Lec 12 – Example 0.1. Sol:

1. Decision Rule: $|\bar{x}-65|>1$

$\bar{X} \sim (\mu, (\frac{\sigma}{\sqrt{n}})^2)$

2. Hypothesis: $H_0: \mu = 65$, $H_1: \mu \neq 65$

3. Calculate standard deviation of $\bar{X} \sim N(65, (\frac{\sigma}{\sqrt{n}})^2 = (\frac{2}{\sqrt{12}})^2)$

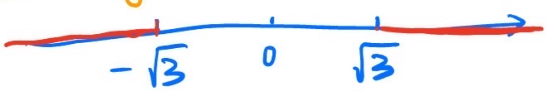
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{12}} = \sqrt{\frac{1}{3}}$$

4. Convert the decision rule to the standard $N(0,1)$

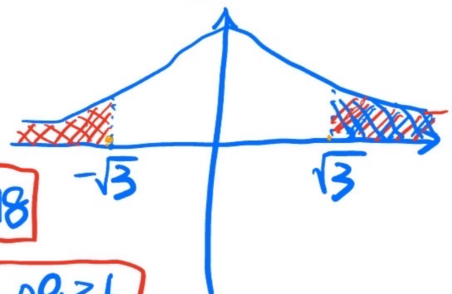
$Z = \frac{\bar{X}-\mu_{\bar{x}}}{\sigma_{\bar{x}}}$

$$|\bar{x}-65|>1 \Rightarrow \left|\frac{\bar{x}-65}{\sqrt{\frac{1}{3}}}\right| > \frac{1}{\sqrt{\frac{1}{3}}} \Rightarrow |Z| > \sqrt{3}$$

$Z = \frac{X-\mu}{\sigma}$



$Z < -\sqrt{3}$ and $Z > \sqrt{3}$



$$P(Z>\sqrt{3}) = 1 - P(Z<\sqrt{3})$$

$$= 1 - P(Z \le 1.732) = 1 - \Phi(Z=1.732) = \boxed{0.1418}$$

$$\alpha = 2P(Z>\sqrt{3}) = 2 \cdot 0.0418 = \boxed{0.0836}$$

Lec 12 - Example 0.4. Sol:

$$\begin{cases} H_0: \mu = 65 \\ H_1: \mu \neq 65 \end{cases}$$

Decision Rule

$$\beta(\mu=64) = P(\, |\bar{x}-\mu| \boxed{>} c \mid \mu = 64)$$

Acceptance Region

$$\rightarrow P(\mu=64) = P(\underbrace{|\bar{x}-65|}\boxed{<} c \mid \mu=64)$$

← Type II error: The probability that we wrongly accept H_0 ($|\bar{x}-65|<c$) when H_0 is false ($\mu = 64$)

$$= P(-c < \bar{x}-65 < c \mid \mu = 64)$$

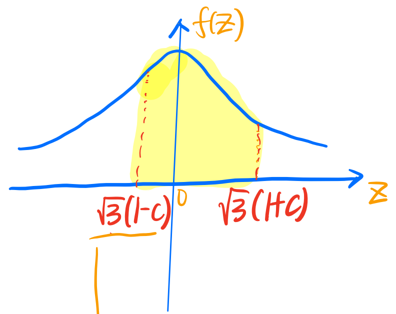
$$= P(65-c < \bar{x} < 65+c \mid \mu = 64), \quad \bar{x} \sim N(64, (\sqrt{\tfrac{1}{3}})^2)$$

Standardization:

$$= P\left(\frac{65-c-\boxed{64}}{\sqrt{\frac{1}{3}}} < \boxed{\frac{\bar{x}-\boxed{64}}{\sqrt{\frac{1}{3}}}} < \frac{65+c-\boxed{64}}{\sqrt{\frac{1}{3}}} \,\middle|\, \mu=64\right)$$

$$= P(\sqrt{3}(1-c) < Z < \sqrt{3}(1+c))$$

$$= \Phi(\sqrt{3}(1+c)) - \Phi(\sqrt{3}(1-c))$$



→ The lower bound could either be larger than 0 or less than 0, depending on $1-c$.