CS 4820, Spring 2019	**Homework 1, Problem 1**
Name: Yanling Wu	
NetID: yw996	
Collaborators:Yexiang Chang(yc2523), Zhizhong Peng(zp83)	

(1) *(5 points)* For each positive integer n, let t_n denote the number of distinct ways to cover a rectangular $2 \times n$ grid with non-overlapping dominoes. What is the value of t_n? Prove the correctness of your answer using mathematical induction.

Figure 1: $t_1 = 1$

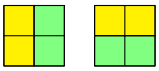

Figure 2: $t_2 = 2$

Figure 3: $t_3 = 3$

Solution:

The value of t_n is $t_n = t_{n-1} + t_{n-2}$, which are Fibonacci numbers.

Proof:

Base case:

for $n = 3$, according to the formula of Fibonacci number, $t_3 = t_2 + t_1 = 1 + 2 = 3$, which is obvious from the question.

Induction Step:

We consider the value of t_n and assuming we have already know all values before t_n. The question that get the value of t_n can be divided to two situations as shown in the figure 4 and figure 5. The situation of figure 4 is a tiling of a $n-1$ board with one vertical domino and that of figure 5 is a tiling of a $n-2$ board with two horizontal dominoes. So $t_n = t_{n-1} + t_{n-2}$ proved.

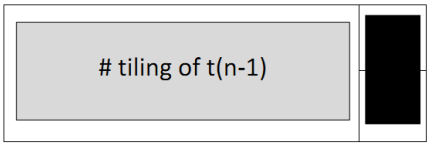

Figure 4: Only using one vertical domino

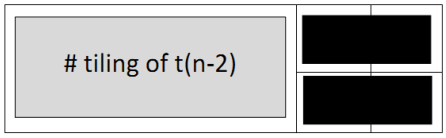

Figure 5: Using two horizontal dominoes