2

SUPERVISOR'S USE ONLY

91262

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Level 2 Mathematics and Statistics, 2012

91262 Apply calculus methods in solving problems

2.00 pm Monday 19 November 2012
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–14 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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You are advised to spend 60 minutes answering the questions in this booklet.

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QUESTION ONE

(a) A function f is given by $f(x) = 2x^3 - 10x + 5$.

Find the gradient of the graph of f at the point where $x = 2$.

(b) For a function p,

$$p'(x) = 4 - 6x^2$$

The graph of p passes through the point (2,5).

Find the function $p(x)$.

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(c) The distance, s, of a tractor from a gate-post, in metres, is modelled by

$$s = 0.1t^2 + 5t,$$

where t is the time in seconds since the tractor passed the gate-post and $0 \leq t \leq 20$.

Give the equation for the velocity of the tractor, and use this to find when the velocity of the tractor is 8 m s^{-1}.

(d) The fuel consumption, f, of a car, in litres per 100 km, is related to the velocity, v, in km h^{-1}, by the formula

$$f = 16 - 0.2v + \frac{v^2}{250}$$

Show that the fuel consumption is a **minimum** when $v = 25$ km h^{-1}.

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(e) A train starts from rest.
Its acceleration t seconds after it starts is given by

$$a = \frac{1}{4}\left(20 - t\right) \text{ m s}^{-2}$$

What distance does the train cover in the first 30 seconds?

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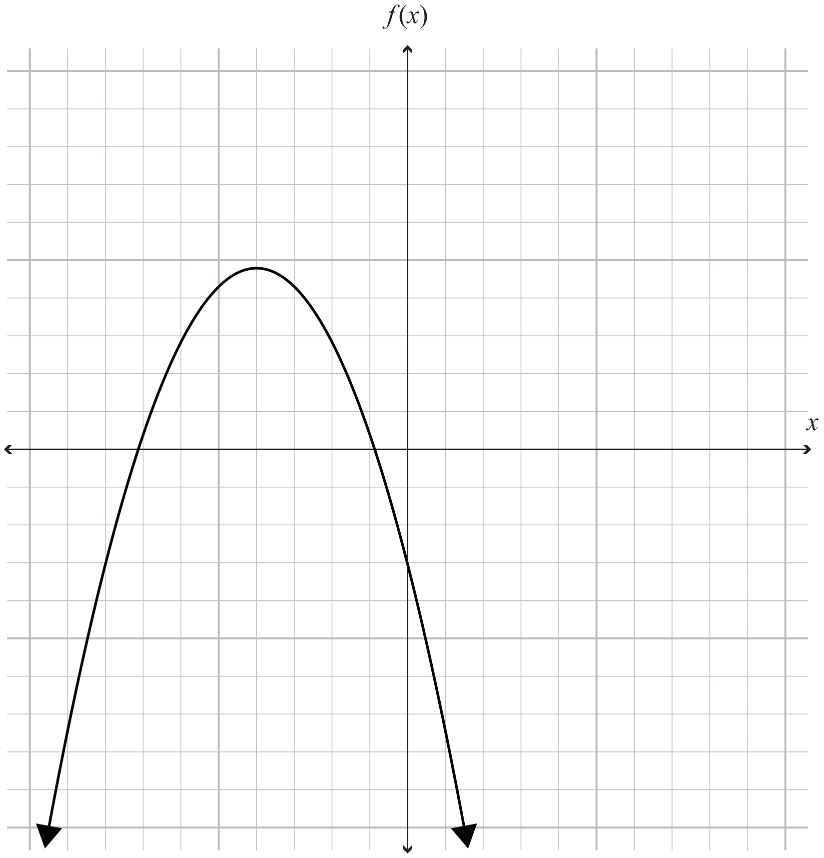
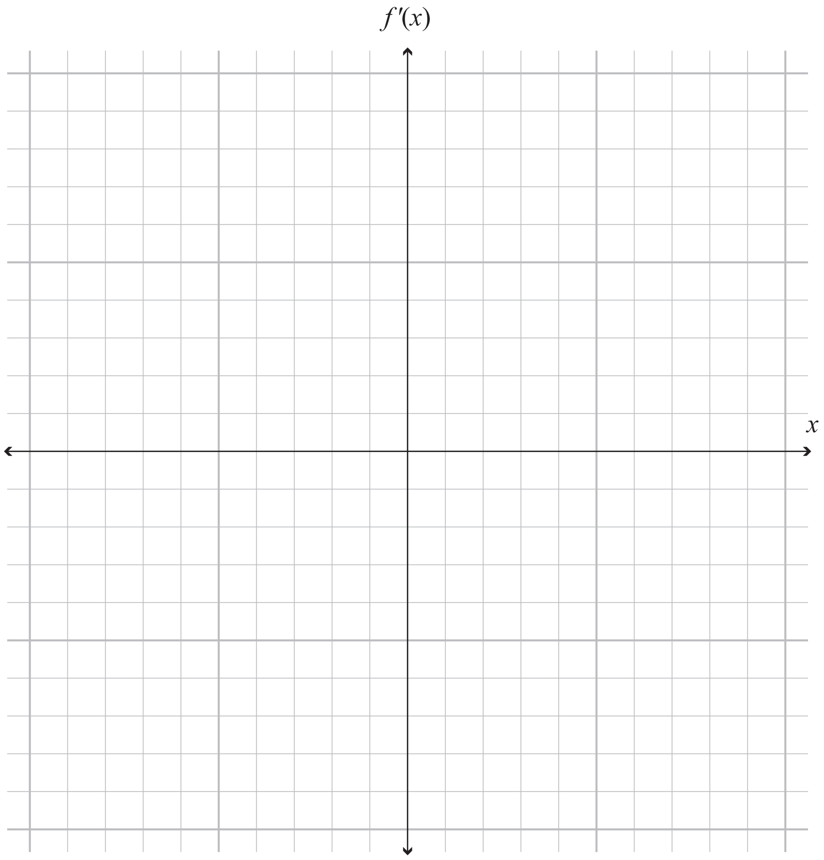
QUESTION TWO

