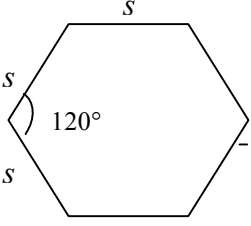
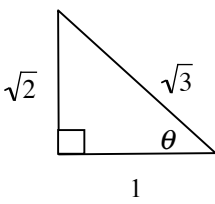


**Assessment Schedule – 2008****Scholarship Mathematics with Calculus (93202)****Evidence Statement**

For up to a maximum of 4 question parts a single minor error in that part may be accepted without loss of marks.

Question	Evidence	Code	Judgement
<b>ONE (a)</b>	 <p>Using 6 equilateral triangles:</p> $\begin{aligned} \text{Area} &= 6 \times \frac{1}{2} s^2 \sin 60^\circ \\ &= 3s^2 \sin 60^\circ \\ &= \frac{3\sqrt{3}}{2} s^2 \end{aligned}$	<b>2</b>	
	<p><b>OR</b></p> <p>With two triangles and a rectangle:</p> $\begin{aligned} \text{Area} &= 2 \times \frac{1}{2} s^2 \sin 120^\circ + 2s^2 \sin 60^\circ \\ &= 3s^2 \sin 60^\circ \\ &= \frac{3\sqrt{3}}{2} s^2 \end{aligned}$ <p>Total area of the hexagonal stack <math>= 16 \times \frac{3\sqrt{3}}{2} s^2</math>  <math>= 24\sqrt{3}s^2</math></p>	<b>1</b>	
<b>ONE (b)</b>	$A = 6hs + \frac{3}{2} s^2 \left( \frac{-\cos \theta}{\sin \theta} + \frac{\sqrt{3}}{\sin \theta} \right) = 6hs + \frac{3}{2} s^2 (-\cot \theta + \sqrt{3} \csc \theta)$ <p>So <math>\frac{dA}{d\theta} = \frac{3}{2} s^2 (\cos \csc^2 \theta - \sqrt{3} \csc \theta \cot \theta)</math> and for max/min</p>	<b>3</b>	
	$\begin{aligned} \frac{3}{2} s^2 (\cos \csc^2 \theta - \sqrt{3} \csc \theta \cot \theta) &= 0 \\ \csc \theta (\cos \csc \theta - \sqrt{3} \cot \theta) &= 0 \\ \cos \csc^2 \theta (1 - \sqrt{3} \cos \theta) &= 0 \end{aligned}$ 	<b>1</b>	
	<p>But <math>\cos \csc^2 \theta \neq 0</math> so</p> $1 - \sqrt{3} \cos \theta = 0$ $\cos \theta = \frac{1}{\sqrt{3}}$ <p><b>OR</b></p> $A = 6hs + \frac{3}{2} s^2 \left( \frac{-\cos \theta}{\sin \theta} + \frac{\sqrt{3}}{\sin \theta} \right)$ $\frac{dA}{d\theta} = \frac{3}{2} s^2 \left( \frac{\sin \theta (\sin \theta) - (-\cos \theta + \sqrt{3}) \cos \theta}{\sin^2 \theta} \right)$	<b>2</b>	
		<b>1</b>	



