CS-E5740 Complex Networks, Answers to exercise set 4

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Problem 1

a) Use the branching process, and also the fact that $\langle k \rangle = \frac{\langle k^2 \rangle}{\langle k \rangle} - 1$, we can derive

$$\begin{aligned} n_d &= \langle q \rangle * n_{d-1} \\ &= (\frac{\langle k^2 \rangle}{k} - 1) * n_{d-1} \\ &= (\frac{\langle k \rangle^2 + \langle k \rangle}{k} - 1) * n_{d-1} \\ &= \langle k \rangle * n_{d-1} \end{aligned}$$

Use the equation above recursively we can get

$$n_{d-1} = \langle k \rangle * n_{d-2}$$

$$\vdots$$

$$n_2 = \langle k \rangle * n_1$$

And finally $n_d = \langle k \rangle^d$

The above equation shows that for ER networks $\langle q \rangle = \frac{\langle k^2 \rangle}{k} - 1 = \langle k \rangle$, use the fact that giant component exits if $\langle k^2 \rangle / \langle k \rangle > 2$, it's obvious that that the giant component appears in large and sparse ER network when $\langle k \rangle > 1$.

b) The results for $\langle k \rangle = 0.5$are shown in Figure 1 and 2. The results for $\langle k \rangle = 1$are shown in Figure 3 and 4. The results for $\langle k \rangle = 2$are shown in Figure 5 and 6.

c) When $\langle k \rangle = 2$, we can notice that both cases when $N = 100000$ or $N = 10000$ the simulated average node count will not grow as fast as the theoretical result. This is mainly because there are some loops occur when the depth increases as we can see in the figures. Once some loops are created, some of the nodes inside the loops will not be active in the branching process so the network will not grown as fast as the theory expect.

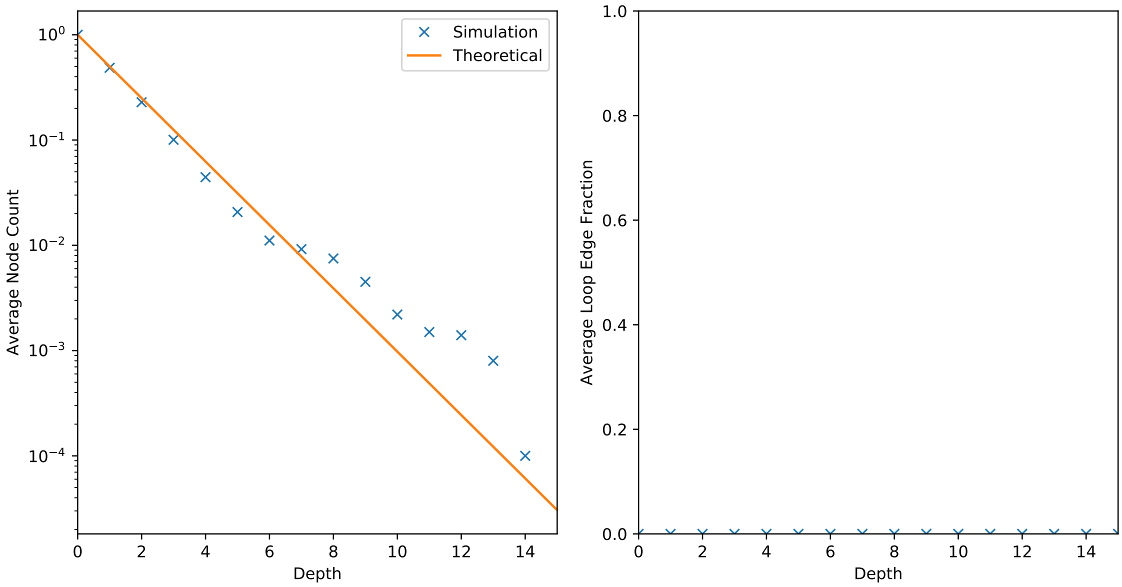

Figure 1: Average node count and average loop edge fraction with $N = 10k$ and $\langle k \rangle = 0.5$