(a) Sketch the gradient function $f'(x)$ for the function $f(x)$ below:





If you need to redraw this graph, use the grid on page 12.

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(b) (i) Find the x-coordinate of the point on the graph of $h(x) = x^2 - 12x$ where the gradient is equal to 4.

(ii) Find the equation of the tangent to the curve of $h(x) = x^2 - 12x$ at the point $(1,-11)$.

(c) $g(x) = x^3 - 9x^2 + 24x - 8$

For what values of x is g a decreasing function?

You must justify your answer, using calculus.

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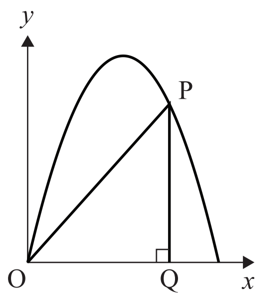
(d) A right angled triangle OPQ is drawn as shown where O is at (0,0).
P is a point on the parabola $y = ax - x^2$
and Q is on the x-axis.



Show that the maximum possible area for the triangle OPQ is $\frac{2a^3}{27}$

QUESTION THREE

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(a) A curve $y = f(x)$ passes through the origin (0,0) and has a gradient function

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - 2x$$

Find the co-ordinates of the point on the curve where $x = 6$.

(b) The volume, V, of a cube changes as the length x of the sides changes.

$$V = x^3$$

Find the rate of change of the volume, with respect to the length of the side, when the side is 5 cm.

(c) Sketch the function $h(x)$ for the gradient function $h'(x)$ below, given that $h(\mathrm{a}) = 0$.

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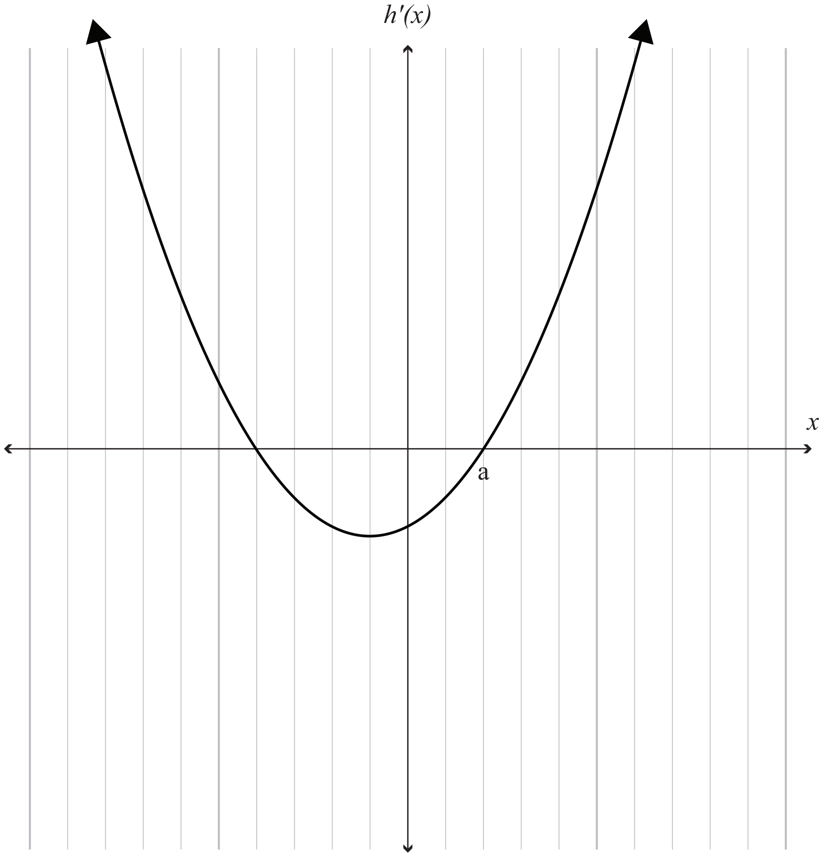

