2 n$~~
when $k = \log_2 n$. ~~reach~~ reach 1
$\therefore T(n) = 1 + 5 \log_2 n$.

$T(n) = 1 + 5 \cdot \log_2 n$

Answer for (a):	~~$T(n) = 1 + 5 \cdot \log$~~

(b) $T(n) = T(n-1) + \frac{1}{n}$, $T(0) = 0$.

$T(1) = T(0) + 1 = 0 + 1 = 1$
$T(2) = T(1) + \frac{1}{2} = 1 + \frac{1}{2} = \frac{3}{2}$
$T(3) = T(2) + \frac{1}{3} = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$
$\vdots$
$T(n) = T(n-1) + \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$

$T(n) = T(n-1) + \frac{1}{n}$
$= T(n-2) + \frac{1}{n} + \frac{1}{n-1}$
$= T(n-3) + \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2}$
when $n = 0$ $T(0) = 0$.
$T(n) = \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \cdots + 1$

Answer for (b):	no solution.

$\therefore T(n) = \sum_{k=1}^{n} \frac{1}{k} \to \ln(n) + C$
it does not have a solution.

when $n=1$
$T(1)=1$

(c) Prove that your answer to part (a) is correct using induction.

Suppose $T(n)=1+5\cdot \ln_2 n$ is true when $n=k$

$\therefore T(k)=1+5\cdot \ln_2 k$

for ~~n=k+1~~ $n=2k$

$T(\cancel{k+1}\ 2k) = T(k)+5$

$= 1+5\cdot \ln_2 k + 5$

$= 1+5(\ln_2 k + 1)$

$= 1+5(\ln_2 k + \ln_2 2)$

$= 1+5\ln_2 2k$

$\therefore$ it is ~~true~~ also true for $n=2k$

6. (10 points) Suppose function call parameter passing costs constant time, independent of the size of the structure being passed.

(a) Give a recurrence for worst case running time of the recursive Binary Search function in terms of n, the size of the search array. Assume n is a power of 2. Solve the recurrence.

$T(n) = T(\cancel{n-1}\ \frac{n}{2}) + d$

$= T(\frac{n}{4}) + 2d$

$= T(\frac{n}{8}) + 3d.$

$= T(\frac{n}{2^k}) + kd.$

reach base case when $k=\log_2 n.$

$\therefore T(n) = C + (\log_2 n)\cdot d$

Recurrence:	$T(n)=T(\frac{n}{2})+d$
Base case:	$T(1)=C$
Recurrence Solution:	$T(n)= C+ \cancel{dn}\,(\log_2 n)\cdot d$

(b) Give a recurrence for worst case running time of the recursive Merge Sort function in terms of n, the size of the array being sorted. Solve the recurrence.

$T(n) = 2T(\frac{n}{2}) + dn$

$\cancel{= 2T(\frac{n}{4}) +}$

$= 2(2T(\frac{n}{4}) + \frac{n}{2}) + dn.$

$= 4T(\frac{n}{4}) + 2dn$

$= 8T(\frac{n}{8}) + 3dn$

$= 2^k T(\frac{n}{2^k}) + kn\cdot d$

when $k=\log_2 n.$

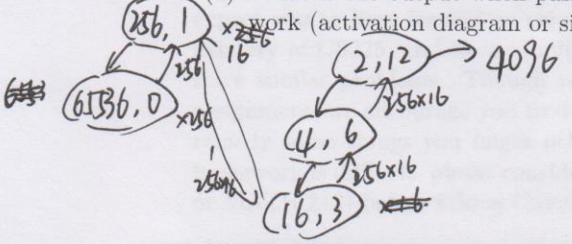
$T(n) = n\cdot C + \log_2 n \cdot dn.$

Recurrence:	$T(n)=2T(\frac{n}{2})+dn$
Base case:	$T(1)=C$
Running Time:	~~$T(n)=n\cdot C+(n-1)dn.$~~ $T(n)=n\cdot C + dn\cdot \log_2 n$

7. (10 points) Consider the pseudocode function below.

```
derp( x, n )
      if( n == 0 )
            return 1;
      if( n % 2 == 0 )
            return derp( x^2, n/2 );
      return x * derp( x^2, (n - 1) / 2);
```

(a) What is the output when passed the following parameters: $x = 2$, $n = 12$. Show your work (activation diagram or similar).



Answer for (a):	4096

(b) Briefly describe what this function is doing.

This function calculate the result of x^n

(c) Write a recurrence that models the running time of this function. Assume checks, returns, and arithmetic are constant time, but be sure to evaluate all function calls. [Hint: what is the *most* n could be at each level of the recurrence?]

$T(n) = T(n/2) + d.$

$T(1) = C$

(d) Solve the above recurrence for the running time of this function.

$T(n) = T(n/2) + d$

$= T(n/4) + 2d$

$= T(n/8) + 3d$

$= T(n/2^k) + kd.$

when $k = \log_2 n$ reach base case

$\therefore T(n) = C + (\log_2 n) d$