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SUPERVISOR'S USE ONLY

TOP SCHOLAR

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Scholarship 2013
Calculus

2.00 pm Monday 18 November 2013
Time allowed: Three hours
Total marks: 40

ANSWER BOOKLET

There are six questions in this examination. Answer ANY FIVE questions.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write ALL your answers in this booklet.

Make sure that you have Formulae and Tables Booklet S–CALCF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Answers developed using a CAS calculator require **ALL commands to be shown**. Correct answers only will not be sufficient.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Figure Five from Question Five is repeated on Page 27 of this booklet so that you can show any working.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

You have three hours to complete this examination.

ASSESSOR'S USE ONLY

This examination consists of six questions.
Answer any FIVE questions.

QUESTION NUMBER

1a

$y = (\phi(e^{-x} - e^{-2x}))^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}\phi(-e^{-x} + 2e^{-2x})(\phi(e^{-x} - e^{-2x}))^{-1/2}$

$\frac{dy}{dx} = 0 = \frac{\frac{1}{2}\phi(-e^{-x} + 2e^{-2x})}{\sqrt{\phi(e^{-x} - e^{-2x})}}$ $\quad x \neq 0$

$-e^{-x} + 2e^{-2x} = 0$

$2e^{-2x} = e^{-x}$

$\cancel{e^{-2x} = 2e^{-x}}$ $\quad \ln 2 - 2x = -x$

$\cancel{-2x = \ln 2}$ $\quad -x \quad \ln 2 = x$

$\cancel{x = \ln 2}$

$\cancel{x = \ln 2}$

$y = \sqrt{\phi(e^{-\ln 2} - e^{-2\ln 2})}$

$= \sqrt{\phi(0.5 - 0.25)}$

$= \frac{\sqrt{1+\sqrt{5}}}{2\sqrt{2}}$

$= 0.636$ (3dp)

at $x = (\ln 2)$ cm $= 0.693$ cm the Drop is ~~0.636cm~~ Wide $2y = 1.27$ cm Wide at that pt

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QUESTION NUMBER

ASSESSOR'S USE ONLY

1b $\frac{d^2y}{dx^2} = 0$

$y^2e^{4x} \neq 0$

$y = 0$ at $x = 0$

$\therefore x \neq 0$

$e^{2x} - 6e^x + 4 = 0$

$e^x = \frac{6 \pm \sqrt{36-16}}{2}$

$= 3 \pm \sqrt{9-4}$

$= 3 \pm \sqrt{5}$

$x = \ln(3+\sqrt{5})$ or $x = \ln(3-\sqrt{5})$

$x = 1.656$ or -0.269 (3dp)

$\therefore x = 1.656^{\#}$ cm or $\ln(3+\sqrt{5})$ cm

as $x \geq 0$

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QUESTION NUMBER | ASSESSOR'S USE ONLY

1c

$V = \int_a^b \pi\phi(e^{-x} - e^{-2x})\,dx$

$= \int_0^{\ln p} \pi\phi(e^{-x} - e^{-2x})\,dx$

$= \pi\phi \int_0^{\ln p} (e^{-x} - e^{-2x})\,dx$

$= \pi\phi\left[-e^{-x} + \frac{1}{2}e^{-2x}\right]_0^{\ln p}$

$= \pi\phi\left(\left(-\frac{1}{p} + \frac{1}{2p^2}\right) - \left(-1 + \frac{1}{2}\right)\right)$

$= \pi\phi\left(-\frac{1}{p} + \frac{1}{2p^2} + \frac{1}{2}\right)$

$= \frac{\pi\phi}{2}\left(\frac{-2}{p} + \frac{1}{p^2} + 1\right)$

$= \frac{\pi\phi}{2}\left(\frac{p^2 - 2p + 1}{p^2}\right)$

$= \frac{\pi\phi}{2}\left(\frac{(p-1)^2}{p^2}\right) = \frac{\pi\phi}{2}\left(\frac{p-1}{p}\right)^2 = RHS$

~~for~~

all positive values of x can be represented as $x = \ln(p)$ and as x increases, p also increases

as $p \to \infty$ $\frac{p-1}{p} \to 1$ so

as $p \to \infty$ $V \to \frac{\pi\phi}{2}(1)^2 = \frac{\pi\phi}{2}$

so a upper limit of $V_c = \frac{\pi\phi}{2}$ exists

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e.g at $x = 1{,}000{,}000$ $p = 2.69 \times 10^{43}$

so the volume of the drop between $x = 0$ and $x = \ln(2.69 \times 10^{43})$ is

$V = \frac{\pi\phi}{2}(0.9999\ldots)^2$ or $\frac{\pi\phi}{2}(1)^2$

as calculators only have the capacity for 10 decimal places.