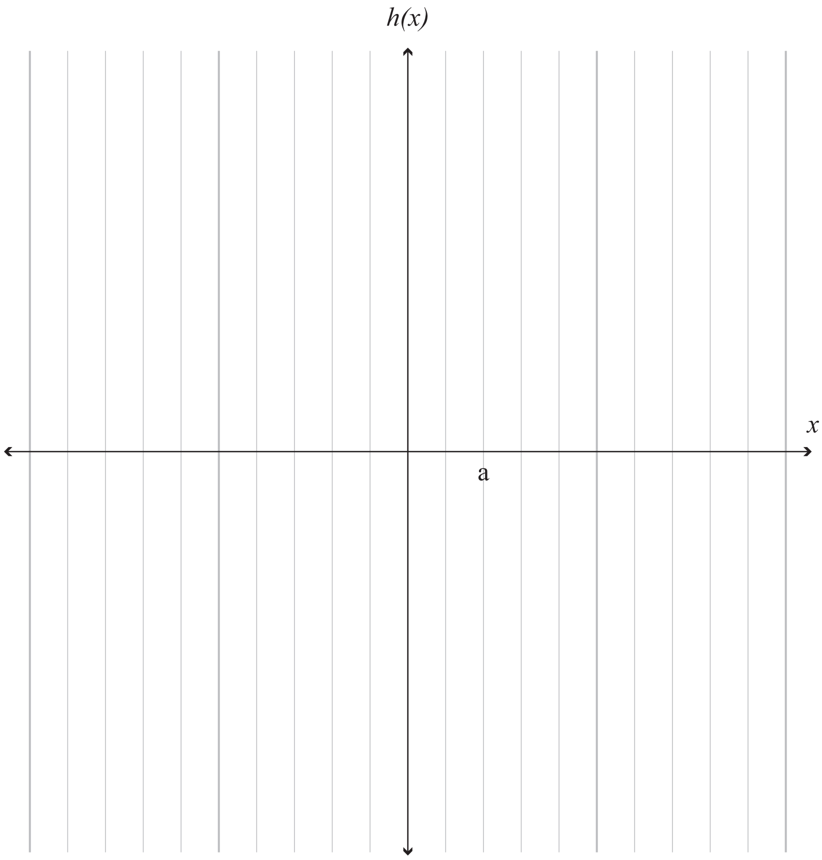

If you need to redraw this graph, use the grid on page 13.

(d) The curve of $f(x) = 2x^3 + Ax + B$ has a tangent with gradient 10 at the point (–2,33).

Find the coordinates of the point on the curve where $x = 4$.

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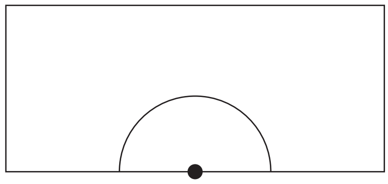
(e) A chemical is dropped into the water in a rectangular swimming pool at a point half-way along its length.



After 0.1 minutes, the chemical spreads in a semi-circular shape with radius r metres, where

$$r = 1 + 2t$$

and t is the time in minutes since the chemical was added to the water.

When the chemical first reaches the far side of the pool, the area of the semi-circle is increasing at the rate of 60 m^2 min^{-1}.

