CS 4820, Spring 2019	**Homework 1, Problem 2**
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(1) *(10 points)* Suppose $n\,(\geq 1)$ people are stranded on a freeway due to a particularly heavy snowstorm. Let us model the freeway as the real line. The locations of the stranded people are given by real numbers. An emergency rescue operation found n hotels to potentially accommodate the n people. Assume that each hotel has the capacity to shelter 1 person. However, due the prevailing road conditions access to these hotels are severely limited. For each hotel H, there is a specific segment of the freeway such that only people stranded in this segment can make it to H. Given as input the locations of the stranded people, and a set of n segments of the freeway (each segment corresponding to a particular hotel), your task is to design an efficient algorithm to decide if it is possible for all n people to find accommodation for the night. (Assume that the input contains real numbers of finite precision, so that any arithmetic operation on two real numbers takes constant time. Neither the list of people's locations nor the list of segments are assumed to be sorted in any particular order.)

Solution:
Algorithm:
Set the M to empty;
Sort the locations of the stranded people in ascending order and save it as L
Sort the set of segments of the hotel based on the the endpoints of one hotel in ascending order, and save this set as H
for every person p in L:
    for every hotel h in H:
        If the location of p is in the scale of the h
            put the (p,h) into set M;
        EndIf
    EndFor
    remove p;
EndFor
If the set H is empty:
    It is possible for all people to find accommodation.
Else
    It is impossible;

Time complexity The time complexity of the worst case should be $O(n^2)$, but time complexity in average should be $O(nlogn)$.

Proof
1. If the algorithm outputs yes, the solution will exist obviously and there will no mistake because there is a if_statement to judge whether the person is located in the area of one hotel in the algorithm. Hence if the output is yes, it exist one solution to satisfy the requirement of this question.

2. If the algorithm outputs no, we need to prove the solution does not exist. We could prove this statement by contradiction. Assume that there exists one situation that the algorithm outputs "not exist" but actually it exists one matching way to make every person to accommodation in one hotel. If this situation appears, there exist two persons p_1, p_2 and two hotels h_1, h_2 thatp_1 can live in either h_1 or h_2 but p_2 only can live in h_2. In the algorithm, h_2 was lived by p_1 and thus p_2 cannot find a hotel to live. Actually, the p_1 should live in h_1 and p_2 can live in h_2. However, When p_1 can live in h_1 and p_2 cannot live in h_1, we can know $p_1 < f(h1) < p_2$, $f(h1)$ means the endpoints of h_1. And Based on the condition that p_2 can live in h_2, we can know $f(h_2) > p_2$. So we could know that $f(h_1) < f(h_2)$. So in my algorithm, p_1 should be accommodated to h_1, which is contradiction.