QUESTION NUMBER

ASSESSOR'S USE ONLY

2a $\langle f(x), g(x) \rangle_0^1 = \int_0^1 kx^2 + (k^2+1)x + k \, dx$

$= \left[\frac{kx^3}{3} + \frac{(k^2+1)x^2}{2} + kx\right]_0^1$

$= \frac{k}{3} + \frac{k^2+1}{2} + k$

$= \frac{3k^2+8k+3}{6}$

$\|f(x)\|_0^1 = \sqrt{\int_0^1 (f(x))^2 \, dx} = \|f(x)\|_0^1 \neq 0$

$= \sqrt{\left[\frac{k^2x^3}{3} + \frac{2kx^2}{2} + x\right]_0^1}$

$= \sqrt{\frac{k^2+3k+3}{3}} \neq 0$

$\|g(x)\|_0^1 = \sqrt{\int_0^1 (g(x))^2 \, dx} \neq 0$

$= \sqrt{\left[\frac{x^3}{3} + \frac{2kx^2}{2} + k^2x\right]_0^1}$

$= \sqrt{\frac{1}{3} + k + k^2} \neq 0$

$\|f(x)\|_0^1 \neq 0$ and $\|g(x)\|_0^1 \neq 0$ for all real values of k

$\cos\frac{\pi}{2} = \frac{\langle f(x), g(x)\rangle_0^1}{\|f(x)\|_0^1 \cdot \|g(x)\|_0^1} = 0$

$\frac{3k^2+8k+3}{6} = 0 \qquad 3k^2+8k+3 = 0$

$k = \frac{-8 \pm \sqrt{64-36}}{6}$

$= \frac{-4}{3} \pm \frac{\sqrt{28}}{6}$

$= -0.451$ or $k = -2.215$ (3dp)

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QUESTION NUMBER

ASSESSOR'S USE ONLY

2b $\langle p(x), q(x) \rangle_0^1 = \int_0^1 27x^2 - 51x + 20 \, dx$

$= \left[9x^3 - \frac{51}{2}x + 20x\right]_0^1$

$= 3.5$

$||p(x)||_0^1 = \sqrt{\int_0^1 (3x-4)^2 \, dx}$

$= \sqrt{\left[3x^3 - 12x^2 + 16x\right]_0^1}$

$= \sqrt{7}$

$||q(x)||_0^1 = \sqrt{\int_0^1 (9x-5)^2 \, dx}$

$= \sqrt{27x^3 - 45x^2 + 25x \, dx}$

$= \sqrt{7}$

$\cos\theta = \dfrac{\langle p(x), q(x) \rangle_0^1}{||p(x)||_0^1 \cdot ||q(x)||_0^1}$

$= \dfrac{3.5}{\sqrt{7} \cdot \sqrt{7}} = \dfrac{3.5}{7} = \dfrac{1}{2}$

$\theta = \frac{\pi}{3}$

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QUESTION NUMBER

ASSESSOR'S USE ONLY

2c let $y(x) = \sin(nx)$ and $z(x) = \sin(mx)$

$\langle y, z \rangle_0^{2\pi} = \int_0^{2\pi} \sin(nx)\sin(mx)\,dx$

~~$= \frac{1}{2}\int_0^{2\pi} \cos(n-m) - \cos(n+m)$~~

$= \frac{1}{2}\int_0^{2\pi} \cos((n-m)x) - \cos((n+m)x)\,dx$

$= \frac{1}{2}\left[\frac{1}{n-m}\sin((n-m)x) - \frac{1}{n+m}\sin((n+m)x)\right]_0^{2\pi}$

