CS-E5740 Complex Networks,
Answers to exercise set 1

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Problem 1

a)

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

b)

$$\rho = \frac{m}{\binom{N}{2}} = \frac{9}{28}$$

c) Degree of node 1 is 1.
Degree of node 2 is 1.
Degree of node 3 is 2.
Degree of node 4 is 5.
Degree of node 5 is 3.
Degree of node 6 is 3.
Degree of node 7 is 2.
Degree of node 8 is 1.
Thus for the degree distribution, $P(1) = \frac{3}{8}$, $P(2) = \frac{1}{4}$, $P(3) = \frac{1}{4}$, $P(5) = \frac{1}{8}$. And for any $k \notin \{1, 2, 3, 5\}$, $P(k) = 0$.

d) The average degree $< k >$ of this network is $< k >= \frac{2m}{N} = \frac{2*9}{8} = \frac{9}{4}$

e) The diameter d of the graph which is $d = max(d_{ij})$ is $d_{28} = 4$.

f) The clustering coefficient for node 1 is $c_1 = 0$
The clustering coefficient for node 2 is $c_1 = 0$
The clustering coefficient for node 3 is $c_1 = \frac{1}{\binom{2}{2}} = 1$
The clustering coefficient for node 4 is $c_1 = \frac{2}{\binom{5}{2}} = \frac{1}{5}$
The clustering coefficient for node 5 is $c_1 = \frac{2}{\binom{3}{2}} = \frac{2}{3}$
The clustering coefficient for node 6 is $c_1 = \frac{1}{\binom{3}{2}} = \frac{1}{3}$
The clustering coefficient for node 7 is $c_1 = \frac{0}{\binom{2}{2}} = 0$
The clustering coefficient for node 8 is $c_1 = 0$
The average clustering coefficient for this network is $c = \frac{1+\frac{1}{5}+\frac{2}{3}+\frac{1}{3}}{8} = \frac{33}{120}$

Problem 2

a) The image is shown in Figure 1

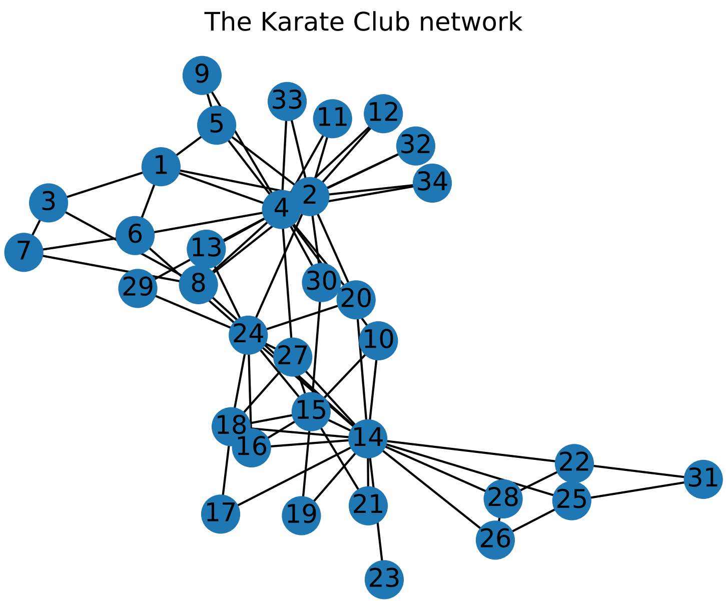

Figure 1: The Karate Club network

b) As Figure 2 shows, The calculated density is 0.13903743315508021. It's exactly the same with the one calculated by NetworkX function.

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D from self-written algorithm: 0.13903743315508021
D from NetworkX function: 0.13903743315508021
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Figure 2: Density Calculation Result

c) As Figure 3 shows, The calculated density is 0.13903743315508021. It's exactly the same with the one calculated by NetworkX function.

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C from self-written algorithm: 0.5706384782076824
C from NetworkX function: 0.5706384782076824
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Figure 3: Density Calculation Result

