Formal Lab Report: Zeeman Effect

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Abstract

This experiment seeks to investigate the Zeeman's effect, and to determine the relationship between the fractional order splitting, Δk, with the charge to mass ratio, e/m_e. Moreover, the final result achieved ($\frac{e}{m_e} = (\mathbf{1.529 \pm 0.043})\ \boldsymbol{x}\ \mathbf{10^{11}}\ \frac{\mathbf{C}}{\mathbf{kg}}$) establishes a strong positive linear relationship between Δk, and e/m_e, which lies within the correct order of magnitude, and only 14.86% from literature value. Further, the investigation explores how this influences the frequency of a spectral line, Δf, and to determine its relationship with Δk, which was determined to be directly proportional, and highlighting a linear relationship between e/m_e and Δf.

Introduction

The Zeeman effect refers to the phenomenon where a single emission line splits into three components (σ^+, π, σ^-) when the source is in the presence of a magnetic field (Feynman et al., 1963). The splitting occurs because of the degeneracy breakdown amongst electrons orbitals, which happens when the orbitals are aligned with the magnetic field (Department of Physics, King's College London, 2023). In the absence of a field, the magnetic quantum number does not have an influence on an orbital's electron energy, yet, when a magnetic field is introduced, orbitals align according to their values causing a slight energy shift (Feynman et al., 1963). The role of quantum numbers is to define an atom's electronic state, which includes factors such as energy levels, orbital type, and total angular momentum ('Quantum Numbers and Electron Configurations,' Purdue University, 2020). When an electron transitions between orbitals it causes a photon to be emitted. The photon's frequency is dependent on the distinct orbital it shifts to, resulting in the observed singlet line splitting (Department of Physics, King's College London, 2023). The exploration of electronic configuration and spectral transitions through quantum parameters allows further understanding of phenomena in atomic physics, including the normal Zeeman effect (Feynman et al., 1963).

The aim of the experiment is to investigate the phenomenon of splitting singlet emission lines into three components, when the source is in the presence of a magnetic field, also known as the Zeeman effect. The realignment of electron orbitals causes a shift in the orbital energies which leads to the spectral line splitting (Department of Physics, King's College London, 2023). Further, the specific objective of this experiment is to electronic charge to mass ratio (e/m_e) from the frequency separation of a singlet spectral line (Cadmium; 643.8nm) in a magnetic field.

Theoretical Background

The Normal Zeeman Effect

The Zeeman effect was first predicted by H.A. Lorenz in 1895 and experimentally confirmed by P. Zeeman describes the splitting of atomic energy levels under influence of an external magnetic field (Department of Physics, King's College London, 2023). Three emission lines are observed at a right angle, whilst double lines are parallel to the magnetic field, this is called the Zeeman effect which can be categorized into normal and anomalous ('Zeeman Effect | Normal, Anomalous & Paschen–Back Effect,' MinutePhysics, 2023).

The anomalous Zeeman effect's total spin (S) is not zero, meanwhile the normal Zeeman effect undertakes transition between atomic states when the total spin of **S is 0** (Department of Physics, King's College London, 2023). The total angular momentum quantum number, J, is defined to be **J = L + S**, of a state is then a pure orbital angular momentum **(J = L)** ('Quantum Numbers and Electron Configurations,' Purdue University, 2020). Then the magnetic moment, μ, can be written as (Department of Physics, King's College London, 2023):

$$\boldsymbol{\mu} = \frac{\mu_B}{\hbar}\boldsymbol{J} \quad (1)$$

Where μ_B (Bohr's magneton) can be expressed as,

$$\mu_B = \frac{\hbar e}{-2m_e} \quad (2)$$

Where $\hbar$ is Planck's constant divide by a factor of 2π, e is the elementary charge, m_e is the mass of an electron.