$= \frac{1}{2}((0-0)-(0-0))$

$= 0$

$\|y\|_0^{2\pi} = \sqrt{\int_0^{2\pi} \sin^2(nx)\,dx}$

$= \sqrt{\int_0^{2\pi} \frac{1-\cos(2nx)}{2}\,dx}$

$= \sqrt{\left[\frac{1}{2}x - \frac{\sin(2nx)}{4n}\right]_0^{2\pi}}$

$= \sqrt{\pi}$

$\|z\|_0^{2\pi} = \sqrt{\int_0^{2\pi} (1-\cos(2mx))/2\,dx}$

$= \sqrt{\left[\frac{1}{2}x - \frac{\sin(2mx)}{4m}\right]_0^{2\pi}}$

$= \sqrt{\pi}$

$\cos\theta = \frac{0}{\sqrt{\pi}\cdot\sqrt{\pi}} = \frac{0}{\pi} = 0$ for all values of n and m

$\theta = \frac{\pi}{2}$ $\therefore \sin(nx)$ and $\sin(mx)$ are orthogonal for all positive integer values of n and m

$\|y\|_0^{2\pi} = \|z\|_0^{2\pi} = \sqrt{\pi} \neq 0$

ns

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y8

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QUESTION NUMBER

ASSESSOR'S USE ONLY

3a i) Where n, m, o, p ... are Integers all Polynomials of the form $P(x) = a_{2n}x^{2n} + a_{2m}x^{2m}$... are even as $(-x)^{2n} = x^{2n}$ for all Integer values of n.

all Polynomials of ~~$P(x) = a_{2n}x^{2n-1} + a_{2m}x^{2m}$~~ the form $P(x) = a_{2n-1}x^{2n-1} + a_{2m-1}x^{2m-1}$... are odd as $(-x)^{2n-1} = (-x)^{2n}(-x)^{-1}$

$= x^{2n} . -x^{-1}$

$= -x^{2n-1}$

$\therefore f(-x) = -f(x)$

all Polynomials which combine the two forms of the above polynomials as if $P(x)$ can be expressed as the sum of the even function: $f(x)$ and the odd function: $g(x)$ then

$P(-x) = f(-x) + g(-x) = f(x) - g(x) \neq P(x)$ or $-P(x)$.

$\therefore$ the function is neither even or odd.

$P(x) = f(x) + g(x)$

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QUESTION NUMBER

ASSESSOR'S USE ONLY

3a ii)

$g = \quad g(-x) = g(x)$

$(g(-x))' = g'(-x) \cdot -1 = -g'(-x) = (g(x))' = g'(x)$

let $g'(x)$ be $f(x)$

$f(x) = -f(-x)$

$= - g'(x)$ or $f(x)$ is Odd.

$(g(-x))' = (g(x))'$

let $h(x) = -x$

$(g(h(x)))' = (g(x))' \qquad h'(x) = -1$

$g'(h(x)) \cdot h'(x) = g'(x)$

$g'(-x) \cdot -1 = g'(x)$

$-g'(-x) = g'(x)$

$\therefore$ the function $g'(x)$ is Odd.

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QUESTION NUMBER

ASSESSOR'S USE ONLY

3b $y = e^{-x}\sin(kx)$

$\frac{dy}{dx} = -e^{-x}\sin(kx) + ke^{-x}\cos(kx)$

$\frac{d^2y}{dx^2} = e^{-x}\sin(kx) + -2ke^{-x}\cos(kx) - k^2e^{-x}\sin(kx)$

$\frac{d^3y}{dx^3} = -e^{-x}\sin(kx) + ke^{-x}\cos(kx) + 2ke^{-x}\cos(kx) + 2k^2e^{-x}\sin(kx)$
$+ k^2e^{-x}\sin(kx) - k^3e^{-x}\cos(kx)$