<p><b>TWO</b> <b>(a)</b></p>	$(z+1)^3 = 8 \text{ so } \left(\frac{z+1}{2}\right)^3 = 1 \text{ and } \frac{z+1}{2} = 1, w, w^2$ $z = 1, 2w-1, 2w^2-1$ <hr/> <p>and the sum of the roots is <math>1 + 2w - 1 + 2w^2 - 1 = 2(1 + w + w^2) - 3 = -3</math>.</p> <p><b>OR</b></p> $(z+1)^3 = 8 \text{ so } z^3 + 3z^2 + 3z + 1 = 8 \text{ and } z^3 + 3z^2 + 3z - 7 = 0$ <hr/> <p>the sum of the roots is <math>-\frac{b}{a} = -3</math>.</p> <p><b>OR</b></p> $z^3 + 3z^2 + 3z - 7 = 0 \text{ so } (z-1)(z^2 + 4z + 7) = 0,$ $z = 1, -2 - i\sqrt{3}, -2 + i\sqrt{3} \text{ (from quadratic formula)}$ <p><b>OR</b></p> $(z+1)^3 = 8 \text{ so } z+1 = 2\text{cis}\left(\frac{2k\pi}{3}\right), \quad k = 0, 1, 2$ $z+1 = 2, 2\left(-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right)$ $z = 1, -2 \pm \sqrt{3}i$	<p><b>2</b></p> <p><b>1</b></p> <p><b>1</b></p>	<p>Or</p> $1, \text{cis}\left(\frac{2\pi}{3}\right), \text{cis}\left(\frac{4\pi}{3}\right)$ <p>etc</p> <p>Accept any 3 correct solutions, including those with no <math>w</math>'s</p>
<p><b>TWO</b> <b>(b)</b></p>	$(z+1)^3 = 8(z-1)^3 \text{ so}$ $z^3 + 3z^2 + 3z + 1 = 8(z^3 - 3z^2 + 3z - 1)$ $7z^3 - 27z^2 + 21z - 9 = 0$ <hr/> <p>by inspection <math>z = 3</math>, (or from <math>z+1 = 2(z-1)</math>) so</p> $(z-3)(7z^2 - 6z + 3) = 0$ <hr/> <p>and <math>z = 3, \frac{3-2\sqrt{3}i}{7}, \frac{3+2\sqrt{3}i}{7}</math>.</p> <p><b>OR</b></p> $\left(\frac{z+1}{2(z-1)}\right)^3 = 1 \text{ and } \frac{z+1}{2(z-1)} = 1, w, w^2$ $z+1 = 2z-2, \quad z = 3$ <p>or</p> $z+1 = 2wz-2w, \quad z = \frac{2w+1}{2w-1}$ <hr/> <p>since <math>w = -\frac{1}{2} + \frac{\sqrt{3}}{2}i</math> [or <math>w = -\frac{1}{2} - \frac{\sqrt{3}}{2}i</math>]</p>	<p><b>3</b></p> <p><b>1</b></p> <p><b>2</b></p> <p><b>1</b></p>	<p>Giving A, B, C</p> <p>Accept in decimals</p> $z = .429 \pm .495i \text{ or equivalent}$

	$z = \frac{2w+1}{2w-1} = \frac{-1+\sqrt{3}i+1}{-1+\sqrt{3}i-1} = \frac{\sqrt{3}i}{-2+\sqrt{3}i}$		
	<hr/> $= \frac{\sqrt{3}i(-2-\sqrt{3}i)}{(-2+\sqrt{3}i)(-2-\sqrt{3}i)}$ $= \frac{3-2\sqrt{3}i}{7}.$ <p>Hence the other root is the conjugate <math>= \frac{3+2\sqrt{3}i}{7}.</math></p> <p>The solution is <math>z = 3, \frac{3-2\sqrt{3}i}{7}, \frac{3+2\sqrt{3}i}{7}.</math></p> <p>[or use <math>z+1=2w^2z-2w^2, z = \frac{2w^2+1}{2w^2-1}</math>, although this is not needed with the above]</p> <p><b>OR</b></p> $(z+1)^3 = 8(z-1)^3 \text{ so } \frac{z+1}{z-1} = 2\text{cis}\left(\frac{2k\pi}{3}\right), \quad k=0,1,2$	2	
	<hr/> $1 + \frac{2}{z-1} = 2\text{cis}\left(\frac{2k\pi}{3}\right), \quad k=0,1,2 \quad \text{and} \quad 1 + \frac{2}{z-1} = 2, \quad 2\left(-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right) = 2, \quad -1 \pm \sqrt{3}i$ $\frac{2}{z-1} = 1, \quad -2 \pm \sqrt{3}i$ $z-1 = 2, \quad \frac{2}{-2 \pm \sqrt{3}i}$	1	
	<hr/> $z-1 = 2, \quad \frac{2(-2 \mp \sqrt{3}i)}{7}$ $z = 3, \quad \frac{3 \mp 2\sqrt{3}i}{7}$	2	

**TWO**  
**(c)**

For  $\left(\frac{1}{z} + 1\right)^3 = 8\left(\frac{1}{z} - 1\right)^3$  this is part (ii) with the transformation  $z = \frac{1}{u}$  where  $z$  is the solution to part (ii) and  $u$  the solution here, and so  $u = \frac{1}{z}$ .