In an external magnetic field B, the magnetic moment has the energy (Department of Physics, King's College London, 2023),

$$E = -\mu \cdot \boldsymbol{B} \quad (3)$$

The angular momentum component in the direction of the magnetic field (J_Z) is written as ('Quantum Numbers and Electron Configurations,' Purdue University, 2020),

$$J_Z = M_J \cdot \hbar\,, \quad with\ M_J = J\,, J-1\,, \ldots\,, -J \quad (4)$$

Thus, leading to the splitting of the term with angular momentum J into 2J+1 equidistant Zeeman components differing by the value of the magnetic quantum number for angular momentum (M_J). The interval energy between adjacent components is expressed as (Department of Physics, King's College London, 2023):

$$\Delta E = \mu B \quad (5)$$

To visualize the normal Zeeman effect better, it can be explained as a diagram. For the sake of this example, the element Cadmium will be taken as reference for the red spectral line, which has a wavelength of 643.8 nm, and its corresponding frequency is 465.7 THz (Department of Physics, King's College London, 2023). An electron shell on its fifth shell has a level $^1D_2$ (J=2, S=0) where the Zeeman effect separates the lines into 5 components ('Zeeman Effect,' HyperPhysics - Georgia State University). Whereas, when the electron transitions to the level $^1P_1$ (J=1, S=0) the Zeeman effects projects three components (as shown in Figure 1) (Department of Physics, King's College London, 2023).

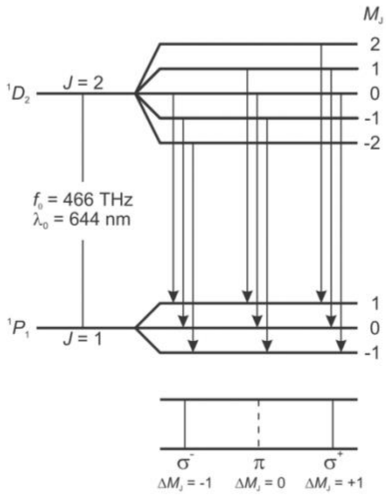

Figure 1: Diagram of the Normal Zeeman effect in transition between levels (Department of Physics, King's College London, 2023).

Electrical dipole radiation is a requirement for optical transition to occur, where magnetic number $\Delta M_J = \pm 1$, for σ components, and 0, for π components ('Zeeman Effect,' Wikipedia, 2023). Resulting in three spectral lines as shown in Figure 1, and the two σ components are shifted by the frequency relation with energy,

$$\Delta f = \frac{\Delta E}{h} \tag{6}$$

Where the π component is not affected by equation 6, and therefore is not shifted.

Angular Distribution and Polarization

The dependence of the angular moment component, ΔM_J , arranged in line with a magnetic field is crucial for the angular distribution and polarization of emitted photons ('Zeeman Effect,' Wikipedia, 2023). Angular distributions are revealed when a magnetic field has a common axis for all atoms of a substance, which experimentally occurs for Cadmium (Department of Physics, King's College London, 2023).

No photons are emitted in the direction of the magnetic field when the angular momentum component ΔM_J is 0, and it represents the small dipole oscillating parallel to the field ('Zeeman Effect,' Wikipedia, 2023). The E-vector oscillating in the dipole's direction, and parallel to the magnetic field results in photons being emitted perpendicularly to the magnetic field, also referred to as linear polarization ('Zeeman Effect,' Wikipedia, 2023).

A large proportion of light travels in the direction of the magnetic field, however when $\Delta M_J = \pm 1$ refers to two parallel dipole oscillating with 90° phase difference ('Zeeman Effect,' Wikipedia, 2023). To dictate what direction light travels when quanta is emitted in a circular-polarized way, the angular momentum component ΔM_J needs to be determined. If $\Delta M_J = +1$ then it travels clockwise, and $\Delta M_J = -1$ it travels anticlockwise (Department of Physics, King's College London, 2023).