$= (3k^2-1)(e^{-x}\sin(kx)) + (3k-k^3)(e^{-x}\cos(kx))$

$3k - k^3 = 0$

$k = 0, +\sqrt{3}, -\sqrt{3}$

$\frac{d^3y}{dx^3} = cy$ where for $k=0$ $c=-1$
or for $k = \pm\sqrt{3}$
$c = 8$

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QUESTION NUMBER

ASSESSOR'S USE ONLY

4a let $z = r\operatorname{cis}\theta$

$z^n = r^n \operatorname{cis} n\theta$

$r = 1$ or 0

$\therefore \operatorname{cis} n\theta = \operatorname{cis}\theta \qquad \operatorname{cis}(n\theta + 2n\pi) = \operatorname{cis}\theta$

$n\theta + 2\pi m = \theta$ where $0 \le m \le n$ and m is an integer

$\theta = \frac{2\pi m}{1-n} \qquad -2\pi \le \theta \le 0$

for $n=2$, $\theta = \underline{0}$ or -2π

for $n=3$, $\theta = 0, \underline{-\pi}$ or -2π

for $n=4$, $\theta = 0, \underline{-\frac{2\pi}{3}}$ or $\underline{-\frac{4\pi}{3}}$

for $n=5$, $\theta = 0, \underline{-\frac{2\pi}{4}}, -\frac{4\pi}{4}, \underline{-\frac{6\pi}{4}}$

for $n=6$, $\theta = 0, \underline{-\frac{2\pi}{5}}, \underline{-\frac{4\pi}{5}}, \underline{-\frac{6\pi}{5}}, \underline{-\frac{8\pi}{5}}$

for $n=7$, $\theta = 0, \underline{-\frac{2\pi}{6}}, -\frac{4\pi}{6}, -\frac{6\pi}{6}, -\frac{8\pi}{6}, \underline{-\frac{10\pi}{6}}$

for $n=8$, $\theta = 0, \underline{-\frac{2\pi}{7}}, \underline{-\frac{4\pi}{7}}, \underline{-\frac{6\pi}{7}}, \underline{-\frac{8\pi}{7}}, \underline{-\frac{10\pi}{7}}, \underline{-\frac{12\pi}{7}}$

for $n=9$, $\theta = 0, \underline{-\frac{2\pi}{8}}, -\frac{4\pi}{8}, \underline{-\frac{6\pi}{8}}, -\frac{8\pi}{8}, \underline{-\frac{10\pi}{8}}, -\frac{12\pi}{8}, \underline{-\frac{14\pi}{8}}$.

$z = 0, \operatorname{cis} 0, \operatorname{cis} -\pi, \operatorname{cis} -\frac{2\pi}{3}, \operatorname{cis} -\frac{4\pi}{3}, \operatorname{cis} -\frac{2\pi}{4}, \operatorname{cis} -\frac{6\pi}{4}, \operatorname{cis} -\frac{2\pi}{5}, \operatorname{cis} -\frac{4\pi}{5}, \operatorname{cis} -\frac{6\pi}{5}, \operatorname{cis} -\frac{8\pi}{5}, \operatorname{cis} -\frac{2\pi}{6}, \operatorname{cis} -\frac{10\pi}{6}$

NB: underlined values of θ are the first time the values occur above (for reference).

(continued from above)

$z = \operatorname{cis} -\frac{2\pi}{7}, \operatorname{cis} -\frac{4\pi}{7}, \operatorname{cis} -\frac{6\pi}{7}, \operatorname{cis} -\frac{8\pi}{7}, \operatorname{cis} -\frac{10\pi}{7}, \operatorname{cis} -\frac{12\pi}{7}, \operatorname{cis} -\frac{2\pi}{8}, \operatorname{cis} -\frac{6\pi}{8}, \operatorname{cis} -\frac{10\pi}{8}$, or $\operatorname{cis}(-14\pi/8)$

there are 23 different solutions for $z^n = z$ where $2 \le n \le a$ and a is a whole number. 22 of the solutions have an absolute value of 1 and 1 has an absolute value of zero

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QUESTION NUMBER

ASSESSOR'S USE ONLY

4bi $\frac{m_0}{m_1} = \left(\frac{1+\frac{\Delta v}{c}}{1-\frac{\Delta v}{c}}\right)^{\frac{c}{2u}}$

$\frac{2u}{c}\ln\frac{m_0}{m_1} = \ln\left(\frac{1+\frac{\Delta v}{c}}{1-\frac{\Delta v}{c}}\right)$

