The Influence of Mass and Size of a Falling Object on its Fall Time and the Dependence of Velocity on Air Resistance.

I. Research Question:

How does the mass and size of a falling spherical object influence the fall time? The fall times are used to determine the dependence of velocity on air resistance.

II. Aim:

To determine the dependence of velocity on air resistance.
We study the free fall of spherical objects with different circumferences and mass. By measuring the free fall time, the data will permit us to determine how air resistance depends on velocity.

III. Rationale:

Some years ago, I was mesmerized seeing Brian Cox's Galileo's experiment. Prof. Cox drops a bowling ball and a feather from a large height in the world's largest vacuum chamber (BBC TWO, 2014). And indeed, the ball and the feather drop with equal speeds and hit the floor simultaneously as expected by Newton's equations. I always wanted to do an experiment like that....

Preparing for this IA I got some more advanced books in physics that discuss air resistance (Feynman et al., 2010, pp. 12-3) (Kittel et al., 1973, p. 216). I would love to have had access to large vacuum chambers or wind tunnels, as these are clearly the best settings to study air resistance on a moving object, but I believe that it is possible to verify some of the hypotheses referred to w.r.t. air resistance in the aforementioned books (Feynman et al., 2010, pp. 12-3) (Kittel et al., 1973, p. 216).

IV. Hypothesis

Air resistance is dependent on velocity: Feynman notes that the air resistance is approximately a constant times velocity squared (Feynman et al., 2010, pp. 12-3). In the Berkeley Physics Course volume 1 generalize the expression of air resistance to velocity to the power x, $F = K V^x$ (Kittel et al., 1973, p. 216). Hodder's physics review textbook also assumes, like Feynman, that air resistance is proportional to velocity squared (Hodder Education, n.d., pp. 16-17).

The main focus of the investigation is to test the v^2 dependence of air resistance. Furthermore, it is investigated whether the data will show indications of a modified power law.

V. Theoretical Background Information

Section IV – 1: Analysis of Forces

The physics of a free-falling body, neglecting air resistance, can be analysed using the SUVAT equations learnt previously in the Mechanics topic in the IB physics curriculum (Hamper, 2014, p.47). The fall time t of the object falling of a height h is be given by

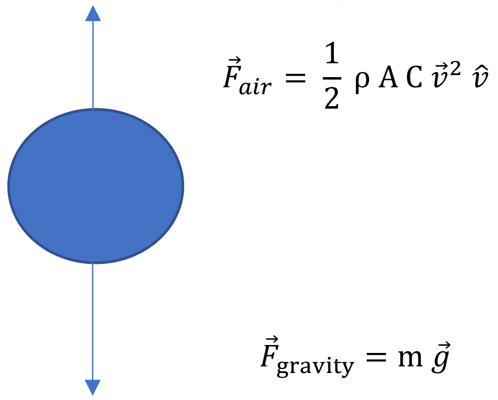
$h = \frac{1}{2} \times g \times t^2$ where g is the gravitational constant, $9.81\frac{m}{s^2}$

Thus, $t = \sqrt{\frac{2h}{g}}$ (1a)

and the terminal velocity is $V = gt = \sqrt{2gh}$ (1b)

Now, let's include air resistance in the problem at hand. In the picture below, the forces acting on a free-falling sphere are shown.

Figure 1: Forces acting on a free-falling object



F(air) is the air resistance hypothesised to be proportional to the velocity squared. The density of air, ρ, is 1.225 grams per m^3. The constant C is the drag coefficient which equals to 0.47 for spherical objects (Hodder Education, n.d., pp. 16-17). The cross-sectional area, A, is the effective surface area of the object. In this case, A = ¼ π D^2, where D is diameter of the spherical object. For convenience the constant K is defined as,

$K = \frac{1}{2}(\rho)(A)(C)$ (2a)

The principal objective of my experiment is to verify the velocity squared dependence of the air resistance formula.

The analysis starts by applying Newton's second law, F = ma, on a falling body of mass m. The force net on the vertical axis acting on the ball can be expressed as:

$$m\,a = m\,g - K\,V^2 \quad (2b)$$

Hence,

$$a = g - \frac{k}{m} \times V^2 \quad (2c)$$

An observation made from the formula (2c) is that the relative importance of the air resistance decreases when the mass of the object increases, as expected.

To proceed with this investigation, it is required to measure the time taken for the object to fall, which will be compared to the theoretical predictions. **These predictions were calculated using an Excel program that I created has been specifically created for the purpose of this experiment.**

Section IV – 2: Creation of the simulation program to calculate theoretical fall times.

For clarification, this program was created specifically for this experiment, it is not an off-the-shelf application.

