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OPERATIONAL RESEARCH (CEF 401)

Linear Programming Problem and Graphical Method to Solve It

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GENERAL INTRODUCTION

Linear Programming (LP) is a mathematical optimization technique with a rich historical background and wide-ranging applications. It has evolved as an essential tool for rational decision-making and resource allocation, shaping various fields. The historical significance of LP can be traced back to the mid-20th century when eminent figures like George Dantzig, Leonid Kantorovich, and Tjalling Koopmans made seminal contributions, laying the foundation for LP as a pivotal mathematical optimization method. In today's context:

i- **Engineering and Operations Research:** It optimizes resource allocation in engineering projects, scheduling, and supply chain operations.

ii- **Agriculture:** LP enhances resource management in agriculture, improving land and crop utilization.

iii- **Public Policy:** Governments use LP for efficient resource allocation in healthcare, education, and infrastructure.

iv- **Environmental Impact:** LP contributes to environmentally responsible practices in waste management and energy allocation.

This project emphasizes the increasing significance of LP and the graphical method as fundamental skills for addressing resource allocation challenges across domains. Proficiency in these techniques equips individuals to make informed decisions and contribute to efficiency and sustainability.

1. Definition of Linear Programming:

Linear programming is a mathematical method used to optimize operations while adhering to certain constraints. Its primary goal is to maximize or minimize a numerical value. Linear programming problems involve linear functions subject to constraints in the form of linear equations or inequalities. The term "linear programming" comprises two words: "linear," indicating the relationship between variables with a degree of one, and "programming," defining the process of selecting the best solution from various alternatives.

2. Components of Linear Programming:

A linear programming problem consists of two fundamental components:

Objective Function: This function defines the primary aim of the optimization, such as maximizing profit or minimizing cost.

Constraints: Constraints represent a set of equalities or inequalities that describe the limitations or restrictions under which the optimization must occur.

3. Linear Programming Formula:

A linear programming problem is composed of the following elements:

Decision Variables: These variables, denoted as *'x' and 'y'*, determine the outcome of the problem and represent the final solution.

Objective Function (Z): Z is the linear function that must be optimized based on the given conditions to yield the final solution.

Constraints: These are the limitations imposed on decision variables, typically in the form of linear equations or inequalities.

Non-Negative Restrictions: Decision variables must always have non-negative values, which are enforced by these restrictions. The general formula of a linear programming problem is as follows:

Objective Function: $Z = ax + by$

Constraints: $cx + dy \leq e$, $fx + gy \leq h$ (or the inequalities can be "$\geq$")

Non-negative restrictions: $x \geq 0$, $y \geq 0$

4. How to Solve Linear Programming Problems:

To solve a linear programming problem effectively, it is essential to follow a structured approach:

Step 1: Identify the decision variables.
Step 2: Formulate the objective function, determining whether it needs to be minimize or maximize.
Step 3: Specify the constraints.
Step 4: Ensure that the decision variables have non-negative values (non-negative restraint).
Step 5: Solve the linear programming problem using either the simplex method or the graphical method.

5. Linear Programming Methods:

There are two primary methods for solving linear programming problems: the simplex method and the graphical method. The graphical method is particularly useful when dealing with problems involving two decision variables.

Problem Description:

The furniture dealer faces a decision-making scenario involving two types of products: tables and chairs. The dealer has a budget of Rs 50,000 for investment and storage space for a maximum of 60 pieces. Tables are priced at Rs 2,500 each, while chairs cost Rs 500. The dealer anticipates earning a profit of Rs 250 for each table and Rs 75 for each chair sold. The objective is to determine the optimal quantity of tables and chairs to purchase within the budget to maximize total profit, assuming that all purchased items can be sold.

Analysis of Problem Statement

1. Dealer's Inventory: The dealer manages a product inventory consisting of two types: chairs and tables.

2. Available Budget: The dealer has Rs 50,000 at their disposal for investment in inventory.

3. Storage Limit: The storage space is limited to a maximum of 60 pieces.

4. Cost and Profit: Each table costs Rs 2,500 and yields a profit of Rs 250 when sold. Each chair costs Rs 500 and generates a profit of Rs 75 per sale.

Objective: Determine the optimal quantity of tables and chairs to purchase with the available budget to maximize the total profit.