$\cancel{\frac{2u}{c}\ln\frac{m_0}{m_1} = \ln\left(1+\frac{\Delta v}{c}\right) - \ln\left(1-\frac{\Delta v}{c}\right)}$

$\frac{u}{c}\ln\frac{m_0}{m_1} = \frac{1}{2}\ln\left(\frac{1+\Delta v/c}{1-\Delta v/c}\right)$

let $\frac{u}{c}\ln\frac{m_0}{m_1} = x$

$\tanh(x) = \frac{e^{\ln\left(\frac{1+\Delta v/c}{1-\Delta v/c}\right)} - 1}{e^{\ln\left(\frac{1+\Delta v/c}{1-\Delta v/c}\right)} + 1}$

$= \frac{\left(\frac{1+\Delta v/c}{1-\Delta v/c}\right) - 1}{\left(\frac{1+\Delta v/c}{1-\Delta v/c}\right) + 1}$

$= \frac{(1+\Delta v/c) - (1-\Delta v/c)}{(1+\Delta v/c) + (1-\Delta v/c)}$

$= \frac{2\Delta v/c}{2}$

$= \Delta v/c$

$\Delta v = c \cdot \tanh(x)$

$= c \cdot \tanh\left(\frac{u}{c}\ln\left(\frac{m_0}{m_1}\right)\right)$

= RHS

QED

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QUESTION NUMBER

ASSESSOR'S USE ONLY

4b ii)

$$\frac{dM}{dv} = \frac{-M}{v(1-\frac{v^2}{c^2})}$$

$$\int \frac{dM}{-M} = \int \frac{dv}{v(1-v^2/c^2)}$$

$$-\ln M + C_1 = \frac{1}{v} \int \frac{A}{(1+\frac{v}{c})} + \frac{B}{(1-\frac{v}{c})} \, dv$$

$A + B = 1$

$-A + B = 0$

$A = B$

$A = \frac{1}{2} = B$

$$-\ln M + C_1 = \frac{1}{2v} \int \frac{1}{(1+\frac{v}{c})} + \frac{1}{1-\frac{v}{c}} \, dv$$

$$-\ln M + C_1 = \frac{1}{2v}\left(c\ln(1+\tfrac{v}{c}) - c\ln(1-\tfrac{v}{c})\right) + C_2$$

$C_1 = C_2 = 0$

$$-\ln M = \frac{c}{2v}\left(\ln(1+\tfrac{v}{c}) - \ln(1-\tfrac{v}{c})\right)$$

$$\ln M = \frac{-c}{2v}\left(\ln\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)\right)$$

QED

2

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QUESTION NUMBER

ASSESSOR'S USE ONLY

6a ~~$(cis\theta)^5 = cis5\theta$~~

$cis5\theta = LHS$
$= (cis\theta)^5$
$= (\cos\theta + i\sin\theta)^5$
$= \cos^5\theta + 5i\cos^4\theta\sin\theta + 10i^2\cos^3\theta\sin^2\theta +$
$10i^3\cos^2\theta\sin^3\theta + 5i^4\cos\theta\sin^4\theta + i^5\sin^5\theta$
$= (\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta) +$
$i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$
$= \cos5\theta + i\sin5\theta$ (As above

$\cos5\theta = Re(cis5\theta) = Re((cis\theta)^5)$
$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$
QED

$\sin5\theta = Im(cis5\theta) = Im((cis\theta)^5)$
$= 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$
QED //

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4

QUESTION NUMBER

ASSESSOR'S USE ONLY

6b Where $x_{n+1} = x_n$, $y_{n+1} = y_n$ and $z_{n+1} = z_n$

(A) $-0.2x_n + 0.7y_n + 0.6z_n = 0$ (D) $10(D) = A$

(B) $0.1x_n - 0.8y_n + 0.4z_n = 0$ (E) $10(E) = B$

(C) $x_n + y_n + z_n = 99$ (F) $10(F) = C$

$$\begin{pmatrix}10A\\10B\\10C\end{pmatrix} = \begin{pmatrix}-2 & 7 & 6\\1 & -8 & 4\\10 & 10 & 10\end{pmatrix}\begin{pmatrix}x_n\\y_n\\z_n\end{pmatrix} = \begin{pmatrix}0\\0\\990\end{pmatrix}$$

let $\begin{pmatrix}-2 & 7 & 6\\1 & -8 & 4\\10 & 10 & 10\end{pmatrix}$ be ~~A~~ D

$$\begin{pmatrix}x_n\\y_n\\z_n\end{pmatrix} = D^{-1}\begin{pmatrix}0\\0\\990\end{pmatrix}$$