Acceleration is by definition is the rate of change of velocity, $a = \frac{dv}{dt}$
Thus, the fundamental equation of this problem is: using equation (2c) above.

$$\frac{dv(t)}{dt} = g - \frac{k}{m} \times v(t)^2 \qquad (3)$$

This is a differential equation for the velocity, if it can be solved for v(t), the time taken for an object to fall a specific height can be calculated. There is a numerical method to solve this differential equation from the HL Math IB textbook (Rondie, 2019, p. 546). An outline of this is given below. Subsequently, the algorithm created is implemented on a Microsoft Excel program.

The discrete version of the equation (3) is:

$$\frac{v(t+\Delta t)-v(t)}{\Delta t} = g - \left(\frac{k}{m}\right) \times v(t)^2$$

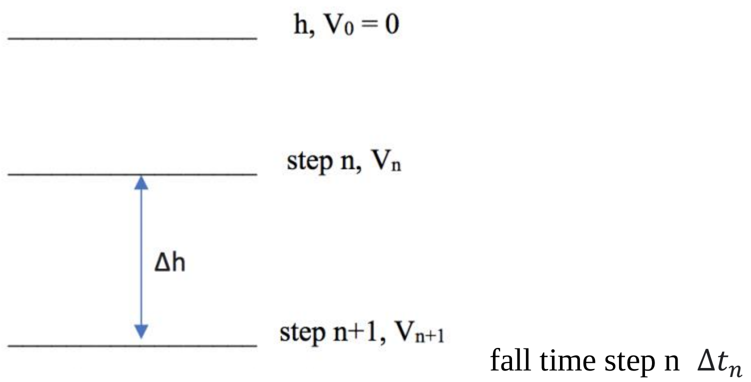
Which can be rewritten as

$$v(t+\Delta t) = g \times \Delta t - \frac{K}{m} \times \Delta t \times v(t)^2 + v(t)$$

This gives an algorithm for calculating v(t) numerically, if v (0) is known, then the v(Δt) can be numerically calculated using this formula. With v(Δt) known, then v(2Δt) can be calculated using the same formula etc. Therefore, continuing this process for an arbitrary number of steps results in a numerical approximation for v(t).

A small variation to this method is made. The object falls from a height, h, with the initial condition is v(0) = 0. Δh is defined as a very small fragment of the height, h. It is more intuitive to use steps of Δh, in height, instead of steps Δt_n, in time.

Figure 2: Illustration of numerical method for solving equation (3)



Firstly, formulas are derived for the case of no air resistance: applying SUVAT, Δh is obtained by:

$$\Delta h = \frac{1}{2} \times g \times \Delta t_n^2 + v_n \times \Delta t_n$$

Using the quadratic formula to solve for Δt_n gives:

$$\Delta t_n = \frac{-V_n}{g} + \frac{\sqrt{V_n^2 + 2 \times g \times \Delta h}}{g} \qquad (4)$$

And the velocity at the next step, n+1, is given by $v_{n+1} = g \times \Delta t_n$ (5)

With this algorithm, velocity can be calculated numerically step by step as a function of time and therefore the fall time can also be calculated numerically for any given height.

Formulas (1a) and (1b) displayed previously can be used to check the implementation of the algorithm.

To include air resistance in the equations (4 & 5) the gravitational force needs to be replaced by the total force (2b & 2c), that is the gravitational force **plus** the air resistive force. Thus, equations 4 and 5 can be modified by substituting g as follows:

$$g \longrightarrow g - \frac{K}{m} \times v_n^2 \qquad (6)$$

This algorithm is implemented in a Microsoft Excel spreadsheet.

In Figure 3 prints of the spreadsheet simulation model are shown. 200 interval steps are used for the calculation. This is largely sufficient; using 50 steps resulted in the same results up to 3 decimal places.

On the left-hand side of Figure 3 the inputs to the model are shown, together with some simple preparatory calculations. The air resistance constant, K, is given by the formula (2) above. The input height is 5.13m. For this particular simulation the circumference of the falling object is 1.222m, and its mass is 0.1kg.

To be considerate of space, only the first 20 and last 15 steps of the simulation are shown here. Numbers in the column "No air resistance, Time (seconds)" are calculated using formula (4) above and numbers in the column "Velocity (m/s)" are calculated using formula (5) above.

In the case with air resistance all quantities are calculated using the same formulas except using the substitution (6) above. The results are shown on the three right most columns which shows Acceleration (equation 6), Time (Δt_n), and Velocity (V_{n+1}).

This particular print out uses a v^2 dependence. The program, however, can handle all hypothesis v^x. The x input is in the top left cell of the screenshot. The screenshot below is an example of what the simulation looks like. This is simply an example, where mass, circumference and the height are all inputs of this algorithm.

Figure 3: **Sample** Screen Shot of the fall simulation spreadsheet.
In this example the mass is 0.1kg, the circumference is 122.2cm and the height is 5.13m.