Mathematical Formulation:

1. Tables-Only Scenario: In one extreme scenario, the dealer could choose to purchase only tables with the entire budget. This scenario would allow the purchase of 20 tables (Rs 50,000 / Rs 2,500 per table) and result in a profit of Rs 5,000 (20 tables * Rs 250 profit per table).

2. Chairs-Only Scenario: Alternatively, the dealer could opt for purchasing only chairs with the budget. This approach would enable the acquisition of 100 chairs (Rs 50,000 / Rs 500 per chair) and yield a profit of Rs 4,500 (100 chairs * Rs 75 profit per chair).

Given that various combinations of tables and chairs could be purchased (as exact quantities are not specified), let's assume the dealer purchases X tables and Y chairs, with X and Y being non-negative.

The following constraints can be derived:

1. Investment Constraints: The dealer's budget constraint is represented as $5X + Y \leq 100$ (since Rs 50,000 is available for investment).

2. Storage Constraints: The total number of units should not exceed the storage space limit, which is expressed as $X + Y \leq 60$.

The dealer's objective is to maximize profit (Z) based on the equation:

Max Z: 250X + 75Y

Subject to Constraints:

- $5X + Y \leq 100$
- $X + Y \leq 60$
- $X, Y \geq 0$

By solving this linear programming problem, the dealer can make informed decisions regarding the optimal quantities of tables and chairs to purchase, ensuring maximum profitability while staying within budget and storage constraints.

5.1. Linear Programming by Graphical Method:

When a linear programming problem involves two decision variables, the graphical method can be employed for a straightforward solution. This method is based on graphically representing constraints and identifying the feasible region where the optimal solution lies. It provides a visual and intuitive way to solve linear programming problems.

Problem Description:

You serve as the manager of a bakery that specializes in producing two distinct products: croissants and muffins. The profit per unit for croissants is \$2, while muffins yield a profit of \$1.5 per unit. The primary objective is to maximize the daily profit while adhering to specific production and distribution constraints. This problem is a classic linear programming scenario commonly encountered in real-life business operations.

The bakery faces the following key constraints:

1. Oven Capacity: The bakery possesses an oven with the capacity to bake a maximum of 150 items each day. Baking slots are allocated differently for croissants and muffins, with each croissant requiring 1 slot and each muffin requiring 1.5 slots.

2. Ingredient Availability: There is a limitation on ingredient resources. Each croissant necessitates 0.1 units of a special flour blend, and each muffin demands 0.2 units. The bakery has a maximum of 12 units of this special flour blend available.

3. Demand: There exist demand constraints for both croissants and muffins. The maximum daily demand for croissants is 80 units, and for muffins, it is 100 units.

Decision Variables:

Let X represent the quantity of croissants to be produced.

Let Y represent the quantity of muffins to be produced.

Objective Function:

Maximize Profit: $Z = 2X + 1.5Y$

Constraints:

1. Oven Capacity: $X + 1.5Y \leq 150$

2. Ingredient Availability: $0.1X + 0.2Y \leq 12$

3. Croissant Demand: $X \leq 80$

4. Muffin Demand: $Y \leq 100$

5. Non-negativity Constraints: $X \geq 0, Y \geq 0$

Graphical Representation:

1. Create the Graph:

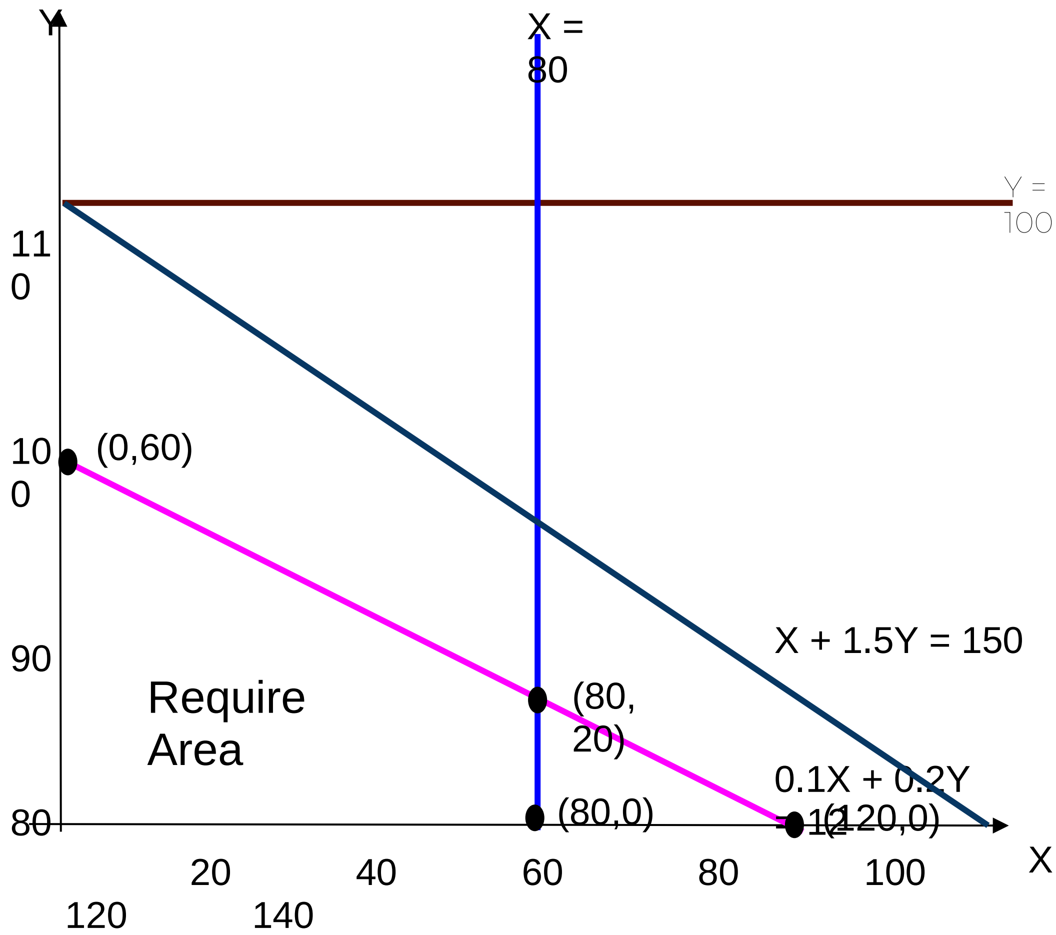
- Label the horizontal axis as "X" for Croissants and the vertical axis as "Y" for Muffins.

2. Plot the Constraints:

- ***Constraint 1***: $X + 1.5Y \leq 150$
  - *When* $X = 0$, $1.5Y \leq 150$, *implying* $Y \leq 100$.
  - *When* $Y = 0$, $X \leq 150$.
- ***Constraint 2:*** $0.1X + 0.2Y \leq 12$
  - *When* $X = 0$, $0.2Y \leq 12$, *indicating* $Y \leq 60$.
  - *When* $Y = 0$, $0.1X \leq 12$, *suggesting* $X \leq 120$.
- ***Constraint 3:*** $X \leq 80$
  - *A vertical line at* $X = 80$.
- ***Constraint 4:*** $Y \leq 100$
  - *A horizontal line at* $Y = 100$.

3. Plot the Feasible Region:

The feasible region is the intersection of all these constraints, forming a triangular region.



- **Identify the Optimal Solution**

The optimal solution will be at the vertex of the feasible region where the objective function is maximized.

This vertex will be the point where the objective function line is tangent to the feasible region.

Point	Z = 2X + 1.5Y
(0,60)	90
(80,0)	160
(80,20)	190

- Label the Solution

190 is the maximum value of Z. Thus, the solution is X = 80 and y = 20

CONCLUSION

In summary, linear programming is not just a mathematical concept but a powerful problem-solving tool with wide-ranging practical applications. It enables businesses to make informed decisions about resource allocation, production levels, and profit maximization. Moreover, linear programming finds use in the intricate designs of engineering projects, aids companies in efficient manufacturing processes, and contributes to the optimization of energy systems in the ever-changing landscape of the energy industry. It also plays a crucial role in the field of transportation, helping organizations save costs and time.

The importance of linear programming is further emphasized by its relevance to operations analysis. Many real-world problems can be effectively framed as linear programming problems, making it a fundamental technique in the study of optimization. Specialized variations, such as network flow and multi-commodity flow problems, have spurred significant research efforts to develop efficient algorithms for their solution.

In a world where efficiency and optimal resource utilization are paramount, linear programming remains a cornerstone of mathematical problem solving and continues to shape our approach to tackling complex, real-world challenges. It stands as a testament to the power of mathematics in aiding decision-making and finding the best solutions in diverse fields and industries.