**Either**

However, when  $z = r\text{cis}\theta$ ,  $\frac{1}{z} = \frac{1}{r}\text{cis}(-\theta)$ ,

$$\text{since } \frac{1}{z} = \frac{1}{r\text{cis}(\theta)} = \frac{\text{cis}(-\theta)}{r\text{cis}(\theta)\text{cis}(-\theta)} = \frac{\text{cis}(-\theta)}{r(c^2 + s^2)} = \frac{1}{r}\text{cis}(-\theta)$$

or  $\frac{1}{r}$  times the solution to (b)

and since  $r = \frac{\sqrt{21}}{7} = \sqrt{\frac{3}{7}}$ ,  $\frac{1}{r} = \sqrt{\frac{7}{3}}$  and the (b) solutions are

$$\sqrt{\frac{3}{7}}\left(\sqrt{\frac{3}{7}} \pm \frac{2}{\sqrt{7}}i\right)$$

Hence the three solutions are  $z = \frac{1}{3}, \sqrt{\frac{7}{3}}\left(\sqrt{\frac{3}{7}} \pm \frac{2}{\sqrt{7}}i\right)$

$$= \frac{1}{3}, 1 \pm \frac{2}{\sqrt{3}}i.$$

**OR**

$$z = \frac{1}{3}, \frac{7}{3-2\sqrt{3}i}, \frac{7}{3+2\sqrt{3}i} = \frac{1}{3}, \frac{7(3+2\sqrt{3}i)}{9+12}, \frac{7(3-2\sqrt{3}i)}{9+12}$$

$$= \frac{1}{3}, \frac{3 \pm 2\sqrt{3}i}{3} = 1 \pm \frac{2\sqrt{3}i}{3}.$$

**OR**

$$u = \frac{1}{z} = \frac{2w-1}{2w+1} \frac{-2+\sqrt{3}i}{i\sqrt{3}} = \frac{-3-2i\sqrt{3}}{-3}$$

and the other complex root is the conjugate  $= \frac{3-2i\sqrt{3}}{3}$ .

The solutions are  $z = \frac{1}{3}, 1 - \frac{2i\sqrt{3}}{3}, 1 + \frac{2i\sqrt{3}}{3}$ .

**OR**

Replacing  $z$  with  $\frac{1}{z}$  in  $7z^3 - 27z^2 + 21z - 9 = 0$  and multiplying through by  $z^3$ .

$$7 - 27z + 21z^2 - 9z^3 = 0 \text{ and use } 9z^3 - 21z^2 + 27z - 7 = 0$$

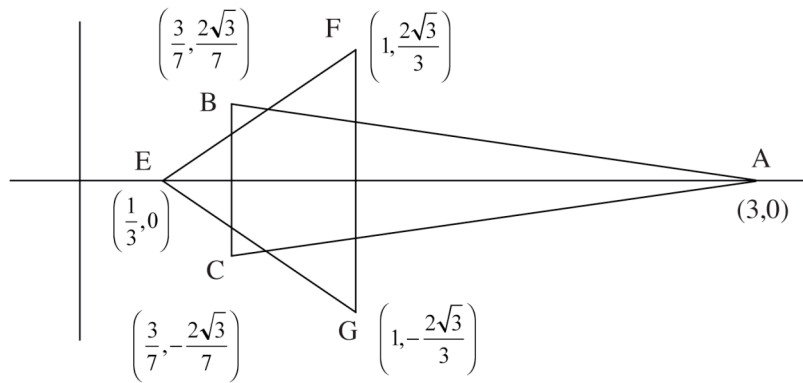
$$(3z-1)(3z^2-6z+7)=0, \text{ and use quadratic formula.}$$