				No Air Resistance			With Air Resistance	
Power of V	2	Intervals	Height (metres)	Time (seconds)	Velocity (m/s)	Acceleration (m/s^2)	Time (seconds)	Velocity (m/s)
Height (metres)	5.13		5.13		0.000			0.000
mass of an object (kg)	0.1	1	5.10	0.0723	0.709	9.810	0.0723	0.709
gravitation (g)	9.81	2	5.08	0.0300	1.003	9.638	0.0300	0.998
height intervals (steps)	200	3	5.05	0.0230	1.229	9.469	0.0231	1.217
		4	5.03	0.0194	1.419	9.304	0.0195	1.398
		5	5.00	0.0171	1.586	9.141	0.0173	1.557
Assumption that air resistance,	-K * v(t)^x	6	4.98	0.0154	1.738	8.981	0.0157	1.698
air resistance constant -K	0.0342	7	4.95	0.0142	1.877	8.824	0.0145	1.826
Drag Coeffienct, C (sphere)	0.47	8	4.92	0.0132	2.006	8.670	0.0136	1.943
rho (at sea level, 15oC)	1.225	9	4.90	0.0124	2.128	8.518	0.0128	2.052
Area (metres^2)	0.119	10	4.87	0.0117	2.243	8.370	0.0121	2.154
Circumference (metres)	1.222	11	4.85	0.0112	2.353	8.223	0.0116	2.249
Diameter (metres)	0.389	12	4.82	0.0107	2.457	8.080	0.0111	2.339
		13	4.80	0.0102	2.558	7.938	0.0107	2.424
		14	4.77	0.0098	2.654	7.800	0.0104	2.505
		15	4.75	0.0095	2.748	7.663	0.0100	2.582
		16	4.72	0.0092	2.838	7.529	0.0098	2.655
		17	4.69	0.0089	2.925	7.398	0.0095	2.726
		18	4.67	0.0086	3.010	7.269	0.0093	2.793
		19	4.64	0.0084	3.092	7.142	0.0090	2.857
		20	4.62	0.0082	3.173	7.017	0.0088	2.920

	185	0.38	0.0027	9.649	0.383	0.0049	5.251
	186	0.36	0.0027	9.675	0.376	0.0049	5.253
	187	0.33	0.0026	9.701	0.370	0.0049	5.255
	188	0.31	0.0026	9.727	0.363	0.0049	5.257
	189	0.28	0.0026	9.753	0.357	0.0049	5.259
	190	0.26	0.0026	9.778	0.351	0.0049	5.260
	191	0.23	0.0026	9.804	0.344	0.0049	5.262
	192	0.21	0.0026	9.830	0.338	0.0049	5.264
	193	0.18	0.0026	9.855	0.332	0.0049	5.265
	194	0.15	0.0026	9.881	0.327	0.0048	5.267
	195	0.13	0.0026	9.906	0.321	0.0048	5.268
	196	0.10	0.0026	9.932	0.315	0.0048	5.270
	197	0.08	0.0026	9.957	0.310	0.0048	5.271
	198	0.05	0.0026	9.982	0.304	0.0048	5.273
	199	0.03	0.0026	10.007	0.299	0.0048	5.274
	200	0.00	0.0026	10.032	0.294	0.0048	5.276
		total time	1.0227	Without Air Resistance	total time	1.3252	With Air Resistance
check for terminal velocity, sqrt(2gh)		10.032					
check for time, sqrt(2h/g)		1.0227					
					time difference	0.3025	

Without air resistance, the results of the simulation model agree with equations (1) above for the terminal velocity in yellow and the fall time in blue, displayed in the image above. This gives us confidence that the algorithm is correctly coded.

The fall time including air resistance is shown in red. For this particular example the object experiences an air resistance used 0.3025 seconds more to hit the floor than the free-falling object.

The simulation program enables us to calculate fall times of any spherical objects of any mass falling off any height, and for any hypothesis V^x.

VI. Variables:

The relevant variables for this experiment are.

Independent Variables:	
Mass of the spherical objects:	66g to 266g 78g to 178g
Circumference of the spherical objects	75.5cm (C1) 122.2cm (C2)

The range of masses and circumferences are chosen using the simulation described above. Such simulation studies are critical to this, because e.g. if the object is too heavy the time difference between fall including friction and free fall become too small to measure. When the object is too light turbulence might disturb its path.

Dependent variable: Time Taken for the object to fall from a specific height(seconds).
This is not going to be an easy measurement. The simulation, explained above, show that the difference with the free fall time is typically 0.3 seconds.

Controlled Variables:

Controlled Variable:	What happens if this isn't controlled:	How to control it:
height (h)	Objects falling at different height won't give consistent values for the fall time.	Ideally this experiment would require a bigger height, however this is the maximum height in a closed space environment within the school. The balls are consistently

		being dropped at the same marked spot.
The drag coefficient (C)	The drag coefficient varies depending on the shape and material of the ball affecting the air resistance value.	The drag coefficient C is 0.47 (Hodder Education, n.d., pp. 16-17). corresponding to a smooth spherical object. The same two smooth spherical objects are used throughout the experiment.
Air turbulence	The movement and path the falling ball will take won't be completely vertical if there is turbulence.	To avoid any turbulence in the environment and to ensure that the ball falls in a straight line, doors and windows are closed in that area.

