ECI-114 Probabilistic Systems Analysis for Civil Engineers, 2024, Summer I
Department of Civil and Environmental Engineering
University of California Davis

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Homework due: July 11th, 2024

Homework # 2: Random Variables and Distributions (total 100 points)

The objective of this assignment is to familiarize yourself with the concepts discussed in class. Remember to be as explicit as possible in your development of the solutions. For instance, explain the variables, the random experiments, etc.

> Note: Please be organized and clear. If we cannot understand your work, we will grade accordingly. For the problems that you only provide a solution without the process we will mark down the grade.

1) *15 points, 5 each.* A quality control study in the Trader Joe's distribution center showed that 1.2% of all frozen bags of broccoli were contaminated. An immediate recall is performed on many thousands of broccoli bags that have been sold at the Davis location, and they are collected and examined for contamination before they are returned to customers. Define (for the 3 questions bellows) the random variable of interest, name its distribution and give the appropriate pmf (in equation form), as well as answering the questions.
(a) In a sample of 100 recalled bags, what is the probability of getting at least 2 contaminated items?
(b) What is the probability that every single bag from the 100 item sample will have to be examined for 1 contaminated item to be found?
(c) What is the expected number of bags that will have to be examined before a contaminated item is found?

2) 10 *points, 5 each*. The West Coast is, on average, hit by 5 hurricanes each year.
(a) What is the probability that fewer than four hurricanes will hit the region in a given year?
(b) Find the expected value and variance for the time *between* hurricanes.

3) *15 points, 5 each*. A student from the ECI 114 class walks to campus every morning and has to stop at the Russell and La Rue intersection. Given his/her interest in statistics and probability, he/she recorded the status of the walk signal (pass/no pass) when he/she arrives at the intersection. After many and many days of walking up to the intersection, the student realized that the walk signal will be in the pass mode 16% of the time. Assume every walk is an independent trial and we are starting from tomorrow. Define the random variable, the distribution, and write the equation for the pmf for the following questions.
(a) What is the probability that the first morning the student arrives to a walk signal is the 3rd morning she/he approaches it?
(b) What is the probability that the student has to wait at the cross walk 11 days in a row?
(c) What is the expected value and standard deviation for the random variable defined in part (b)?

4) *15 points, 5 each.* An automated container is used to dispense liquid soap in the bathrooms of a school. The amount of soap dispensed by the container is uniformly distributed with an average of 0.5 mL and a standard deviation of 25 microliters (1 milliliter = 1000 microliters).
(a) What is the interval over which the amount of soap dispensed from the container ranges?

> (Hint, the uniform distribution is symmetric about its mean, which should help you calculate the range)

(b) What is the probability the container will dispense between 0.46 and 0.53 mL?
(c) A certain test is invalidated if more than 0.51 mL of liquid is dispensed. What is the probability of performing an invalid test using this container?

5) *25 points, 5 each.* Assume X~N(13, 4). Recall, the notation X~N(μ, σ) says, "X is a Normally distributed random variable with a mean μ and standard deviation σ." Determine:

(a) $Pr[X \le 9]$
(b) $Pr[X > 25]$
(c) $Pr[3 \le X \le 14]$
(d) Calculate the value of x, given, $P[X<x] = 0.33$
(e) Calculate the value of x, given, $P[x < X < 9] = 0.1$

6) *20 points, 5 each*. For a continuous random variable Y, the PDF has the form:

$$f(y) = \begin{cases} c\,y^2 & (0 \le y < 1) \\ 0.5\,y & (1 \le y < 2) \\ 0 & else \end{cases}$$

(a) Find the value of c the value c.
(b) Find E[Y] and Var[Y]
(c) Find $Pr[0.5 \le Y \le 1.25]$
(d) Construct the cumulative distribution function of Y, and use it to check your answer in (c).