Mathematical Proof of the Fractional Splitting Order being 1/2, ($\Delta k = 1/2$):

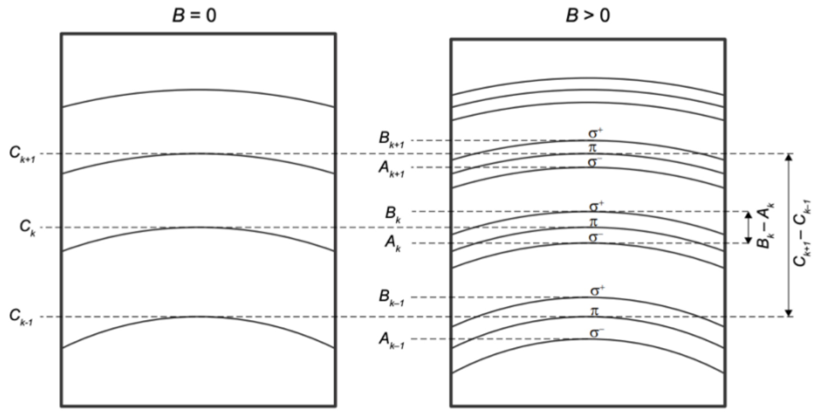

Figure 2: Demonstration of the Zeeman Effect. Where C_k represents the position of the unsplit spectral line for the π component, and A_k and B_K represent the positions of the split lines for the σ^+ and σ^- components (Department of Physics, King's College London, 2023).

Therefore, the fractional splitting order, Δk is expressed as (Department of Physics, King's College London, 2023),

$$\Delta k = \frac{B_k - A_k}{C_{k+1} - C_{k-1}} \tag{7}$$

If the fractional splitting order, Δk is equal to 1/2, that implies that,

the line $A_k = B_{k-1}$, and $B_k = A_{k+1}$

due to the normal Zeeman effect occurring and splitting the fringes when a magnetic field is present. So, it can be said that the line B_k now resides in between C_{k+1} and C_k because the spacing between each "curved line" is equal, meaning that,

$$B_k = \frac{C_{k+1} + C_k}{2}$$

And similarly, for,

$$A_k = B_{k-1} = \frac{C_k + C_{k-1}}{2}$$

Hence,

$$B_k - A_k = \frac{C_{k+1} + C_k}{2} - \frac{C_k + C_{k-1}}{2} = \frac{C_{k+1} - C_{k-1}}{2}$$

Plugging this back into equation 7 gives,

$$\Delta\mathrm{k} = \frac{\frac{C_{k+1} - C_{k-1}}{2}}{C_{k+1} - C_{k-1}} = \frac{1}{2}$$

QED.

Experimental Procedure

Apparatus

The experiment could only be conducted with the following equipment: a Cadmium Lamp with holding plates, Clamps, Pole Pieces, Positive Lens, f = 150mm for a Condenser Lens and an Imagine Lens, Fabry-Perot etalon, Hall Probe, Colour Filter (red) in holder, Ocular with Line Graduation, Ruler, iPhone, Laptop. The experimental set up can be seen below in Figure 3.

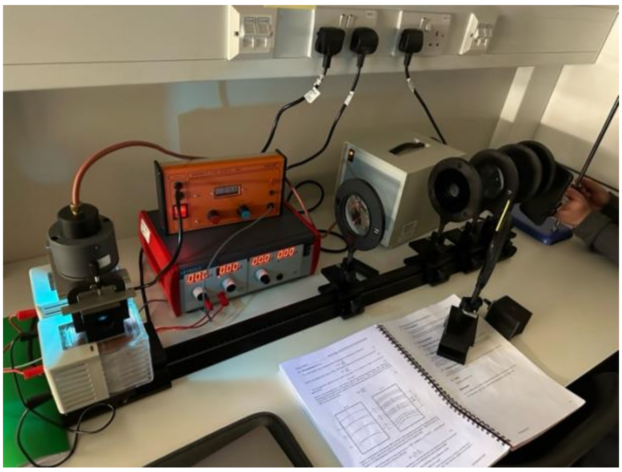

Figure 3: Experimental Set Up

Risk Assessment

For starters, there are some delicate equipment being used so do not touch the glass or any electronic parts when dealing with the Cadmium Lamp, Hall Probe, or the Fabry Perot Etalon. They must be dealt with care as the apparatus' are very fragile.