VII. Apparatus:

Two spherical objects:

- 24-inch Beach balls from the brand Elcoho
- Plastic Size 5 Football from the brand called PVC
- A measuring tape (±1mm) is used to measure the circumferences.
- A weight scale (±0.5g) is used to measure the mass of these balls

- A set of disc shaped masses is used to enhance to weight of the spherical objects. These weights are attached to the ball using duct tape.
- A laser measuring tool (±1mm) is used to measure the height.

An iPhone 11 on camera stand is used to record the fall. Logger Pro® 3 – Vernier programme is used for video analysis, permitting time measurements. The video takes 30fps. Resulting in an accuracy of 0.033sec. Later on, a method will be presented to improve this accuracy. In the video a metre stick is placed into the background to gauge height in the video. The video can capture this length to an accuracy of ±1cm.

VIII. Methodology:

Experimental Set Up:

Experiment is conducted in the school hall as shown in Figure 4. Against the staircase there is a metre stick to gauge height in the video analysis later on. When dropping the ball, it is consistently dropped from the same height. The iPhone is on the stand is at 3 metres from impact.

Procedure:

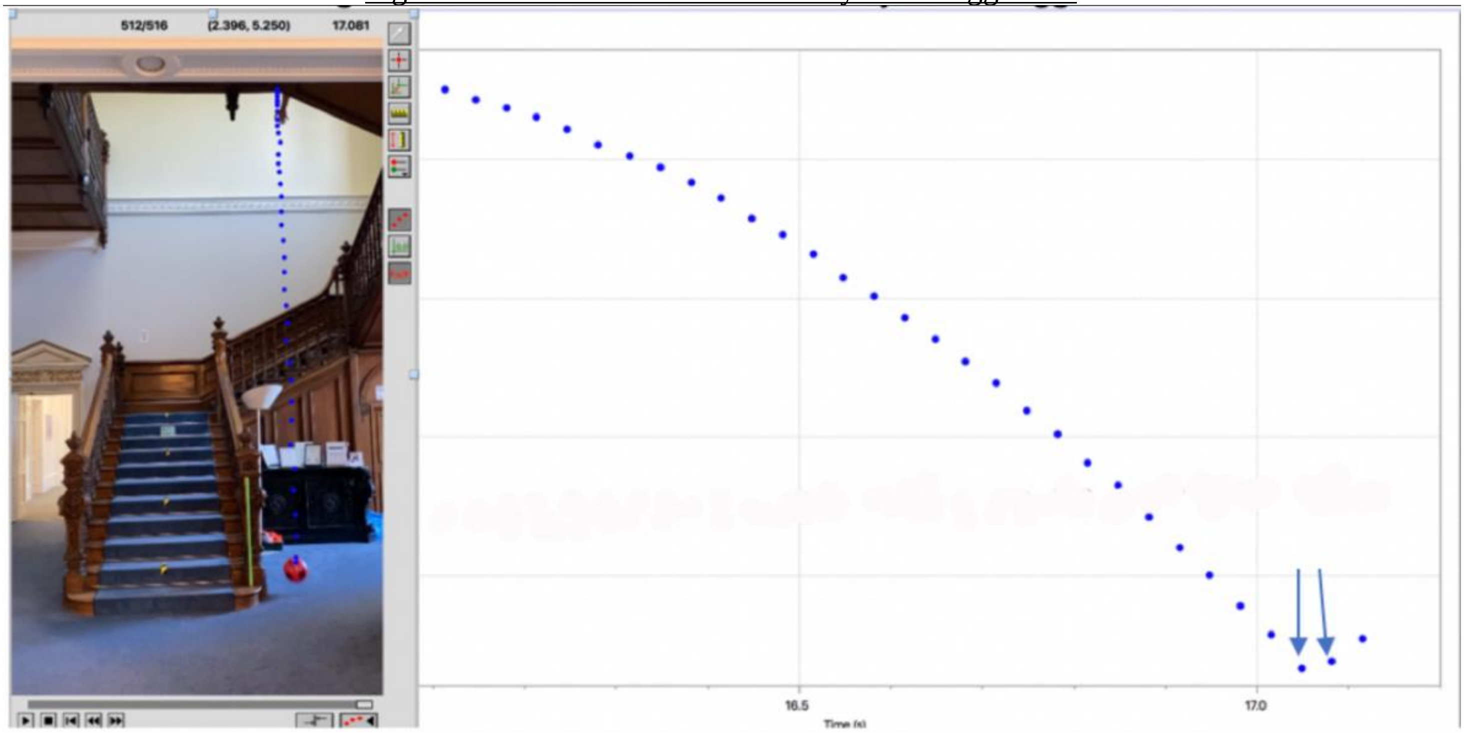
1. Put the iPhone on the camera stand while it is recording.
2. Bring the ball up the stairs and hold it against the landing where you previously marked with tape.
3. Drop the ball carefully without exerting any lateral forces on it to ensure it falls down in a straight line.
4. Go back down the stairs, end the video.
5. Repeat these procedures applying the disc shaped masses on the ball with duct tape as the image below.

