CS-E5740 Complex Networks, Answers to exercise set 2

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September 28, 2019

Problem 1

a)

Ensemble i	Probability	Average Degree	Average Cluster Coefficient	Diameter
1	$(1-p)^3$	0	0	0
2	$p(1-p)^2$	2/3	0	1
3	$p(1-p)^2$	2/3	0	1
4	$p(1-p)^2$	2/3	0	1
5	$p^2(1-p)$	4/3	0	2
6	$p^2(1-p)$	4/3	0	2
7	$p^2(1-p)$	4/3	0	2
8	p^3	2	1	1

Now we can use the derived data on the table to calculate the properties we want to know.

- **Average Degree**
  Using $p = 1/3$
  Then $< k >= \sum_1^N \pi_i < k_i >= 2/3$
- **Average Cluster Coefficient**
  Using $p = 1/3$
  Then $< c >= \sum_1^N \pi_i < c_i >= 1/27$
- **Average Diameter**
  Using $p = 1/3$
  Then $< d^* >= \sum_1^N \pi_i < d*_i >= 25/27$

b) Still we can use the table from a) to calculate.

Average Degree
Then $< k >= \sum_1^N \pi_i < k_i >= p(1-p)^2 * 3 * 2/3 + (1-p)p^2 * 3 * 4/3 + 2 * p^3 = 2p$

- **Average Cluster Coefficient**
  Then $< c >= \sum_1^N \pi_i < c_i >= p^3$

- **Average Diameter**
  Then $< d^* >= \sum_1^N \pi_i < d_i^* >= 1 * (1-p)^2 p * 3 + 2 * (1-p)p^2 + 1 * p^3 = 3p - 2p^3$

Problem 2

a) - **First Factor**
   Each node can have maximum $N-1$ links, thus it can be abstracted as having $N-1$ independent trails where each trail has the probability of success p.
   - **Second Factor**
   If there are k successes (links) in $N-1$ trails, then p^k is the probability of having k successes.
   - **Third Factor**
   Since there are k successes (links) in $N-1$ trails, then there are $N-1-k$ fails, and $(1-p)^{N-1-k}$ is the probability of having $N-1-k$ successes.
   Multiplying the second factor and the third factor together formulates the probability of having one node that has k links.

b) The definition of the Clustering Coefficient of a node is:

$$\frac{\#edges\,between\,its\,neighbors}{\#possible\,edges\,between\,its\,neighbors}$$

This actually represent the probability of having an edge between a node's neighbors, in other words, having a link, which is exactly p.

c) Actually the clustering coefficient of a node only depends on the local connectivity of its neighbors. So even if $k \Longrightarrow \infty$, the clustering coefficient of a node is still p.

d) The image is shown in Figure 1 and 2.