Find the width of the pool. (Area of a circle = πr^2)

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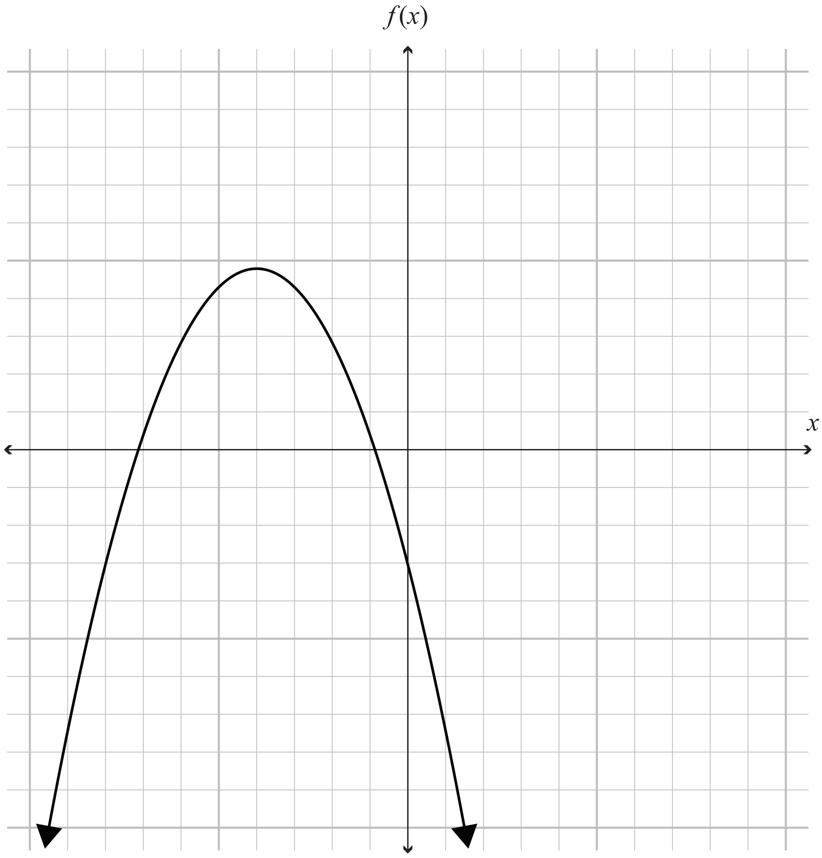
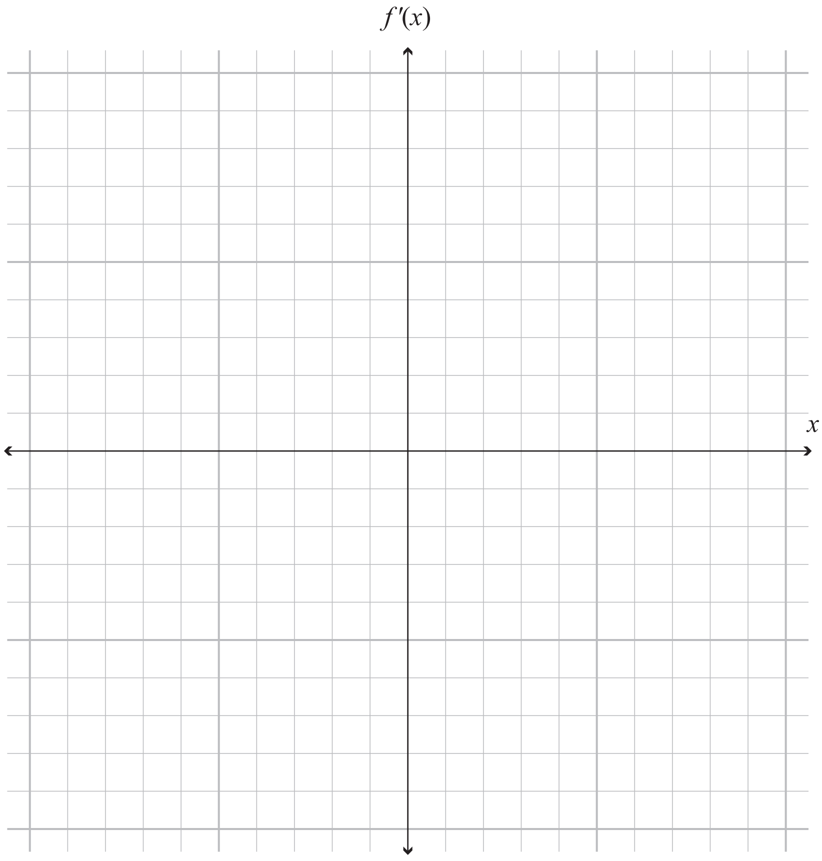
If you need to redraw your graph from Question Two (a), draw it on the grid below. Make sure it is clear which graph you want marked.





If you need to redraw your graph from Question Three (c), draw it on the grid below. Make sure it is clear which graph you want marked.