**3****1**Accept  
unsimplified here

Or equivalent

Giving E, F, G

**1**



Ratio of areas  $\frac{\Delta ABC}{\Delta EFG} = \frac{\frac{1}{2} \cdot \frac{4\sqrt{3}}{7} \cdot \frac{18}{7}}{\frac{1}{2} \cdot \frac{4\sqrt{3}}{3} \cdot \frac{2}{3}} = \frac{9 \cdot 18}{2 \cdot 49} = \frac{81}{49}$ .  $[ABC = \frac{36\sqrt{3}}{49}, EFG = \frac{4\sqrt{3}}{9}]$

**1**

For the diagram  
(infer with a  
correct answer)



	$\frac{\frac{1}{2}P}{P - \frac{1}{2}P} = Me^{P \times \frac{1}{2P}t} = Me^{\frac{t}{2}}$ $\frac{\frac{1}{2}P}{P - \frac{1}{2}P} = \frac{1}{199}e^{\frac{t}{2}}$ <hr style="border-top: 1px dashed black;"/> $1 = \frac{1}{199}e^{\frac{t}{2}}$ $\frac{t}{2} = \ln(199), \quad t = 2 \ln(199) = 10.586, \text{ or } 11 \text{ days}$ <p><b>OR</b></p> $N = \frac{\frac{1}{199}Pe^{P \times \frac{1}{2P}t}}{1 + \frac{1}{199}e^{P \times \frac{1}{2P}t}} = 0.5P$ $\Rightarrow \frac{e^{0.5t}}{199 + e^{0.5t}} = 0.5$ <hr style="border-top: 1px dashed black;"/> $\Rightarrow 0.5e^{0.5t} = 199 \times 0.5$ $\Rightarrow e^{0.5t} = 199$ $t = 10.586 \text{ days, ie } 11 \text{ days}$	<p style="text-align: center;"><b>2</b></p> <p style="text-align: center;"><b>2</b></p>	
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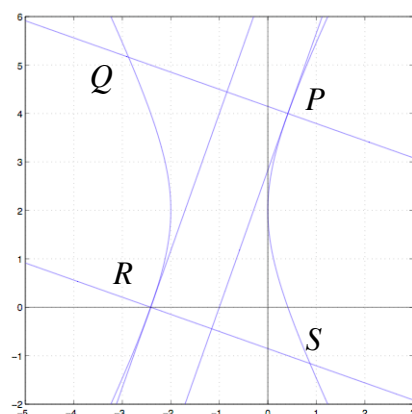