Figure 4: Spherical object with attached weight disc and the experimental hall

In total 70 trials were recorded (with varying circumference and mass).

Video analysis:

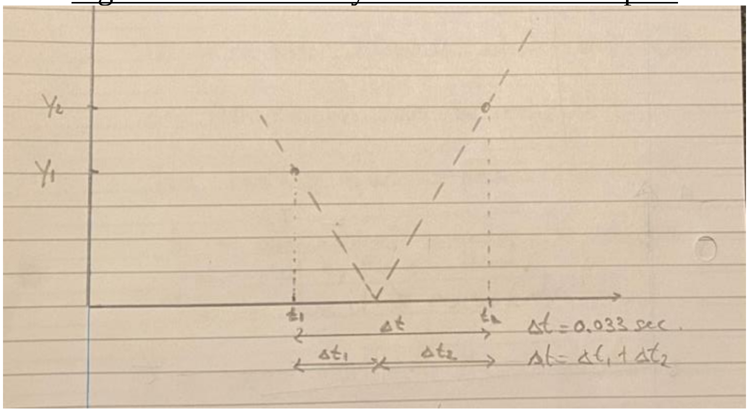
Figure 5: Screenshot from video analysis in LoggerPro.



The blue dots in LoggerPro programme show the position of the ball every 0.033 seconds (the time scale in the plot above is not the fall time but the internal time of the video, the ball does not start falling at t = 0).

The ball hit the ground between the two points indicated by the arrows in figure 5, bottom right hand side. The time difference between these two points is 0.033 seconds. The following procedure is used to estimate when the ball hits the floor in between these two measurements in order to improve the accuracy of time measurements

Figure 6: Further analysis of the time of impact



With reference to figure 6; $Y_1 = V_1 \Delta t_1$, $Y_2 = V_2 \Delta t_2$, where V_1 is the average velocity over time interval Δt_1, and V_2 is the average velocity over time interval Δt_2. t_1 is the time of the ball just before impact, and t_2 is the time of the ball just after impact.

Assumed is $|v1| = |v2| = v$. So, $Y_1 = V_1 \Delta t_1$, $Y_2 = V_2 \Delta t_2 = v (\Delta t - \Delta t_1)$
And therefore, $Y_1 - Y_2 = v \Delta t_1 - v (\Delta t - \Delta t_1) = 2 v \Delta t_1 - v \Delta t$.

Hence, $$\Delta t1 = \frac{1}{2}(\Delta t + \frac{Y1 - Y2}{v})$$

Note that if $Y_1 = Y_2$, $\Delta t_1 = \Delta t_2$ as expected; if e.g. Y_1 or Y_2 are too close to 0, then the same can be said for Δt_1 or Δt_2 and the impact point of the ball is close to t_1, or t_2. Then $\Delta t_1 = \frac{1}{2}\Delta t_2$.
In figure 6, Y_2 is bigger than Y_1 but it can be smaller as well. The figure just illustrates the derivation of Δt_1. However, this derivation is general.

The accuracy of this procedure is limited because of the assumptions on the velocity close to impact. The velocity after impact is expected to be somewhat smaller as the ball does not bounce back elastically. Can we reasonable assume that the ball lands in the first half or second half of (t_2-t_1)? Yes. Can we reasonably determine if it lands on the first quarter or the second quarter of ($t_2 - t_1$)? Yes, it is thought so. Therefore, it is not unreasonable to assume that it is possible to pinpoint the impact to a time interval of one-quarter of Δt, i.e. $\pm\frac{0.033}{4}$ sec which is roughly ±0.008sec. This is an estimated guess.

IX. Environmental, ethical and safety issues

There are no environmental impacts nor ethical issues.

However, there are a few minor safety issues. One of them being that the person conducting this experiment could have tripped on the staircase as he or she walked up and down the stairs 70 times.

Dropping balls with metal weights down from the landing could have dropped on a passer-by. However, the weight is too small to seriously harm anyone, and the experimenter did an alerting counting for anyone nearby.

