ECI-114 Probabilistic Systems Analysis for Civil Engineers, 2024, Summer I
Department of Civil and Environmental Engineering
University of California Davis

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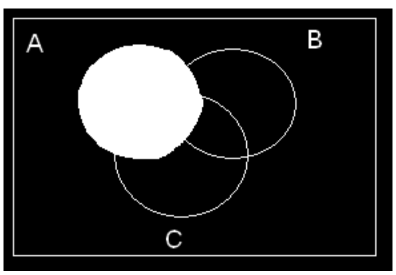
Homework due: **Jul 3rd, 2024**

Homework # 1: Statistics and Probability (total 120 points)

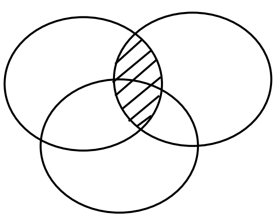
The objective of this assignment is to familiarize yourself with the statistics and probability concepts discussed in class.

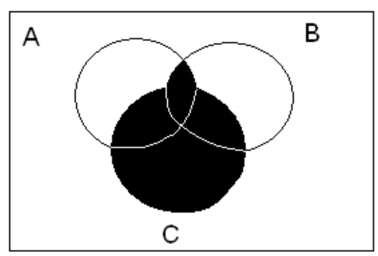
Note: Please be organized and clear. If we cannot understand your work, we will grade accordingly. For the problems that you only provide a solution without the process we will mark down the grade.

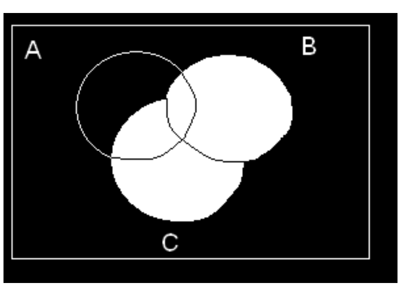
1) ***8 points, 2 each.*** **Write down in set notation (intersection, union, conditional, etc.), the events represented by the White (or diagonal lines) shaded areas in the following Venn Diagrams.**

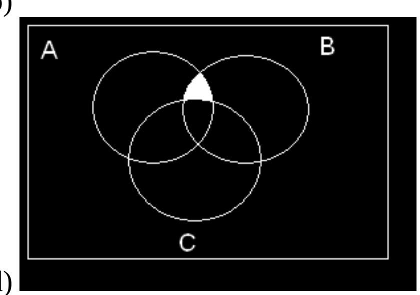
Example: 

represents the event A

a) 

b) 

c) 

d) 

2) ***10 points, 2 each.*** **A random experiment has 8 possible outcomes. The sample space (S) is {r, s, t, u, v, w, x, y}, each one equally likely. Let A be the event {r, s, t}, and let B denote the event {u, v, w, x}. Determine the following probabilities:**

a) P(A) b) P(B) c) P(A') d) P(A∩B) e) P(AUB)

3) 10 *points, 2 each.* A sample space of random experiments is {r, s, t, u, v} with probabilities of 0.01, 0.6, 0.05, 0.04, 0.3 respectively. Let A denote the event {r, u}, B denote the event {s, t, v}, and C denote the event {r, s, t}. Determine the following probabilities:

a) P(A) b) P(B) c) P(C') d) $P(A \cap C)$ e) P(AUB)

4) 8 *points, 2 each*. An engineering company wants to build a new tall structure in the middle of downtown Davis. However, upon a preliminary inspection of the 1000 steel beams they were given by a contractor, they are concerned about the quality of the pieces. Two concerns they have relate to the quality of the pieces (they need to meet their high-strength standard), and if the flaws within the pieces remain below the detection/inspection limit. They assigned one of the staff engineers to conduct the tests, and the results are summarized in the following table:

	Meets High Strength Standard	
	Yes	No
No Flaws Detected	688	75
Flaws Detected	216	21

Using the inspection results, help the company answer the following questions.

a) If a steel beam is randomly selected, what is the probability that it meets the high strength standard?
b) If a steel beam is randomly selected, what is the probability that it meets the high strength standard and has no detectable flaws?
c) If a steel beam is randomly selected and it is known to have flaws detected, what is the probability that it has not met the high strength standard?
d) If the event that a flaw is detected is denoted as event FD, and the event that a beam does not meet the high-strength standard is NHS, how would you write the probability you calculated in part (c) in set notation (intersection, union, conditional, etc.)?