It should be noted that the Cadmium Lamp gets very hot, so be cautious and do not touch the lamp.

Unused ferromagnetic parts such as steel screws or certain electronic devices can interfere with the magnetic field, by weakening or demagnetizing it. So, they must be kept at a distance from the apparatus that makes a magnetic field.

Methodology

Initially, it is crucial to ensure that the experiment is set up correctly as shown in Figure 3, and if in doubt, ask the demonstrator to check if the task was done correctly.

Afterwards, a preliminary experiment undergoes to calibrate the magnetic field. The aim is to find the proportionality factor, α, between the power supply current I and the magnetic field, which can later be used to calculate B directly from I. The experimenter needs to set the Hall probe using the mechanical clamp between the pole pieces. Record the magnetic field as a function of the current, along with its corresponding currents in increments of 0.2A ranging from 0A to 2A.

Further, the following procedure will undergo to carry out the experiment.

1. Check if the polarization filter in the direction of the lens, if it is, remove the filter.
2. The experimenter needs to take a picture of the view of the lens for each corresponding current from 5.5A to 7.5A in increments of 0.5A.
3. It is necessary to ensure that there are three pictures being taken, one without the polarization filter, when with the filter set at 0, and when set at 90 (to view the different components of the Normal Zeeman Effect) for each current.
4. Then, analysis the pictures and measure the distance between the σ^+ and σ^- components (Polarization Filter set at 0), and the distance between the π lines adjacent to the central π line (No Polarization filter) (see equation 7).
5. Finally, Use the data collected to determine e/m_e and to calculate statistical analysis.

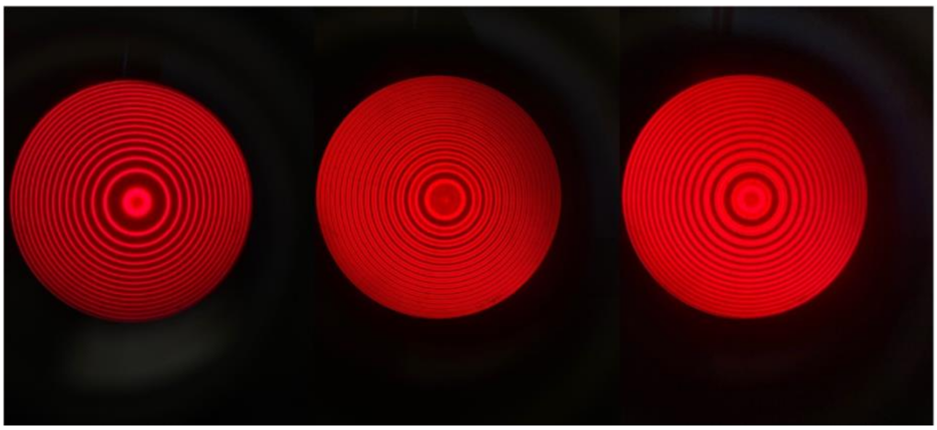

Figure 4: Cadmium Spectral Lines with no polarization filter and with it at 0°, and 90°

As seen in Figure 4 above, the Cadmium light undergoes the normal Zeeman effect as the images showcase different components of the spectral lines due to the polarization filter. The right-hand side of Figure 4 contains three components (σ^+, π, σ^-), however it is quite unclear and blurry compared to the image in the left-hand side, which only has one component (π). So, for step 4 in the methodology, it is evident that it easier to measure the distance between the adjacent π lines from the central π line from the left-hand side of Figure 4. The middle image of Figure 4 highlights two components (σ^+ and σ^-), which is used to measure the distance between them in the procedure for the experiment.