X. Raw Data

Table 1: The Measured Fall Time for A Ball with a Circumference Of 75.5cm at 5 Different Masses										
Mass (g)	Trial 1 (s)	Trial 2 (s)	Trial 3 (s)	Trial 4 (s)	Trial 5 (s)	Trial 6 (s)	Trial 7 (s)	Trial 8 (s)	Trial 9 (s)	Trial 10 (s)
78g	1.152	1.156	1.146	1.137	1.106	1.106	1.138	1.180	1.115	1.119
103g	1.139	1.148	1.086	1.152	1.105					
128g	1.140	1.080	1.147	1.105	1.122	1.079	1.088	1.102	1.053	
153g	1.143	1.109	1.113	1.118	1.108	1.110				
178g	1.010	1.082	1.014	1.047	1.015	1.041	1.072	1.041	1.042	1.017

Table 2: The Measured Fall Time for A Ball with a Circumference Of 122.2cm at 5 Different Masses							
Mass (g)	Trial 1 (s)	Trial 2 (s)	Trial 3 (s)	Trial 4 (s)	Trial 5 (s)	Trial 6 (s)	Trial 7 (s)
66g	1.569	1.690	1.674	1.673	1.641	1.708	1.709
116g	1.338	1.308	1.376	1.304	1.346		
166g	1.238	1.190	1.215	1.225	1.214		
216g	1.181	1.175	1.176	1.213	1.213		
266g	1.179	1.149	1.109	1.074	1.076	1.078	

The numbers of trials for each set are not equal as sometimes video recording did not turn out to be usable.

XI. Processed data

In table 3 below the processed data are shown. The standard deviation of the time measurements is also shown.

Table 3 (Processed Data): Average Measured Time with Different Masses

Table 3a: Circumference of 75.5cm Ball		
Mass (g)	Measured Time (s)	Error in Time (STDEV)
78	1.136	0.024
103	1.126	0.029
128	1.102	0.031
153	1.117	0.013
178	1.038	0.025

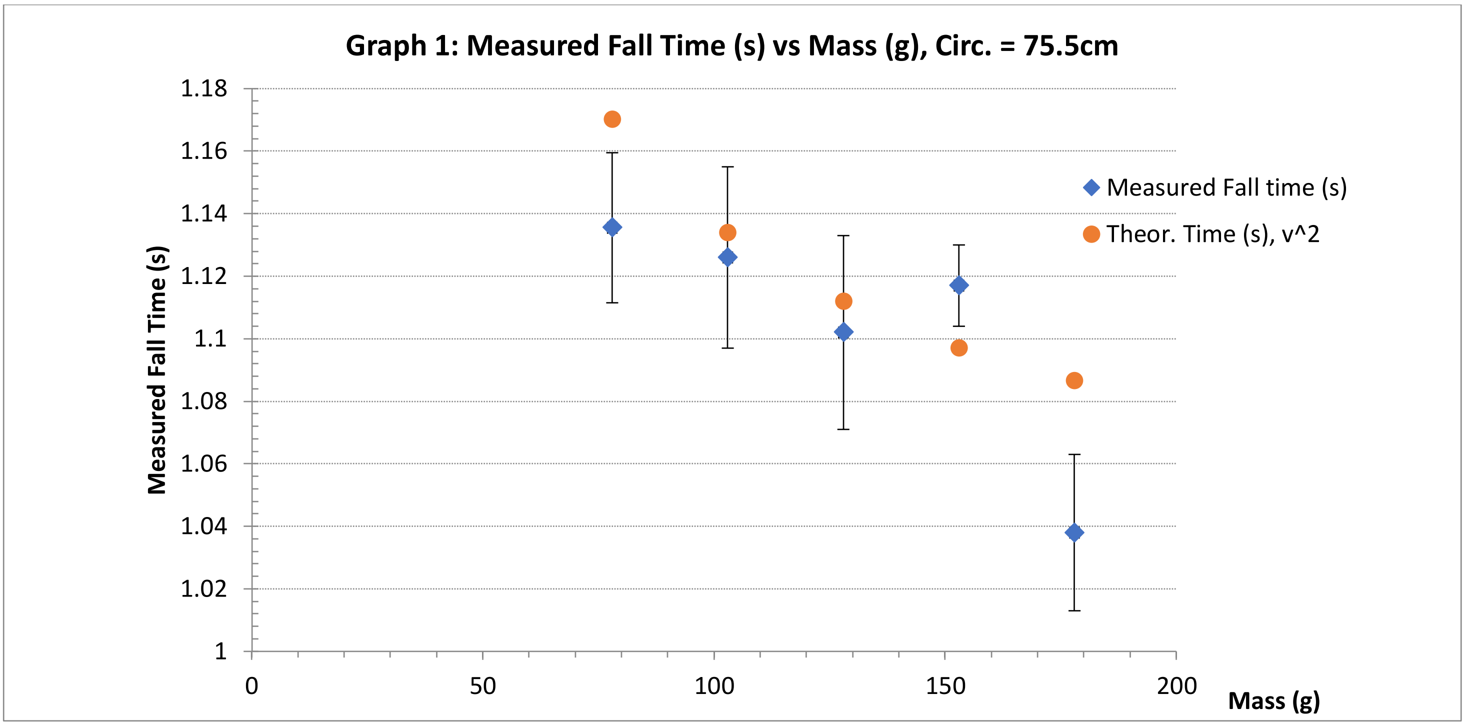
Table 3b: Circumference of 122.2cm Ball		
Mass (g)	Measured Time (s)	Error in Time (STDEV)
66	1.666	0.049
116	1.335	0.030
166	1.217	0.017
216	1.191	0.018
266	1.112	0.044