D^{-1} $(D \mid I) = (I \mid D^{-1})$

$$\left(\begin{array}{ccc|ccc}-2 & 7 & 6 & 1 & 0 & 0\\1 & -8 & 4 & 0 & 1 & 0\\10 & 10 & 10 & 0 & 0 & 1\end{array}\right)$$

Solved on Calculator: $x_n = 76$, $y_n = 14$ and $z_n = 9$

$x_{n+1} = 0.8 \times 76 + 0.7 \times 14 + 0.6 \times 9 = 76$

$\therefore x_{n+1} = x_n$

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QUESTION NUMBER

ASSESSOR'S USE ONLY

6b cont.

$y_{n+1} = 0.1 \times 76 + 0.2 \times 14 + 0.4 \times 9 = 14$

$y_{n+1} = y_n$

$z_{n+1} = 0.1 \times 76 + 0.1 \times 14 = 9$

$= z_n$

$$\therefore \begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \begin{pmatrix} 76 \\ 14 \\ 9 \end{pmatrix}$$

The Calculus teacher will give out 76 easy homework questions, 14 difficult homework questions and 9 impossible homework questions each week once the numbers stabilise.

Solved on paper:

$-2x_n + 7y_n + 6z_n = 0$ (A)

$1x_n - 8y_n + 4z_n = 0$ (B)

$\cancel{10x_n + 10y_n + 10z_n = 990}$ (C)

$x_n + y_n + z_n = 99$ (F)

$$\cancel{\begin{pmatrix} -2 & 7 & 6 \\ 1 & -8 & 4 \\ 10 & 10 & 10 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 990 \end{pmatrix}}$$

$$\cancel{\text{let} \begin{pmatrix} -2 & 7 & 6 \\ 1 & -8 & 4 \\ 10 & 10 & 10 \end{pmatrix} \text{ be } D}$$

QUESTION NUMBER

ASSESSOR'S USE ONLY

6b cont.

$$\cancel{(D \mid I_3) = (I_3 \mid D^{-1})}$$

$$\cancel{(D \mid I_3) \sim \left(\begin{array}{ccc|ccc} -2 & 7 & 6 & 1 & 0 & 0 \\ 1 & -8 & 4 & 0 & 1 & 0 \\ 10 & 10 & 10 & 0 & 0 & 1 \end{array}\right)}$$

$$\cancel{\sim \left(\begin{array}{ccc|ccc} -1 & -1 & 10 & 1 & 1 & 0 \\ 1 & -8 & 4 & 0 & 1 & 0 \\ 10 & 10 & 10 & 0 & 0 & 1 \end{array}\right)}$$

$$\cancel{\sim \left(\begin{array}{ccc|ccc} -1 & -1 & 10 & 1 & 1 & 0 \\ 1 & -8 & 4 & 0 & 1 & 0 \\ 0 & 0 & 110 & 10 & 10 & 1 \end{array}\right)}$$

$\sim$

Cont. on Page 18.

QUESTION NUMBER

ASSESSOR'S USE ONLY

6b ~~$a_{y_n} + 9_{z_n} =$~~

cont. (A) $-2x_n + 7y_n + 6z_n = 0$

(B) $x_n - 8y_n + 4z_n = 0$

(F) $x_n + y_n + z_n = 99$

(A) + (B) + (F) - $11z_n = 99$

$z_n = 9$

(A) $2x_n - 7y_n = 54$

(B) $-x_n + 8y_n = 36$

(A)+2(B) $9y_n = 126$

$y_n = 14$

$x_n = \frac{54 + 7 \times 14}{2}$

$= 76$

(76, 14, 9). Same as calculator answer and response.