Results and Discussion

The data collected is presented in Figure 5 below, where the error propagation is derived from the statistical analysis of the combination of experimental errors. For the general case it can be expressed as,

$$(\Delta U)^2 = \sqrt{\left(\frac{\partial f}{\partial x}\,\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\,\Delta y\right)^2} \tag{8}$$

Where U is a function of x and y.

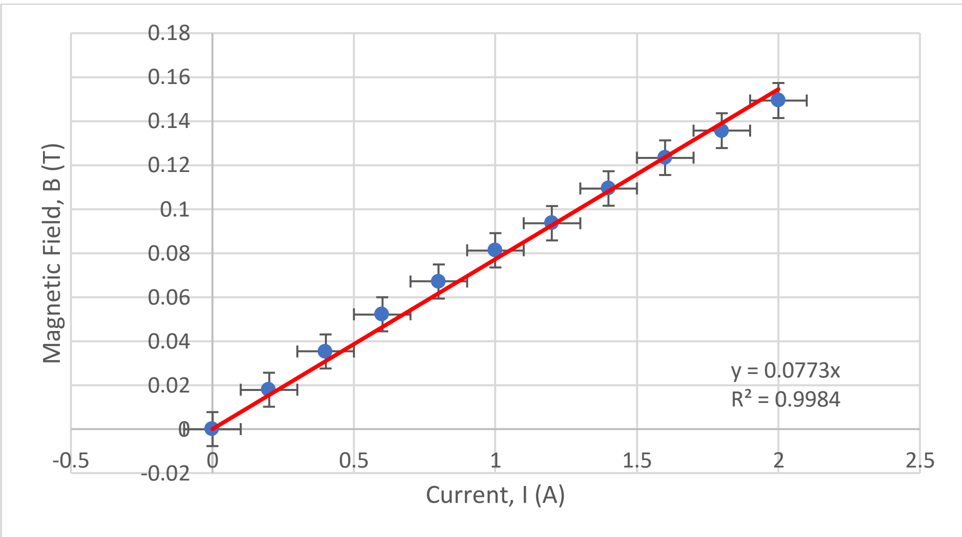

Figure 5: Graph of Magnetic Field, B, plotted against Current, I.

Figure 5 is evidence that the magnetic field, B, is directly proportional to the current, I. Further, the R^2 correlation is approximately 1 showcasing that there is a strong positive linear relationship between the two parameters. The error bars were calculated using equation (8), and they rely within a good range of the linear fit implying minimal error.

This relation between the current and the magnetic field allows the experimenter to extrapolate a value of the field, B, for any current I, using linear regression on Microsoft Excel through the function ‘LINEST’.

Moreover, it is necessary to calculate the fractional order splitting, Δk, to be able to proceed and determine the charge to mass ratio, e/m_e, of the normal Zeeman effect. After completing step 4 in the methodology, to calculate Δk, the ratio must be taken between the distance of the σ^+ and σ^- components, and the distance between the π lines adjacent to the central π line. It should be noted that the measurements of the distances for the different components of the Zeeman effect were taken from a ruler. In addition, when measurements are taken from both ends of the ruler it is necessary to use equation 8 to regulate the error propagation which will contribute to the uncertainty on Δk. The uncertainties for Δk was determined through error analysis (equation 8), and the magnetic field B was extrapolated for the corresponding current values from linear regression on Microsoft Excel, as previously explained. All the processed data is provided in the table below.