<p><b>FOUR (a)</b></p>	$h(x) = [f(x)]^{g(x)}$ $\ln[h(x)] = g(x) \ln[f(x)]$ $\frac{h'(x)}{h(x)} = g'(x) \ln[f(x)] + g(x) \frac{f'(x)}{f(x)}$ <hr style="border-top: 1px dashed black;"/> $h'(x) = h(x) \left[ g'(x) \ln[f(x)] + g(x) \frac{f'(x)}{f(x)} \right]$ $= [f(x)]^{g(x)} \left[ g'(x) \ln[f(x)] + g(x) \frac{f'(x)}{f(x)} \right]$ <p><b>OR</b></p> $h(x) = e^{\ln([f(x)]^{g(x)})} = e^{g(x) \ln[f(x)]}$ $h'(x) = e^{g(x) \ln[f(x)]} \left( g'(x) \ln[f(x)] + g(x) \frac{f'(x)}{f(x)} \right)$ $h'(x) = [f(x)]^{g(x)} \left[ g'(x) \ln[f(x)] + g(x) \frac{f'(x)}{f(x)} \right]$	<p><b>2</b></p> <hr style="border-top: 1px dashed black;"/> <p><b>-1</b></p>	<p>Do not allow only <math>h'(x)</math> on LHS</p>
<p><b>FOUR (b)</b></p>	$p(x) = x \ln(\ln x)$ $p'(x) = x \frac{1}{\ln x} + \ln(\ln x)$ $= \frac{1}{\ln x} + \ln(\ln x)$ <hr style="border-top: 1px dashed black;"/> <p>If <math>f(x) = \ln x</math> then <math>f'(x) = \frac{1}{x}</math>  and <math>g(x) = x</math> then <math>g'(x) = 1</math></p> <p>So <math>h'(x) = (\ln x)^x \left[ g'(x) \ln[f(x)] + g(x) \frac{f'(x)}{f(x)} \right] = (\ln x)^x \left( 1 \times \ln(\ln x) + x \times \frac{1}{x} \times \frac{1}{\ln x} \right)</math></p> $= (\ln x)^x \left( \ln(\ln x) + \frac{1}{\ln x} \right)$ <hr style="border-top: 1px dashed black;"/> <p>Hence</p> $\int (\ln x)^x p'(x) dx = \int (\ln x)^x \left[ \ln(\ln x) + \frac{1}{\ln x} \right] dx$ $= \int h'(x) dx = (\ln x)^x + C$ <p><b>OR</b></p> $p(x) = \ln(\ln x)^x$ $e^{p(x)} = (\ln x)^x$ <p>but <math>\frac{d(e^{p(x)})}{dx} = p'(x) e^{p(x)}</math></p> <p>So <math>\int p'(x) e^{p(x)} dx = e^{p(x)} + C</math> and <math>\int (\ln x)^x p'(x) dx = e^{p(x)} + C = (\ln x)^x + C</math></p>	<p><b>3</b></p> <hr style="border-top: 1px dashed black;"/> <p><b>-1</b></p> <hr style="border-top: 1px dashed black;"/> <p><b>-2</b></p>	<p>+C required</p>

<b>FOUR</b> <b>(c)</b>	<p> <math>g(x) = -(2x - a)^4 + bx + c</math>            For a factor <math>x - 1</math>, <math>g(1) = -(2 - a)^4 + b + c = 0</math> or  <math>b + c = (2 - a)^4</math> </p> <hr/> <p> <math>-(2x - 1)^4 + 8x - 7 = 0</math> so <math>a = 1, b = 8, c = -7</math>  <math>b + c = 1</math> and <math>(2 - a)^4 = 1</math> so <math>x - 1</math> is a factor.  <math>-(2x - 1)^4 + 8x - 7 = (x - 1)(fx^3 + gx^2 + hx + j)</math>  <math>-16x^4 + 32x^3 - 24x^2 + 8x - 1 + 8x - 7 = (x - 1)(fx^3 + gx^2 + hx + j)</math> </p> <hr/> <p> <math>-16x^4 + 32x^3 - 24x^2 + 16x - 8 = (x - 1)(fx^3 + gx^2 + hx + j)</math>            and comparing coefficients  <math>f = -16, j = 8, -h + j = 16, h = -8, -g + h = -24, g = 16.</math>  <math>-(2x - 1)^4 + 8x - 7 = (x - 1)(-16x^3 + 16x^2 - 8x + 8)</math>  <math>= 8(x - 1)(-2x^3 + 2x^2 - x + 1) = 8(x - 1)(x - 1)(-2x^2 - 1)</math>  <math>-8(x - 1)^2(2x^2 + 1)</math>            So for <math>-(2x - 1)^4 + 8x - 7 = 0</math>,  <math>-8(x - 1)^2(2x^2 + 1) = 0</math>  <math>x = 1</math> (twice) or <math>x = \pm\sqrt{\frac{-1}{2}} = \pm\frac{\sqrt{2}}{2}i</math> </p>	<b>3</b>	Or equivalent
		<b>1</b>	
		<b>2</b>	For $x - 1$ and expansion (independent marks) OR using long division