d) The image is shown in Figure 4

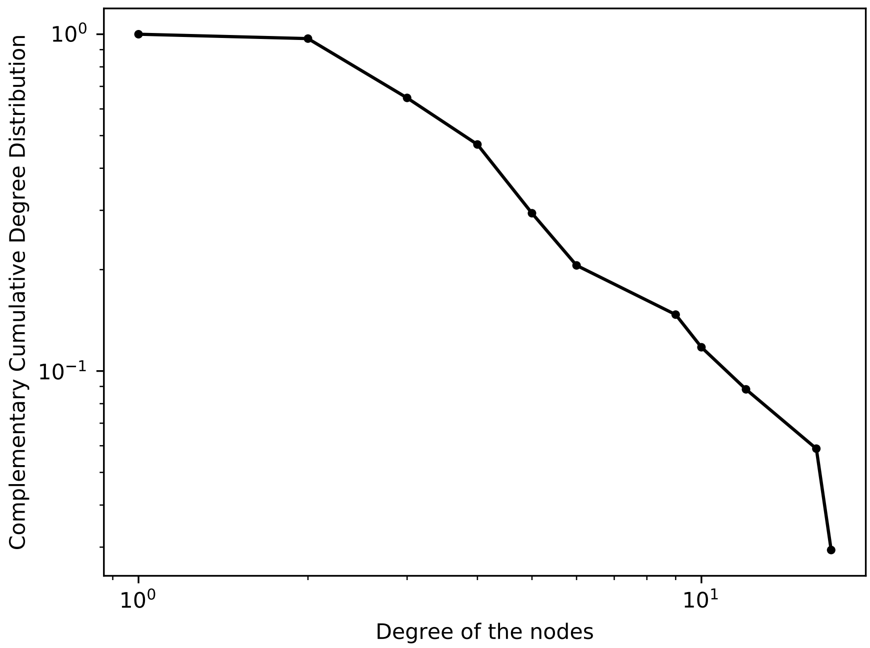

Figure 4: Complementary Cumulative Degree Distribution

e) The calculated average shortest path length is 2.408199643493761.

f) The image is shown in Figure 5

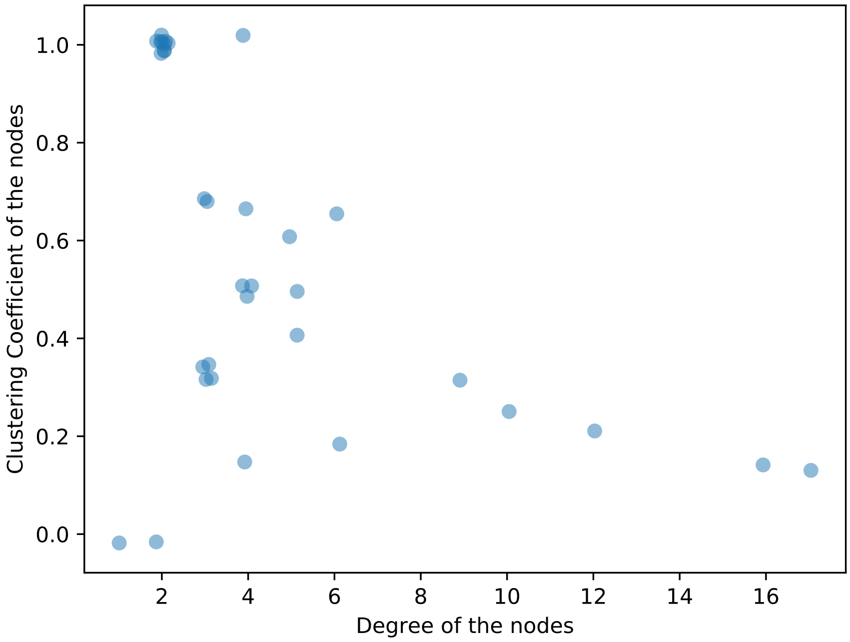

Figure 5: A scatter plot of C_i as a function of k_i

Problem 3

a) The number of walks of length two between node pairs $(1, 1)$ is 3.
The number of walks of length two between node pairs $(1, 2)$ is 1.
The number of walks of length two between node pairs $(1, 3)$ is 0.
The number of walks of length two between node pairs $(1, 4)$ is 1.
The number of walks of length two between node pairs $(2, 2)$ is 2.
The number of walks of length two between node pairs $(2, 3)$ is 1.
The number of walks of length two between node pairs $(2, 4)$ is 1.
The number of walks of length two between node pairs $(3, 3)$ is 1.
The number of walks of length two between node pairs $(3, 4)$ is 1.
The number of walks of length two between node pairs $(4, 4)$ is 2.
And A is

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

A^2 is

$$\begin{bmatrix} 3 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

The result of matrix A^2 is exactly the same with the results by hand computation!

b) A^3 is

$$\begin{bmatrix} 2 & 4 & 3 & 4 \\ 4 & 2 & 1 & 3 \\ 3 & 1 & 0 & 1 \\ 4 & 3 & 1 & 2 \end{bmatrix}$$

So $(A^3)_{3,4} = 1$.
Also if we want to calculate by hand the number of walks of length three between node pairs $(3, 4)$, then we need to find first the possible walks of length 2 from node 2 to any node $x \in \{1, 2, 3, 4\}$, then if node x has a possible walk of length 1 to node 4, this is thus a possible walk of length three from node 3 to node 4.
Using this method, we can find that:
The number of walks of length two between node pairs $(3, 1)$ is 0.
The number of walks of length two between node pairs $(3, 2)$ is 1.
The number of walks of length two between node pairs $(3, 3)$ is 1.
The number of walks of length two between node pairs $(3, 4)$ is 1.
And then only node 2 is connected with node 4, so that the only possible walk is from node 3 to node 1, then node 2 and finally node 4.

c) The statement we need to prove is that given a general network with adjacency matrix A, the element $(A^m)_{ij}$, $m \in \mathcal{N}$ corresponds to the number of walks of length m between nodes i and j.

We make use of mathematical induction. First in a) the statement holds for m = 1 by analyzing the elements of the matrix A^1. Next, assume that the statement holds for a general m, this statement should also hold for $m + 1$. Because the element $a^{m+1}_{i,j}$ is calculated by the sum of $\sum_{k=1}^{4} a^m_{i,k} \times a^1_{k,j}$. This does exactly the inference of number of walks of length $m+1$ from node i to node j based on how many walks are there of length m from node i to node k and the connectivity between node k and node j. Consequently, the statement is true.