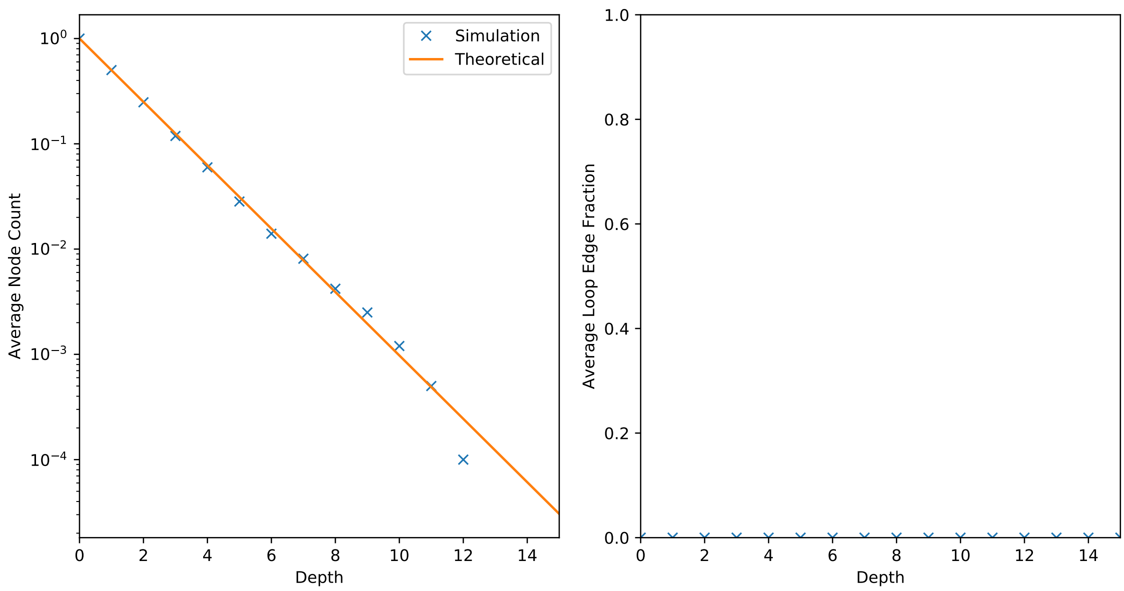

Figure 2: Average node count and average loop edge fraction with $N = 100k$ and $\langle k \rangle = 0.5$

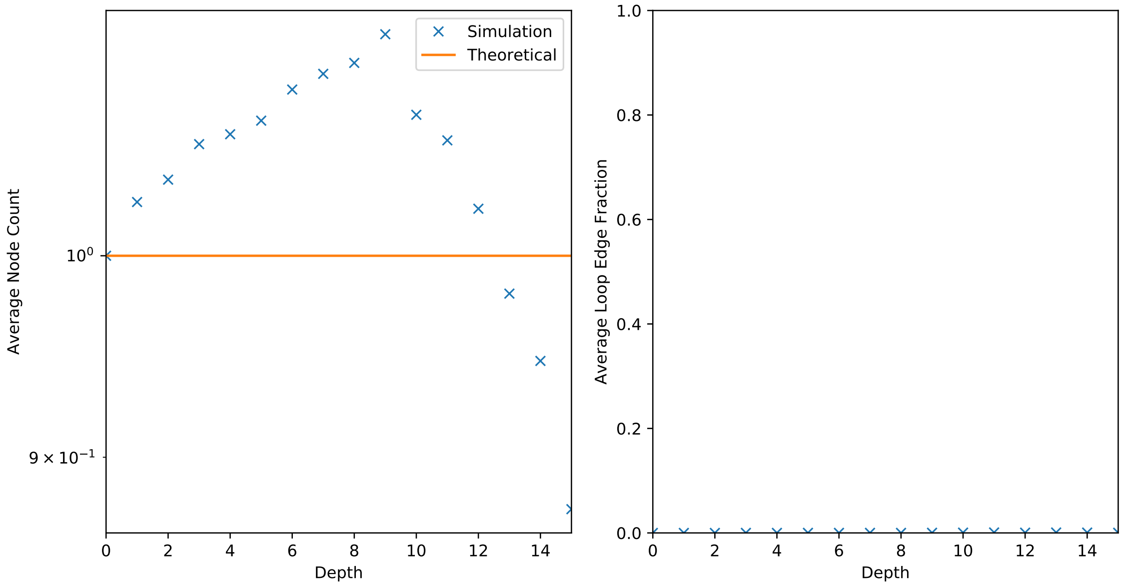

Figure 3: Average node count and average loop edge fraction with $N = 10k$ and $\langle k \rangle = 1$