Table 1: Processed Data for the Magnetic Field B, and the Fractional Order Splitting Δk along with its Uncertainties

Current, I (A)	Magnetic Field, B (T)	Uncertainty on B (T)	Fractional Order Splitting, Δk	Uncertainty on Δk
5.50	0.42	0.01	0.22	0.01
6.00	0.46	0.01	0.23	0.01
6.50	0.50	0.01	0.24	0.01
7.00	0.54	0.01	0.25	0.01
7.50	0.58	0.01	0.26	0.01

It is clear that the current and magnetic field have a linear relationship as previously explained, and the uncertainties on B are considered to be relatively insignificant to the magnetic field. Similarly, it is evident that the magnetic field, B, and the fractional order splitting, Δk, are directly proportional to one another, which will be shown in the graph below (Figure 5). In addition, for the uncertainty on Δk is small relative to the fractional order splitting, Δk, and minimal errors mean that the results are precise, the accuracy of data will be further discussed in the evaluation below.

Calculating The Charge to Mass Ratio, e/m_e

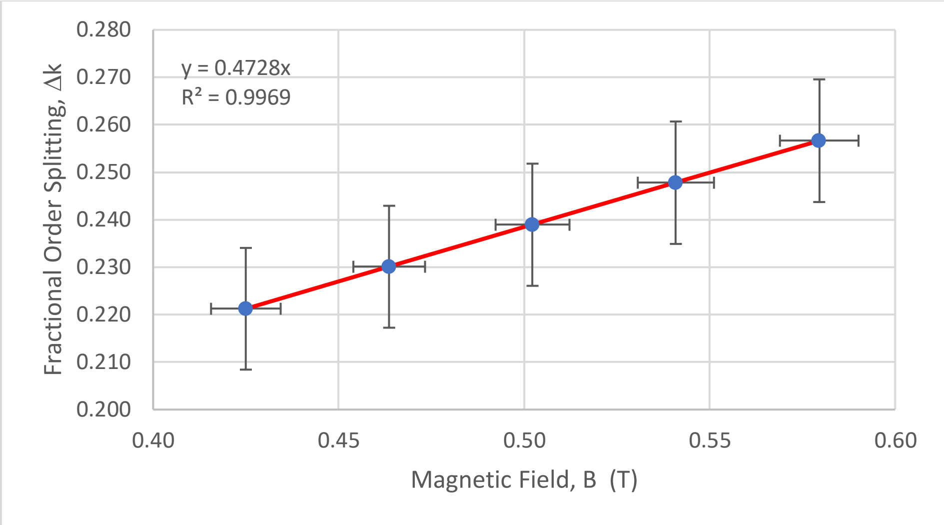

Figure 6: Plotting the Fractional Order Splitting, Δk, against the Magnetic Field, B (T).

Clearly, the fractional order splitting, Δk, has a strong positive linear relationship with the magnetic field, B, which is highlighted through the R^2 correlation approximately being 1. The error bars were calculated from equation (8) and their values are shown in Table 1. The equation of the line is given, and the slope, m, is 0.4728. Analogous to y = 0.4728x, for Δk and B gives (Department of Physics, King's College London, 2023),

$$\Delta k = \frac{dn}{2\pi c}\frac{e}{m_e}B \tag{9}$$

Where d is the thickness (4 mm), n is the refractive index of the medium (1.457), c is the speed of light (3 x 10^8 m/s), e is elementary charge (1.6 x 10^{-19} eV) and m_e is the mass of an electron (9.11 x 10^{-31} kg).

The gradient in Figure 6 is defined as,

$$gradient, m = \frac{dn}{2\pi c}\frac{e}{m_e} = 0.4728 \tag{10}$$

And through statistical analysis, the linear regression, 'LINEST' function in Microsoft Excel, it calculates the uncertainty on the gradient, Δm to be 0.0133, and manipulating modified version of equation 10 for the uncertainty of the charge to mass ration gives $\Delta\left(\frac{e}{m_e}\right)$,

$$\Delta\left(\frac{e}{m_e}\right) = \frac{2\pi c}{dn}\,\Delta\mathrm{m} \;=\; 4.288\,x\,10^9\ \frac{\mathrm{C}}{\mathrm{kg}} \tag{11}$$