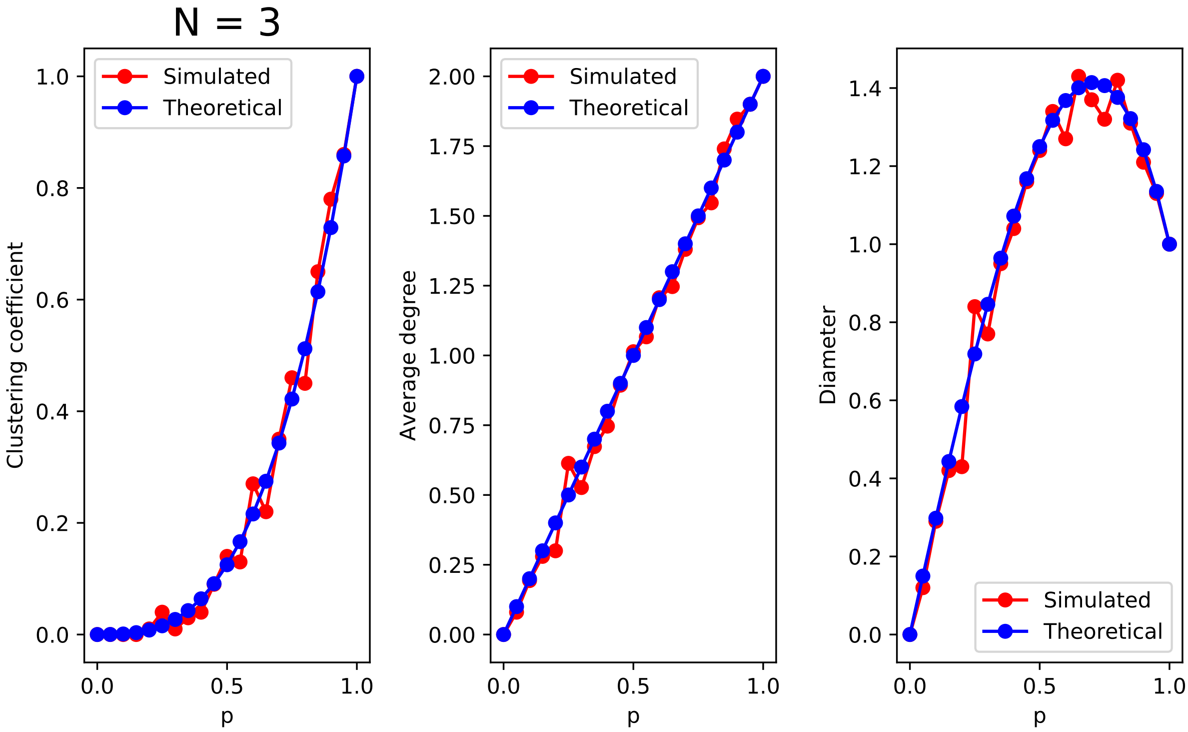

Figure 1: Properties of ER network with 3 nodes

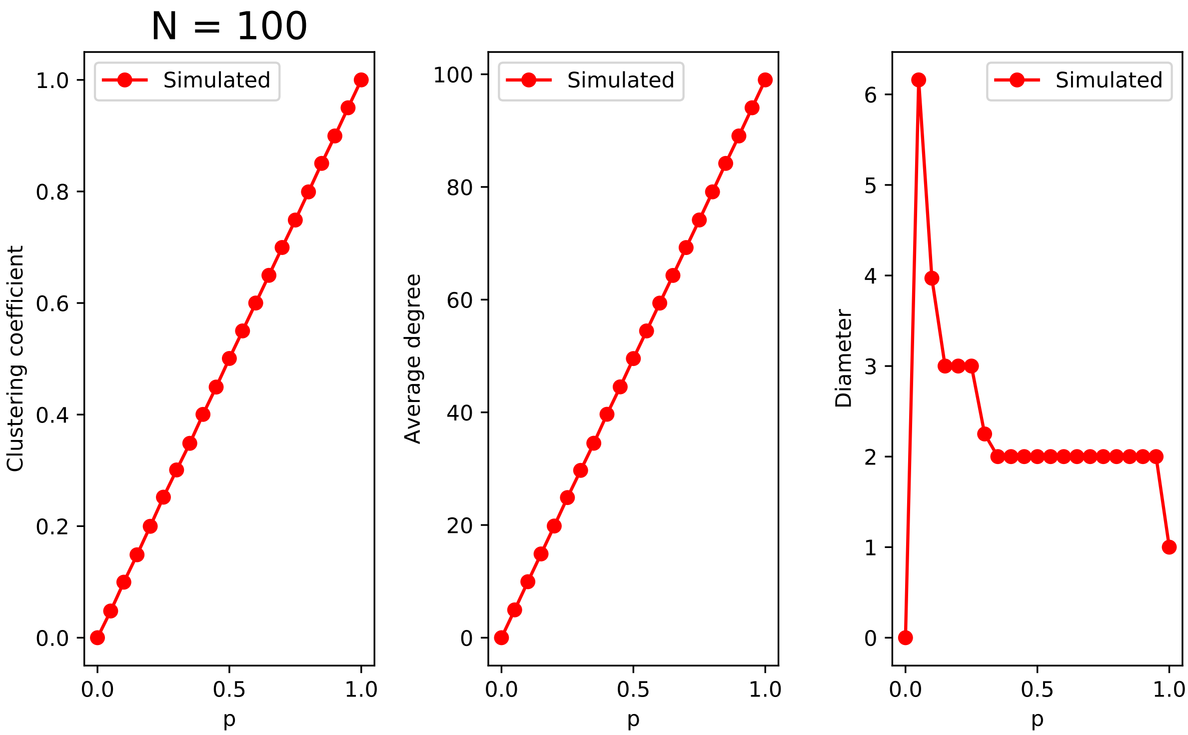

Figure 2: Properties of ER network with 100 nodes

Problem 3

a) For the WS Network using $N = 15, m = 2p = 0.1$ total number of links is 30 and the number of rewired links is 3. For the WS Network using $N = 100, m = 2p = 0.5$, total number of links is 200 and the number of rewired links is 104. Their images are shown in Figure 3 and 4