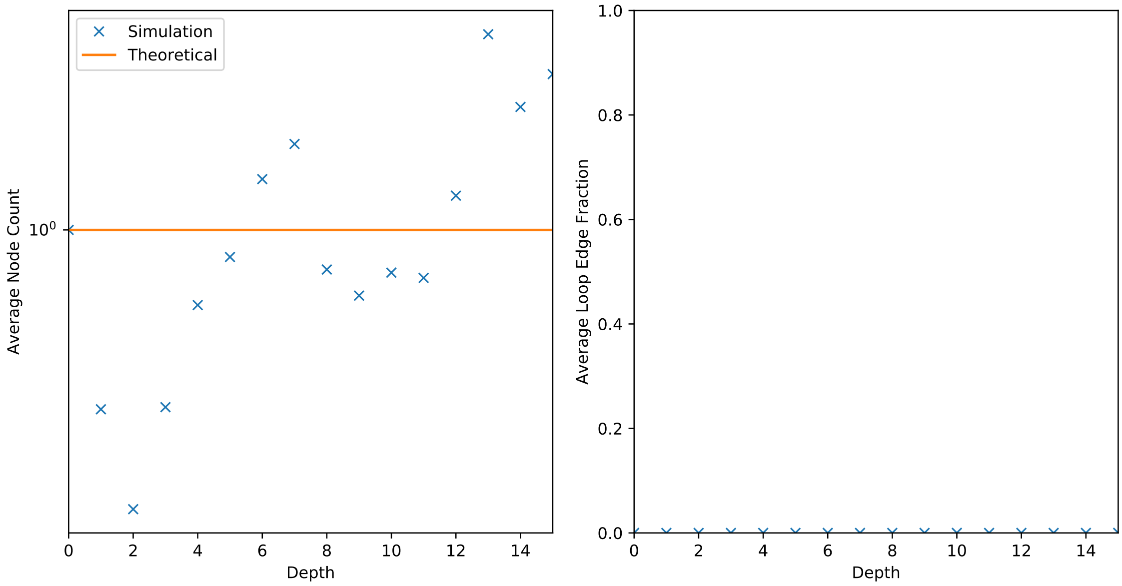

Figure 4: Average node count and average loop edge fraction with $N = 100k$ and $\langle k \rangle = 1$

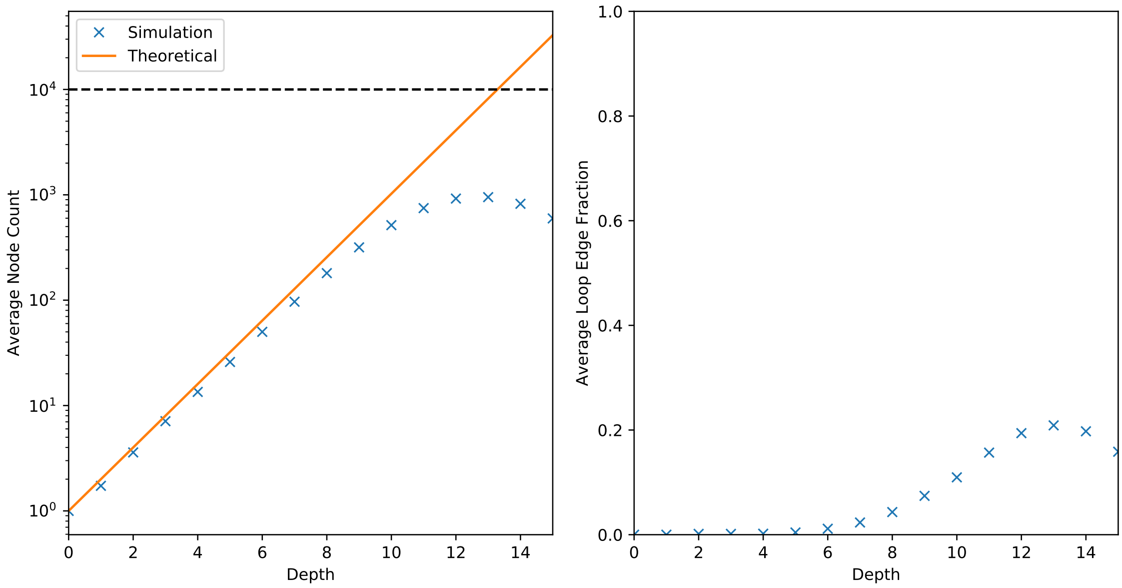

Figure 5: Average node count and average loop edge fraction with $N = 10k$ and $\langle k \rangle = 2$

