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Homework due: July 30th, 2024

Homework # 4: Estimators (100 pts)

The objective of this assignment is to familiarize yourself with the concepts discussed in class.

Remember to be as explicit as possible in your development of the solutions.

Note: Please be organized and clear. If we cannot understand your work, we will grade it accordingly. For the problems that you only provide a solution without the process we will mark down the grade.

For this assignment you need to:

- 20 points. Suppose that a random variable X represents the output of a civil engineering process, and that X is uniformly distributed. The pdf of X is equal to 1 for any positive x smaller than or equal to 2, and it is 0 otherwise. If you take a random sample of 12 observations, what is the approximate probability distribution of $\bar{X} - 10$? (You need to find the mean and variance of this quantity and state your assumptions.)

Hint:

- **Determine the mean and variance of the uniform distribution X .**
- **Use the Central Limit Theorem to approximate the distribution of the sample mean for 12 observations.**

- 10 points. The heights (in inches) of 15 students from the entire UC Davis student body are sampled. The standard deviation for height in the entire UC Davis student body is known to be 3.
(a) What is the variance of the sample mean?

- 20 points, 10 each. For two independent random variables with mean μ and variance σ^2 , we have the following estimators of μ :

$$\hat{\Theta}_1 = \frac{X_1 + X_2}{2} \quad \text{and} \quad \hat{\Theta}_2 = \frac{X_1 + 2X_2}{4}$$

- Find out if they are unbiased estimators of μ ?
- Find their variances, and determine which one is better in terms of the relative efficiency.

Hint:

- **To check if an estimator is unbiased, calculate its expected value and see if it equals the population mean.**
- **For the variances, use the properties of variances for linear combinations of independent random variables.**
- **Relative Efficiency = $\frac{\text{Var}(\hat{\mu}_1)}{\text{Var}(\hat{\mu}_2)}$. The estimator with the smaller variance is the more efficient one.**

- 30 points, 10 points each. Measurements of a critical dimension on a sample of automotive driveshafts are taken with the following results (in units of mm):

10.14	10.4	9.87	10.02	9.99	10.03
10.09	10.11	9.93	9.99	10	10.02

The intended μ is 10, but is known to drift from this mean during production. The measurement follows a normal distribution.

- Assume that the standard deviation for the production process is known as $\sigma = 0.10$ mm. Calculate the 95% confidence interval for the mean for the production process.
- Assume that the standard deviation for the production process is not known. Calculate the 95% confidence interval for the mean.
- Find β (the probability of a Type II error) for the case where a 95% confidence interval is used, the true mean 10.2, and σ is known.

Hint:

- For known σ , use the Z-score to calculate the confidence interval.
- For unknown σ , use the t-distribution with the appropriate degrees of freedom.
- For the Type II error, calculate the probability of not rejecting the null hypothesis when the true mean is 10.2.

- 20 points. Suppose that X is a discrete uniform random variable, with pmf equal to $\frac{1}{4}$ for values of $x = 1, 2, 3$, and 4 . Determine the probability that the sample mean is greater than 1 but less than 3, if you select a random sample of size $n = 40$.

Hint:

- Calculate the mean and variance of the discrete uniform distribution.
- Use the Central Limit Theorem to approximate the distribution of the sample mean.
- Convert the range of interest into Z-scores and use the standard normal distribution to find the probability.