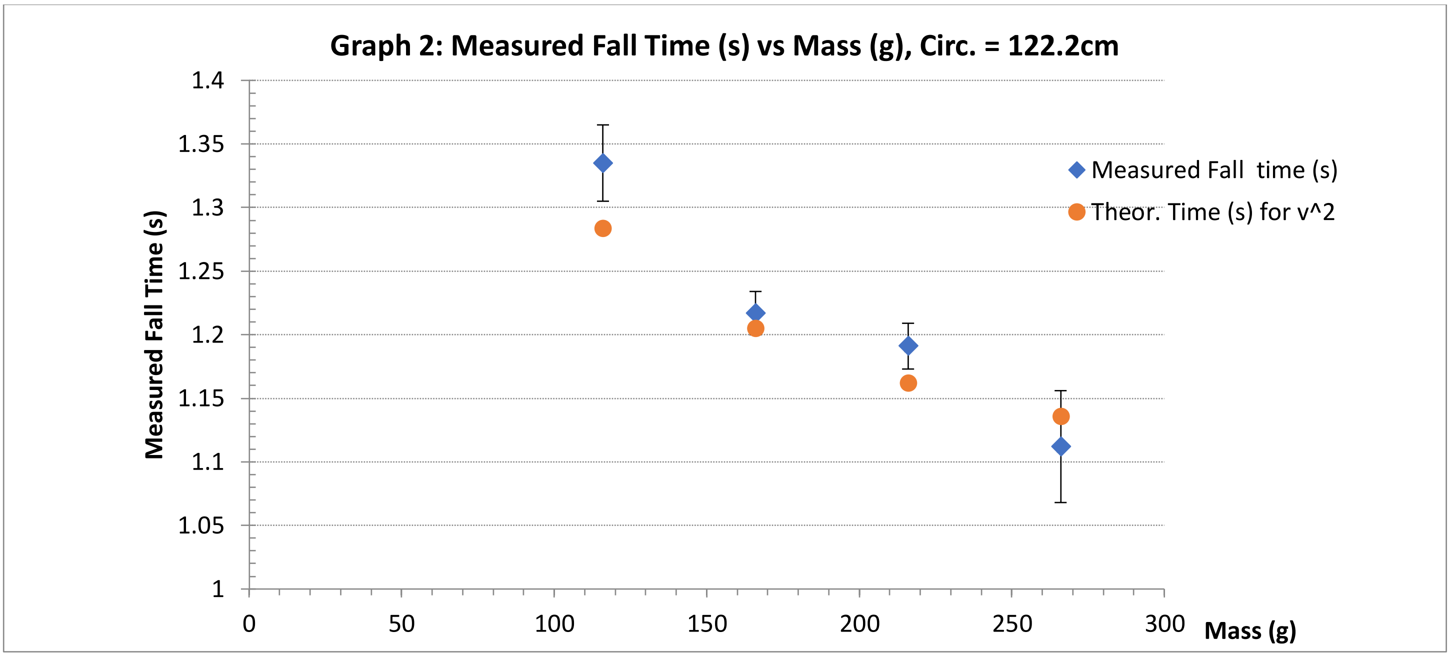
The data for the ball with circumference of 122.2cm, and a mass of 66g was removed for the rest of the processing data. When re-watching the videos, it was noted that the ball swirled a lot and didn't fall in a straight line, probably caused by air turbulence. Leading to the decision of removing this data point for further analysis as it was unreliable.

In graphs 1 and 2, measured fall time are shown together with the theoretical predictions for the fall time, assuming a velocity squared dependence for the air resistance. Graph 1 is for the spherical object with a circumference of 75.5cm.

The error bars shown represent the total error on the time measurements. **There were two dominant sources of error. First the statistical standard deviation on the data points, and secondly, the error on the exact**

measurement of the time of impact as discussed below figure 6. The impact time error is 0.008sec and standard deviation are shown in the table 3. Since the impact error and standard deviation are independent errors the total error (Lyons, 1999, p. 16) is $\sqrt{STDEV^2 + 0.008^2} \approx STDEV$, because the impact time terror is much smaller than the STDEVs.





Graph 1 and 2 summarizes the measurements and its errors. Moreover, it shows the theoretical calculations of the fall time including air resistance, assuming that it is proportional to v^2. It shows nicely that when the mass increases the relative importance of air resistance becomes smaller and the measured time becomes closer to the free fall time (Equation 1a) of $t = \sqrt{2h/g}$ which is 1.023 sec.

An observation made from Graphs 1 and 2 is that the measured fall time is broadly consistent with the fall times calculated using the velocity squared hypothesis for the air resistance. The theoretical values are relatively close to the measurements including error bars.