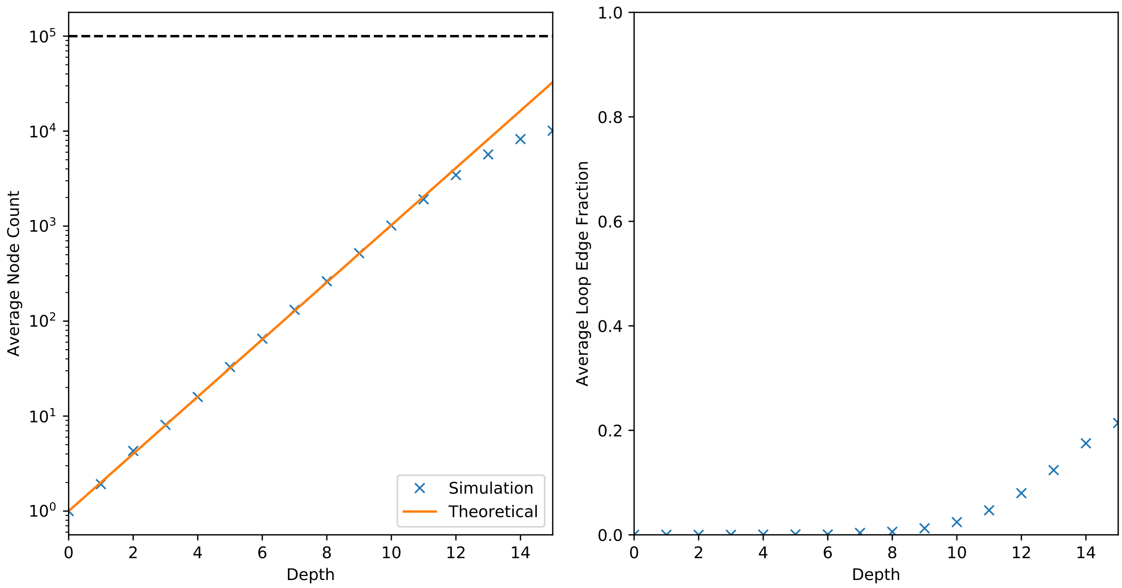

Figure 6: Average node count and average loop edge fraction with $N = 100k$ and $\langle k \rangle = 2$

d) The result is shown in Figure **??**.