5) *12 points, 4 each*. Provide a reasonable description (e.g. S ={…}) of the sample space for the following random experiments. Define the events included for each experiment:

a) Each of 4 bridges on a highway corridor is analyzed and then classified as above or below specification. (4 points)
b) In a simple game of darts, 3 darts are thrown at a board and can land in the center section or outer section of the board. (4 points)
c) In a potato chip manufacturing plant the quality control engineer draws a chip at random and classifies it as high quality or mediocre, and also as large, medium or small in size. (4 points)

6) *8 points, 4 each*. Suppose P(A|B) = 0.2, P(B) = 0.8, and P(A) = 0.64.

a) What is $P(A \cap B)$? b) What is P(B|A)?

7) ***12 points, 4 each.*** **In the current Request for Proposals (RFP) process by the US Department of Transportation, civil engineering departments from 7 Universities, are offering bids on 4 distinct/different research center funding contracts (which are different in the level of funding and the duration). Any University can only be awarded at most 1 grant to open the research center.**

a) Please explain how would you estimate the different ways in which the contracts could be awarded? Estimate the number.
b) Under the assumption that the simple events are equally likely (each University has the same probability of being awarded a contract), find the probability that University No. 2 is awarded one.
c) Suppose that Universities 5 and 6 have submitted bids, if the contracts are awarded at random by the DOT, find the probability that both of these Universities receive contracts. (Hint, you can do this by analyzing the conditional probabilities, or by using counting methods)

8) 8 ***points, 4 each*****. You won the lottery, and have money to design and build a building. In doing so, you need to make some color decisions. You can only choose green, blue, or brown for the windows; a stone for the exterior façade of white, gray, or brown color; and finally a paint color for the front doors which can be either yellow, blue, red, or orange.**

a) Define the number of options (sample space) you have in this problem by using a tree diagram.
b) Verify that you calculated the right number of objects in the sample space by applying a counting technique.

9) ***12 points, 4 each.*** **There is a national civil engineering competition and I need to send 20 students to participate. However, the rules allow for teams of 4 students so I need to divide the students in 5 teams.**

a) How many ways do I have to form the teams?
b) Additionally, to increase the chances I would like to have some sort of hierarchy in the team. Therefore, I decided that a member from each team would be a boss, and each remaining student in the team would be the strategist, the planner, or the auditor. How many total ways could these teams be formed, assuming that students have already been assigned to the teams?
c) What would change in your analyses, if you consider the problem as a sequence of steps? (Note, assign students, and then assign the roles)

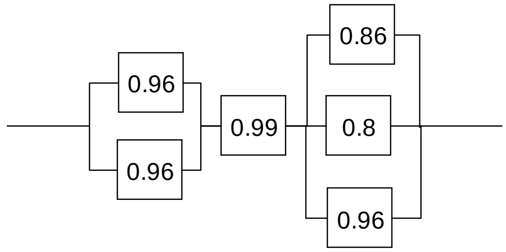
10) ***10 points, 5 each.*** **Before unveiling the new Tesla Model 3, the company wants to reduce the complexity of manufacturing the new model. At the moment, they are thinking of offering among 6 different colors, and 4 different optional item (sunroof, enhance navigation system, spoiler, and an extended battery pack).**

(a) How many ways can users select options for the new model?
(b) The company decides that if they reduce the number of ways cars can be ordered their plant would increase efficiency significantly, but they don't want to eliminate any of the offered options. You have some good ideas on how to change the way cars can be ordered, here is what you come up with:

(i) The "extended battery" package: which includes the extended battery and spoiler. With this package the customer can add sunroof, nav., and any color.
(ii) The "basic" edition: No extended battery, no spoiler, no sunroof, no enhanced nav. and only 4 nonmetallic colors.
(iii) The "intermediate" edition: No extended battery and no spoiler, but the customer can choose any color, a sunroof, and enhanced navigation.

How many total ways can vehicles be ordered now?

11) 8 *points.* The following circuit operates only when there is a path of functional devices from left to right. Calculate the overall probability that the circuit remains functional (its reliability), and indicate where in the circuit the greatest risk of inoperability occurs. The reliability of each device is indicated in the squares.



12) *10 points, 2 each.* The values for a random variable X and associated probabilities are displayed in the following table.

x	-3	-2	-1	0	1	2	3
p(x)	0.5	0.01	0.04	0.03	0.23	0.1	0.09

(a) Based on the table of x and p(x) above, show p(x) meets the criteria for a probability mass function.
Using the values defining p(x) in the table above, determine the following probabilities
(b) $\Pr(X = -2 \text{ or } X = 2)$
(c) $\Pr(X \leq 4)$
(d) $\Pr(X \geq 0 \text{ or } X = -3)$
(e) $\Pr(-3 < X < -1)$

13) 4 *points*. Find a story in history on math geeks who've used their knowledge in statistics and probability in playing the lottery, or on other forms of gambling. Briefly summarize the story (in 150 words), and point out which concepts of statistics they have applied.