CS-E5740 Complex Networks,
Answers to exercise set 3

Yangzhe Kong, Student number: 765756

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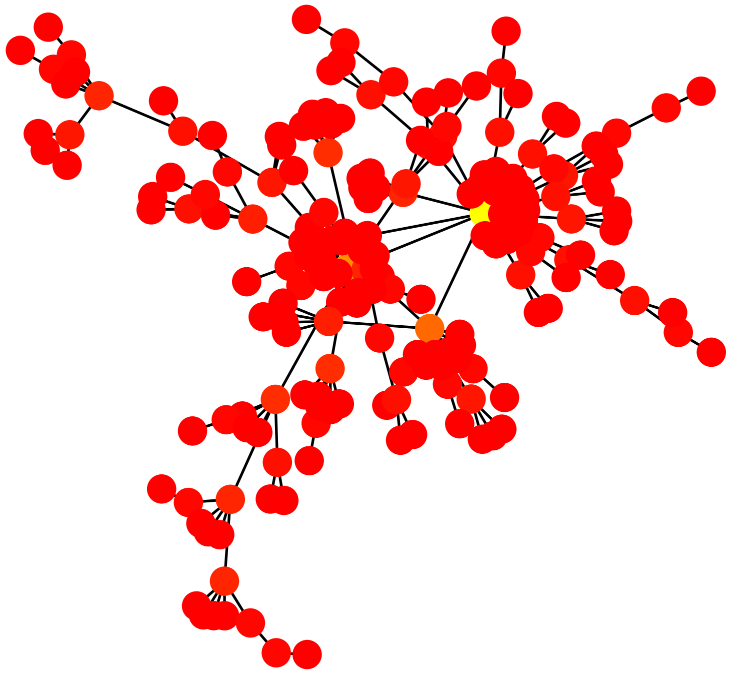
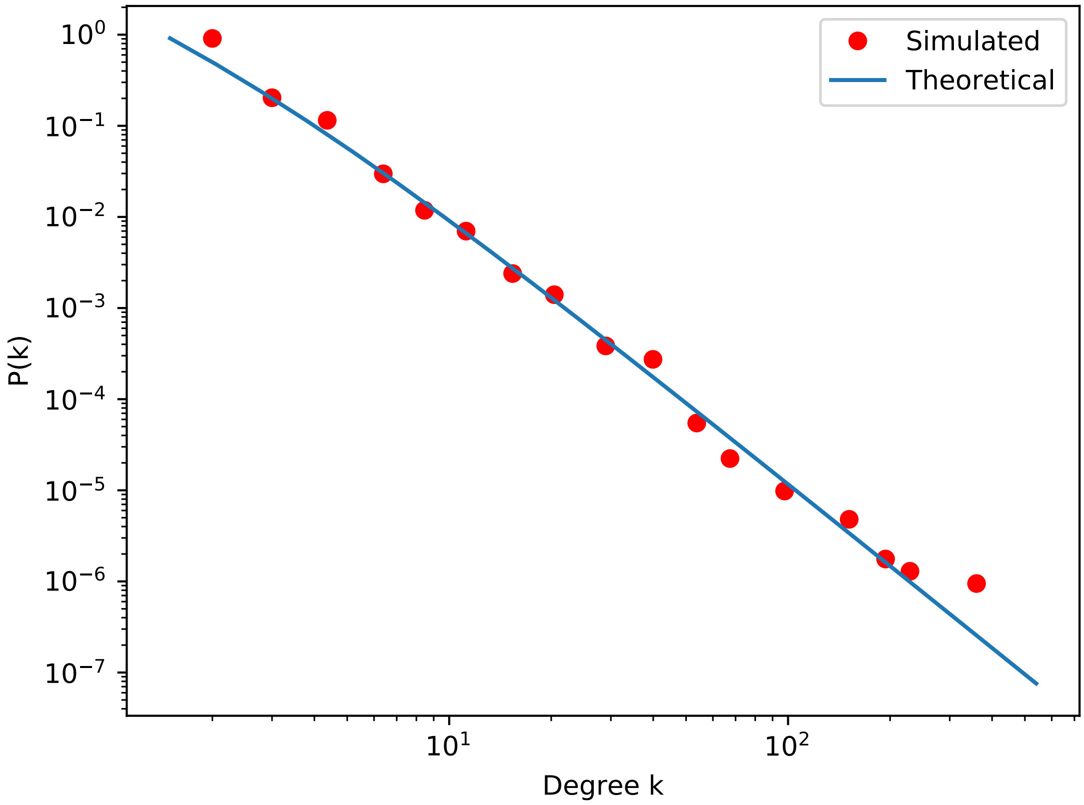
Problem 1

a) the degree of the node with the highest degree in my generated network is 35, the total number of links in my generated network is 200.
The visualization of the network is shown below. As we can see there are some nodes with many connections,while most of the nodes have few connections.



b) The comparison of the experimental and theoretical distributions is shown below.



Problem 2

a) We can derive that:

$$\begin{aligned}\Pi_k &= n_{k,N} \times \Pi_i \\ &= n_{k,N} \times \frac{k}{\sum_{j=1}^{N} k_j}\end{aligned}$$

where only $\sum_{j=1}^{N} k_j$ is unknown. Actually $\sum_{j=1}^{N} k_j = 2mt$ because each time we add a node, we also add m edges, which adds totally $2m$ degrees to the network.

So then

$$\begin{aligned}
\Pi_k &= n_{k,N} \times \Pi_i \\
&= n_{k,N} \times \frac{k}{\sum_{j=1}^{N} k_j} \\
&= n_{k,N} \times \frac{k}{2mt} \\
&= N(t)p_{k,N} \times \frac{k}{2mt} = \frac{tp_{k,N}k}{2mt} \\
&= \frac{p_{k,N}k}{2m}
\end{aligned}$$

b) Because the only vertex with degree m would be the node just added into the network. All previous added nodes will be connected to other nodes, which means that have degree bigger than m as the network grows.
the net change of the number of vertices of degree k as the network grows in size from N to N + 1,

$$\begin{aligned}
n_k^+ - n_k^- &= (N+1)p_{k,N+1} - Np_{k,N} \\
&= \begin{cases} \frac{1}{2}(k-1)p_{k-1,N} - \frac{1}{2}kp_{k,N} & k > m \\ 1 - \frac{1}{2}kp_{k,N} & k = m \end{cases}
\end{aligned}$$

c)

$$\begin{aligned}
p_k &= (N+1)p_k - Np_k \\
&= (N+1)p_{k,N+1} - Np_{k,N} \\
&= \frac{1}{2}(k-1)p_{k-1,N} - \frac{1}{2}kp_{k,N} \\
&= \frac{1}{2}(k-1)p_{k-1} - \frac{1}{2}kp_k
\end{aligned}$$

which means that

$$p_k = \frac{k-1}{k+2}p_{k-1}$$

Also similarly we can derive that

$$\begin{aligned}
p_m &= (N+1)p_m - Np_m \\
&= (N+1)p_{m,N+1} - Np_{m,N} \\
&= 1 - \frac{1}{2}mp_m
\end{aligned}$$

which means that

$$p_m = \frac{2}{m+2}$$

d) Using all the equations we derived above, we can write that

$$p_k = \begin{cases} \frac{2}{2+m} \prod_{i=0}^{k-m-1} \frac{m+i}{m+3+i} & k > m \\ \frac{2}{2+m} & k = m \end{cases}$$