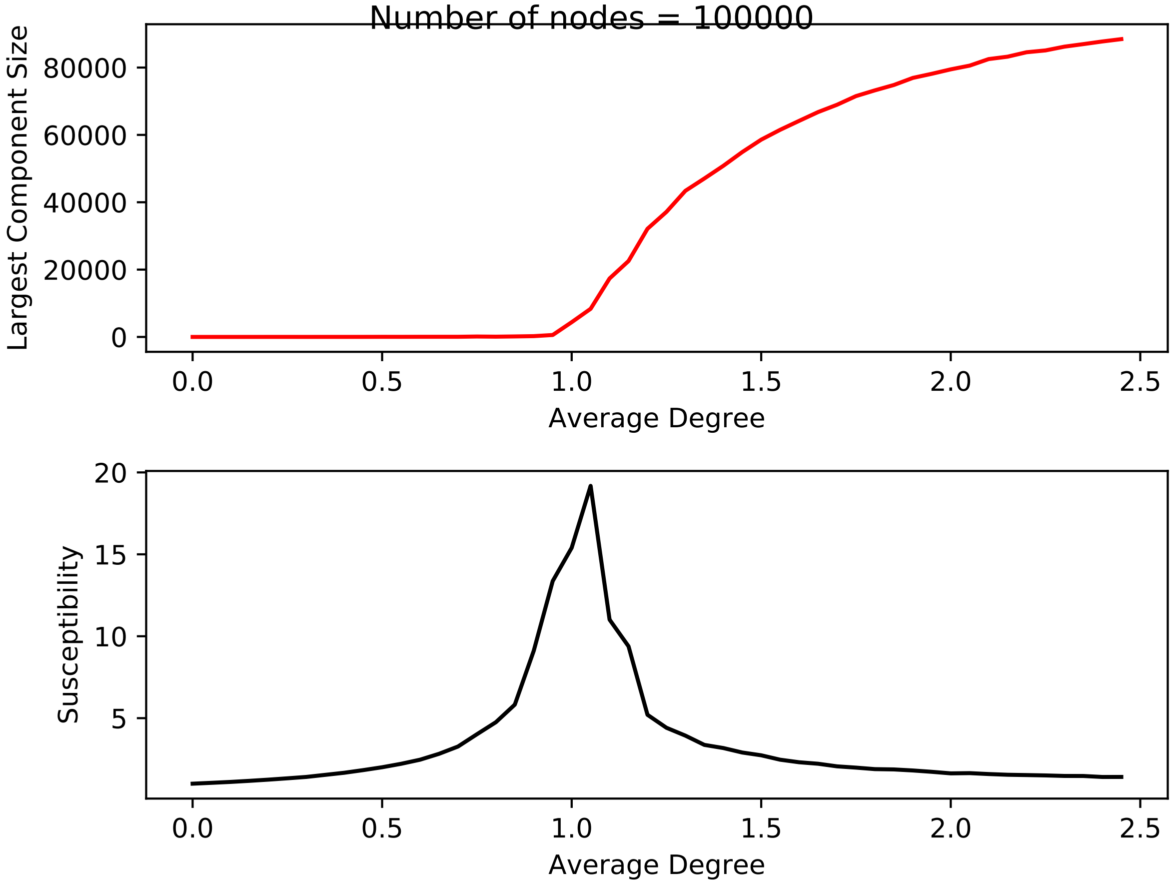

Figure 7: Largest Component Size and Susceptibility with different $\langle k \rangle$

e) The visualization of Susceptibility is also shown in Figure **??**. We can observe that in both graph, when $\langle k \rangle$ approaches to 1, there's a sudden increase. As the definition of Susceptibility goes, this exactly shows that percolation transition happens when average degree $\langle k \rangle \approx 1$

Problem 2

a) The comparison is hown in Figure 8

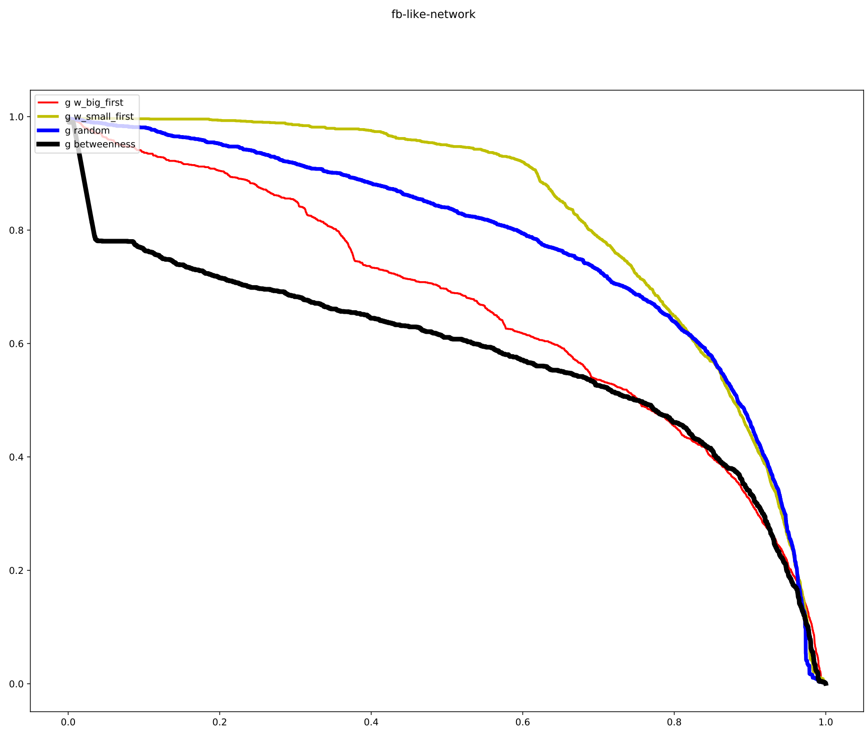

Figure 8: Comparison between four strategies

b) Most vulnerable: descending order of edge betweenness centrality
It's beccause that betweenness centrality measure the importance of edges in a given network. It mainly assign higher values to the edges between clusters. In other words, removing these edges with high betweenness centrality value instead of other 3 strategies will have high probability to break up large components in the network.
Least vulnerable: ascending link weight

c) The stronger links are more important for the integrity because these links most likely occur in large components in the network. So cutting down strong links are more likely to break up large components. On the contrary, removing weak links won't have that much influence since by doing that only limited nodes are affected.

d) Random removal removes edges randomly so the effectiveness of this strategy is pretty poor compared to removal in descending order of edge betweenness strategy.
Removal in descending order of edge betweenness strategy is deterministic instead of random. It removes the edges with larger betweenness value first, which means to remove the important edges first.