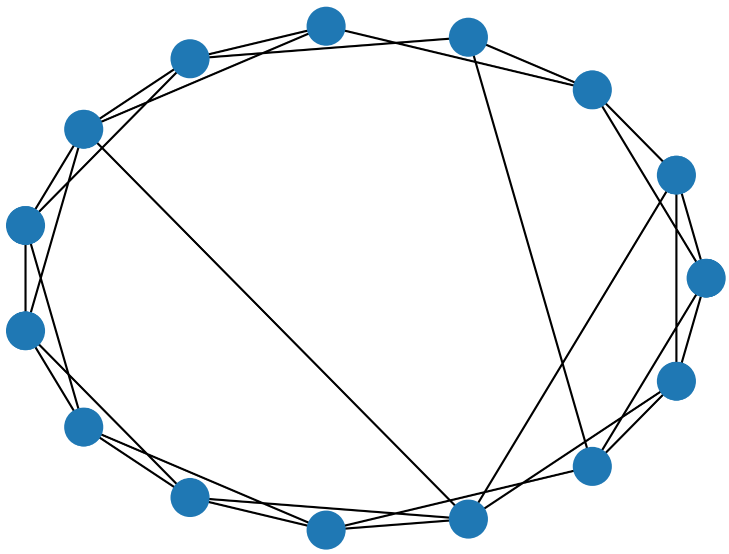

Figure 3: The WS Network using $N = 15, m = 2p = 0.1$

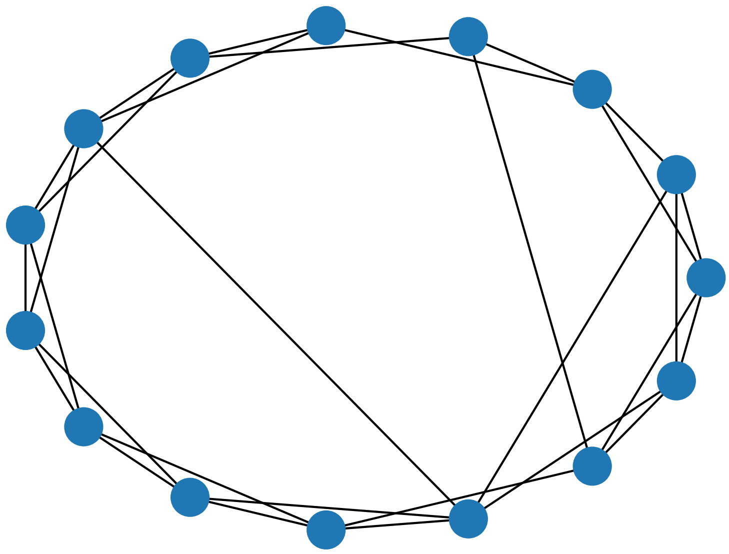

Figure 4: The WS Network using $N = 100, m = 2p = 0.5$

b) The image is shown in Figure 5.

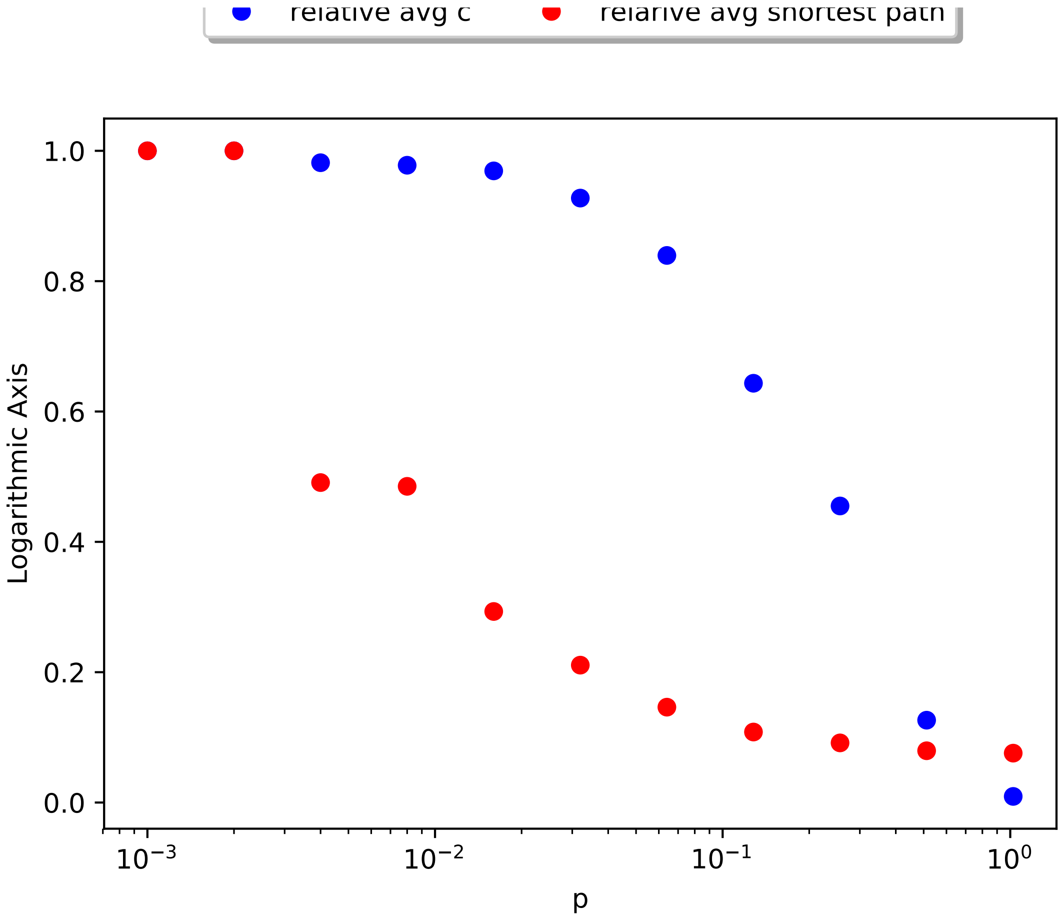

Figure 5: Relative Average Clustering Coefficient and Shortest Path Length