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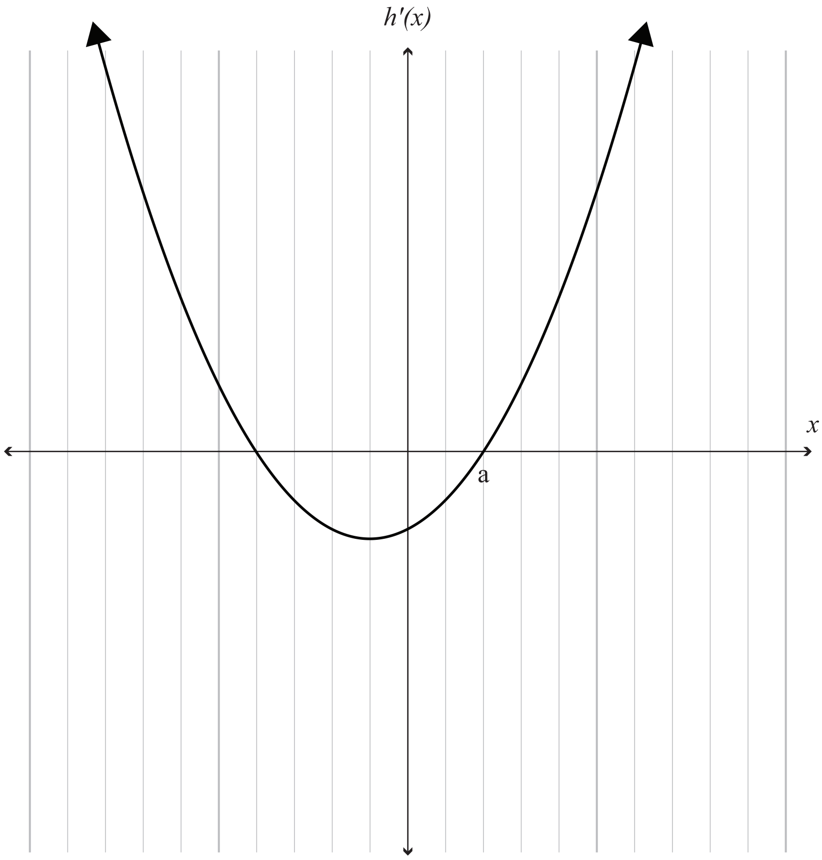

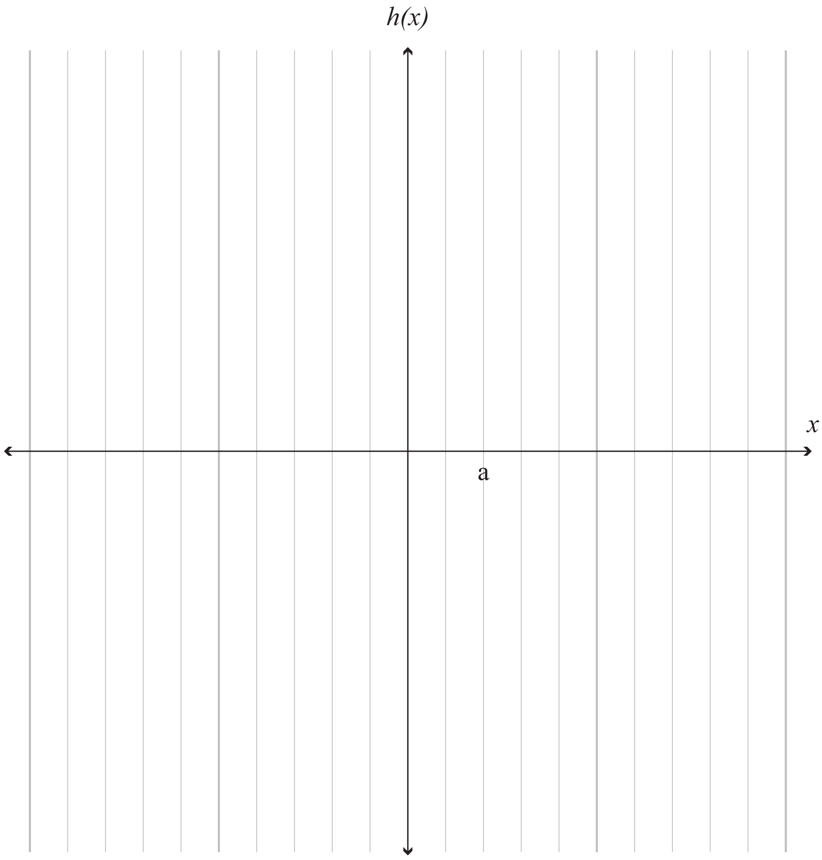

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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