In table 4 predictions are shown for various additional hypothesis for the air resistance along with the measured data. Fall times were calculated for the following assumptions for velocity dependence of the air resistance: V^1, V1$^{.5}$, $V^{1.9}$, $V^{2.1}$, $V^{2.5}$ in order to compare it to our hypothesis of V^2.

Table 4: Comparing Measured Time with Theoretical Times of V^x							
Circumference 75.5cm Ball							
Mass (g)	Measured Time (s)	Theoretical Time (s) V^1	Theoretical Time (s) $V^{1.5}$	Theoretical Time (s) $V^{1.9}$	Theoretical Time (s) V^2	Theoretical Time (s) $V^{2.1}$	Theoretical Time (s) $V^{2.5}$
78	1.136	1.052	1.087	1.148	1.17	1.196	1.331
103	1.126	1.045	1.071	1.117	1.134	1.154	1.264
128	1.102	1.040	1.061	1.098	1.112	1.111	1.220
153	1.117	1.037	1.055	1.086	1.097	1.098	1.190
178	1.038	1.035	1.050	1.077	1.087	1.098	1.168
Circumference 122.2cm Ball							
116	1.335	1.075	1.140	1.247	1.284	1.352	1.515
166	1.217	1.059	1.103	1.178	1.205	1.235	1.391
216	1.191	1.050	1.084	1.141	1.162	1.186	1.316
266	1.112	1.045	1.072	1.118	1.135	1.156	1.267
Residual Sum of the Squares		0.1410	0.0152	0.0146	**0.0083**	0.0110	0.1956
R^2 Test		-346%	51.9%	53.8%	**73.8%**	65.3%	-519%

All theoretical calculations in table 4 are calculated using our simulation which allows variation of mass, circumference and velocity assumptions. The residual sum of the squares (see section XII) is the sum over all measurements in table 4.

XII. Statistical Evaluation of the Data

Measured data and theoretical predictions are often compared using regression analysis (Wathall et al., 2019, p. 349). Regression involves the calculation of the so-called Residual Sum of the Squares (RSS), which is be defined as $RSS = \Sigma_i\,(t_i - y_i)^2$, where t_i is the measured time and y_i is the theoretical prediction and the sum is taken for all the data points all object sizes and masses. **In a regression analysis the best fit to the data is that that minimizes the sum of the squares.** This procedure is correct even if the relation is not linear.

In the last line of table 4 RSS is shown for each different assumption and it appears that the initial hypothesis, V^2, is the best fit to the data because it gives the lowest value of 0.0083 for RSS. The RSS for V^1 and $V^{2.5}$ are clearly much higher, and our data is clearly inconsistent with the hypotheses of V^1 and $V^{2.5}$. The RSS for $V^{1.9}$ and $V^{2.1}$ are relatively close to the best fit value of V^2.

To evaluate the strength of the regression a quantity called R^2 is calculated. It is defined as $R^2 = 1 - \frac{RSS}{TSS}$ where TSS = $\Sigma_i\,(t_i - t_{mean})^2$, and t_{mean} is the average of the time measurements for each of the balls (Khan Academy, n.d.). Strictly speaking R^2 is valid only for linear regression, which is not the case here. However, looking at the graphs 1 and 2 above, it is somewhat close to a linear relation. Thus, R^2 could be useful in this scenario. **The R^2 of the data turns out to be 0.738, which means that about 74% of the total variance is explained by the model. The residual variance is roughly 26% of the total variance. This indicates us that the V^2 hypothesis seems to be valid to a reasonable extent.**

XIII. Conclusion

It has been found that the data is consistent with the hypothesis of velocity squared dependence on air resistance for a falling spherical object. Hence, the fall time is determined by a = g - $\frac{k}{m}$ x V^2, where K = $\frac{1}{2}$(ρ) (A) (C), A being the cross-sectional area (equations 2 above). It should be noticed that V^1 and $V^{2.5}$ hypotheses for dependence of air resistance are clearly excluded by our data. Hypothesis V^x with $1.5 \leq x \leq 2.1$ cannot be clearly excluded by our data.

XIV. Evaluations & Improvements

One of the strengths of this experiment was the development of a versatile computer simulation for the fall time, the employment of which was crucial for obtaining the above results.

It appeared that the error on the measurements is largely dominated by the statistical standard deviation, so an obvious improvement would be to take more data, e.g. 4-5 times as much.

In the statistical analysis above, error on the time measurements wasn't included. The statistical analysis could be improved by including error in the regression analysis.

Data acquisition using videos was cumbersome, a more automated device for measuring time (e.g. laser triggered) would be an enormous help. Such a device would also reduce substantially time need to process the raw data.

An increased height to drop objects from would be welcome. Measured would become more precise on a relative basis and the range of velocities would be extended.

As mentioned in the introduction, access to a wind tunnel would have been ideal, an experiment would have been totally different, not including time measurements but rather measuring air resistance directly.

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