It is evident that equation 10 must be re-arranged to determine the charge to mass ratio, $\frac{e}{m_e}$. The charge to mass ratio along with its uncertainty is,

$$\frac{\boldsymbol{e}}{\boldsymbol{m_e}} \pm \Delta\left(\frac{\boldsymbol{e}}{\boldsymbol{m_e}}\right) = (\mathbf{1.529} \pm \mathbf{0.043})\ \boldsymbol{x}\ \mathbf{10^{11}}\ \frac{\mathbf{C}}{\mathbf{kg}} \tag{12}$$

The literature value of equation 12 can be found by inserting the constants for the elementary charge and the mass of an electron which gives,

$$\frac{\boldsymbol{e}}{\boldsymbol{m_e}}(\boldsymbol{Literature}) = \mathbf{1.757\ x\ 10^{11}}\frac{\mathbf{C}}{\mathbf{kg}} \tag{13}$$

It is evident that the literature value falls outside the uncertainty range for the experimenter's data. Consequently, this disagreement between the experimental data and literature value, come to the percentage difference to be 14.86%.

While the experimental data and literature value have a notable difference, it is worthy to note that the experimental result lies within the same order of magnitude and are only off by 14.86%. On one hand this divergence can be regarded as acceptable and not extreme given the complexities of the experimental set up.

On the other hand, it is pivotal to acknowledge that the uncertainties alone do not take account for this difference. There are a number of systematic errors that influence this, such as the angle the Hall Probe was positioned in the Cadmium Lamp to measure the magnetic field, playing a significant role ('Hall Effect Magnetometer,' Electricity - Magnetism). The fragile nature of the Hall Probe can drastically impact the accuracy of the result, as its vulnerability to the variations in the magnetic field readings is dependent on its positioning ('Hall Effect Magnetometer,' Electricity - Magnetism). This aspect will be further explored in the evaluation section discussing the Hall Probe's significant impact on the results.

Calculating Frequency Differences between the σ components in three fractional order splitting's

The frequency between the σ components can be expressed as the following, if f_0 is the frequency of the undisplaced singlet (Department of Physics, King's College London, 2023),

$$\Delta f(\sigma^+ - \sigma^-) = f(\sigma^+) - f(\sigma^-) = (f_0 + \frac{eB}{4\pi m_e}) - (f_0 - \frac{eB}{4\pi m_e}) = \frac{eB}{2\pi m_e} \tag{14}$$

Using the relation between Δk and B (equation 9), equation 14 can be re-arranged as,

$$\Delta f(\sigma^+ - \sigma^-) = \frac{c}{dn}\ \Delta\mathrm{k} \tag{15}$$

So, equation 15 is used to find the frequency for 3 distinct values of Δk. Further, If the variables $\Delta f(\sigma^{+} - \sigma^{-})$ are redefined as Δf, and wavelengths $\Delta\lambda(\sigma^{+} - \sigma^{-})$ redefined as $\Delta\lambda$, then these frequencies can be re-written as wavelengths below.

$$[\lambda_0 - \Delta\lambda] = \frac{c}{[f_0 - \Delta f]} \tag{16}$$

Now, a relation between the fractional splitting order Δk, the frequency between the σ components, Δf, and the corresponding Separation, $\Delta\lambda$, can be established. Their corresponding uncertainties are calculated through statistical analysis (equation 8) and are all shown in the table below.

Table 2: The Fractional Splitting Order Δk, Separation, $\Delta\lambda$, corresponding Frequency, Δf, along with their uncertainties.