<p><b>FIVE</b> <b>(a)</b></p>	<p><math>x = \sec \theta - 1, y = 2 \tan \theta + 2</math></p> <p>Since <math>\sec^2 \theta = 1 + \tan^2 \theta</math></p> $(1+x)^2 = 1 + \left(\frac{y-2}{2}\right)^2$ $(y-2)^2 = 4(1+x)^2 - 4$	<p><b>2</b></p>	<p>1 only for wrong signs.</p> <p>Or equivalent</p> <p>Accept</p> $y = \pm 2\sqrt{(x^2 + 2x)} + 2$
<p><b>FIVE</b> <b>(b)</b></p>	<p><b>Either</b></p> $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2 \sec^2 \theta}{\sec \theta \tan \theta} = \frac{2 \sec \theta}{\tan \theta}$ <p>so the gradient of the normal is <math>-\frac{\tan \theta}{2 \sec \theta} = -\frac{\sin \theta}{2}</math>.</p> <p>When <math>\theta = \frac{\pi}{4}, -\frac{\sin \theta}{2} = -\frac{1}{2\sqrt{2}}</math>.</p> <hr/> <p><b>Or</b></p> $(y-2)^2 = 4(1+x)^2 - 4 \text{ so } 2(y-2)\frac{dy}{dx} = 8(1+x)$ $\frac{dy}{dx} = \frac{4(1+x)}{y-2} \text{ and at } x = (\sqrt{2}-1, 4) \frac{dy}{dx} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$ <p>So the gradient of the normal is <math>-\frac{1}{2\sqrt{2}}</math>.</p> <hr/> <p>Equation of the normal at <math>\theta = \frac{\pi}{4},</math></p> $y - (2+2) = -\frac{1}{2\sqrt{2}}(x - \sqrt{2} + 1)$ $y - 4 = -\frac{1}{2\sqrt{2}}(x - \sqrt{2} + 1)$ <hr/> $(x+1) = -2\sqrt{2}(y-4) + \sqrt{2}$ <p>So this meets the curve again where</p> $(-2\sqrt{2}(y-4) + \sqrt{2})^2 = \frac{(y-2)^2 + 4}{4}$ $4(-2\sqrt{2}y + 9\sqrt{2})^2 = (y-2)^2 + 4$ $4(8y^2 - 72y + 162) = y^2 - 4y + 8$ $31y^2 - 284y + 640 = 0 \text{ but } y = 4 \text{ is a solution, so}$ $(y-4)(31y-160) = 0 \text{ and } y = \frac{160}{31}, a = 160, b = 31$	<p><b>3</b></p> <p><b>1</b></p> <p><b>1</b></p> <p><b>2</b></p>	<p>Or equivalent</p> <p>Accept implicit <math>a, b,</math> but exact answer only.</p>

**FIVE**  
(c)



R is the point where

$$\theta = \frac{3\pi}{4}$$

$$x = -\sqrt{2} - 1, \quad y = 0$$

**EITHER**

**Either**

QR is parallel to PS, so by symmetry the  $y$  coordinate of S is given by:

$$y - 4 = 0 - \frac{160}{31}$$

$$y = \frac{124 - 160}{31} = -\frac{36}{31}.$$

**Or**

Equation of normal through R is

$$y - 0 = -\frac{1}{2\sqrt{2}}(x + \sqrt{2} + 1)$$

and this meets the curve again where

$$(-2\sqrt{2}y - \sqrt{2})^2 = \frac{(y - 2)^2 + 4}{4}$$

$$8(-2y - 1)^2 = (y - 2)^2 + 4$$

$$8(4y^2 + 4y + 1) = y^2 - 4y + 8$$

$$31y^2 + 36y = 0$$

$$y = 0 \text{ or } y = -\frac{36}{31}$$

$$\text{but } y = 0 \text{ at R and so } y = -\frac{36}{31}.$$

**OR**

By symmetry S can be obtained by rotating Q through  $180^\circ$  about the centre of the hyperbola,  $(-1, 2)$ .

Hence, by similar triangles

$$\frac{160}{31} - 2 = 2 - y \text{ and so } y = 4 - \frac{160}{31} = \frac{124 - 160}{31} = -\frac{36}{31}.$$

**3**

**1**

For coordinates of R

**2**

For correct use of similar triangles, etc

**2**

Or equivalent substitution

Exact answer required, not decimals.