Δk	Δf (Hz)	$\Delta\lambda$ (nm)
0.221 ± 0.013	$(1.139 \pm 0.066) \times 10^9$	0.263 ± 0.015
0.239 ± 0.013	$(1.230 \pm 0.066) \times 10^9$	0.244 ± 0.013
0.257 ± 0.013	$(1.321 \pm 0.067) \times 10^9$	0.227 ± 0.011

It is evident that as the fractional splitting order Δk increases so does the frequency Δf, and consequently the separation, $\Delta\lambda$, decreases demonstrating a linear relationship between Δk and Δf, and therefore, an inversely proportional relationship between Δk and, $\Delta\lambda$**.** As expected because the relation between $\Delta\lambda$ and Δf are inversely related from basic principle of physics. The relative uncertainties displayed in table 2, are minimal on the three parameters suggesting good precision in the results. Consequently, equation (9) illustrates a linear relationship between Δk and e/m_e. Furthermore, a relation between e/m_e can be described to be directly proportional with the frequency of a spectral singlet line, establishing a linear relationship between the two parameters.

Evaluation and Improvements:

The evaluation and improvements section seek to critically assesses components of the report and the experiment such as the methodology, results, and potential systematic errors that impact the reliability and accuracy of the results obtained. This extensive analysis seeks to recognise areas of improvements and develop the overall validity of the findings.

For starters the strengths of the experiment will be discussed. Firstly, it is necessary to address the linear relationship between B and I in Figure 5, for Δk and B in Figure 6, and Δk and Δf in Table 2 which all confirm with the theoretical hypothesis in physics phenomena, as predicted. Secondly, the minimal uncertainties highlight precise data measurements meaning there was a strong and successful statistical analysis conducted. Thirdly, the experimenter provided a detailed methodology, as well as analysing potential risks that could occur when conducting the experiment. In doing so, this ensures the experiment's reproducibility and safety. Finally, the final results obtained for the charge to mass ratio, e/m_e is of the correct magnitude and are only off by 14.86% from literature value. The exploration of the experiment was successfully investigated as the experimenter successfully established a relationship between the fractional splitting order, Δk, and the charge to mass ratio, e/m_e.

Nevertheless, every experiment exhibits weaknesses, and the most significant flaw that resides within this lab is systematic errors. Majority of the unaccounted-for errors stems from the Hall Probe's sensitivity, as it is a delicate instrument since a slight change on the angle its being held at can completely change the final results (Sanfilippo and Soulages, 2009). As the magnetic field B is affected, Δk will be impacted, and the same can be said for the frequency, Δf, the separation, $\Delta\lambda$, and as well e/m_e.

For example, the relation between the magnetic field B, and the angle the probe is positioned as can be expressed as (Sanfilippo and Soulages, 2009),

$$V_H = A\ x\ Bcos(\theta) \tag{17}$$

Where, V_H is the potential difference from the hall probe, A is constant, and B is the magnetic field.

If the angle is not parallel and off by 30° the data measurements will be affect by about 15%! Thus, it is crucial that the probe is stable, consistent, and parallel ($\theta = 0$) at all times.

Additionally, the Hall Probe's sensitivity to external factors such as changes in temperature, or electromagnetic interferences will contribute to inaccuracies in the magnetic field's measurement (Sanfilippo and Soulages, 2009). Therefore, having electronic devices or Power supplies must be kept at a distance (EM controlled environment), and a regulated temperature room is a necessity.

Moreover, variations in stability of Cadmium lamp causes inaccuracies of results, and Len's imperfections: distortions or aberrations, affect quality of observed spectral lines. For improvements, experimenter must ensure stable and consistent light, as well as regular lens maintenance.

Conclusion

In summary, the investigation of the Zeeman Effect was successfully conducted, determining $\boldsymbol{\frac{e}{m_e} = (1.529 \pm 0.043)\ x\ 10^{11}\ \frac{\mathrm{C}}{\mathrm{kg}}}$, and it is 14.86% off from literature value. Further, investigation was done for Δk's linear relationship with e/m_e and frequency of a spectral line, Δf, establishing a linear relationship between e/m_e and Δf, which are all expressed in Tables 1 and 2. The consistent linearity along with minimal uncertainty supports the validity of the results obtained for all parameters, and addressing systematic errors that influence said results, reinforcing a successful experiment.

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