# GESIS Spring Seminar 23

Comparative Social Research with Multi-Group SEM

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Day 2 - 28.02.2023

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## Packages

```
# basic tools
library(tidyverse)
library(sjmisc)

# checking data
library(MVN)
library(correlation)
library(performance)
library(parameters)
library(insight)

# building the models
library(lavaan)

# cheking the models
library(semTools)

# model visualization
library(semPlot)
library(semptools)
```

# Data

```
ESS07 <- read.csv("../data/ESS07.csv")
```

# Confirmatory measurement analysis (CFA)

Many variables in the social and behavioral sciences represent hypothetical constructs (e.g., traits, attitudes, values)

Direct access to hypothetical constructs is rare. Many constructs can only be captured by (multiple) observable indicators (e.g., in a questionnaire)

It is assumed that the such indicators are manifestations of hypothetical constructs - Manifest indicators: directly observable - Latent variables: not directly observable

We need a model that is able to conceptualize the relationship between manifest indicators and latent variables

The classic test theory Novick (1966)

## Procedure

Step 1: Translation of the theoretical model into a measurement model

- We need a model that is able to conceptualize the relationship between manifest indicators and latent variables

Step 2: Checking whether it is a reflexive or formative construct

- Reflective measurement: indicators are "effects" of the latent variable
- Formative measurement: latent variable is "caused" or composed by the manifest indicators

Step 3: Specification of the model (Choose a method to assign a scale to the latent variable)

- Reference indicator method: factor loading of one indicator is fixed to 1.0
- Fixed factor method: the factor variance is fixed to 1.0

Step 4: Checking the model assumptions (ML estimation requirements)

- Data are continuous and multivariate normal (case of non-normal data standard errors and chi-sqare-statistics should be adjusted - e.g., robust ML = MLR)
- Sample size is sufficiently large (ratio of number of cases and number of parameters - N:q rule => 20:1 or 10:1)

Step 5: Checking wether the model is identified

- Complexity of a CFA models is limited by the total number of observations (p) and the number of unknown model parameters (q)
- Degrees of freedom must be at least zero: df = p - q >= 0
- j = number of indicators
- p = j * (j + 1) / 2
- q = counting of the parameters to be calculated (factor loading, residual error, factor variance)
- df = 0 => model is idenitfied and saturated / df > 0 => model is overidentified

Step 6: Checking the fit measurements

| Index | Typ | Theoretical range | Cut-off | N sensitive | Penalty for complexity |
|---|---|---|---|---|---|
| X2/df | badness | >= 0 | < 5 | yes | no |
| CFI | godness | 0.0 – 1.0 | >= 0.95 (>= 0.90) | no | yes |
| TLI | godness | 0.0 – 1.0* | / | no | yes |

| Index | Typ | Theoretical range | Cut-off | N sensitive | Penalty for complexity |
|---|---|---|---|---|---|
| SRMR | badness | >= 0 | < 0.08 | yes | no |
| RMSEA | badness | >= 0 | <= 0.05 (<= 0.08) | yes to small N | yes |
| PCLOSE | badness | 0.0 – 1.0 | >= 0.95 | yes | / |

*negative values indicate extremely misspecified model; when exceeds 1, model is extremely well-fitting

Step 7: Checking for misspecification

- What causes model misfit? => Indicator choice, Factor choice, Violations of assumptions (e.g., multivariate normality) and Causal structure (e.g., restrictions)
- How to diagnose model misfit? => Parameter estimates (Heywood cases?), Residual matrices (i.e., differences between observed and estimated covariances), Modification indices (approximation of the reduction of chi-square if a single constrained parameter is freely estimated)

Step 8: Identifying the mean structure (optional)

- Fixing the intercept of the reference indicator to zero
- Fixing the latent mean to zero

## CFA model of Universalism

### Step 1

Our example measurement model Universalism comes from the Theory of Basic Human Values according to Schwartz (1992) and comprises three facets - Concern, Nature, Tolerance.

Concern: Commitment to equality, justice and protection for all people

Nature: Preservation of the natural environment

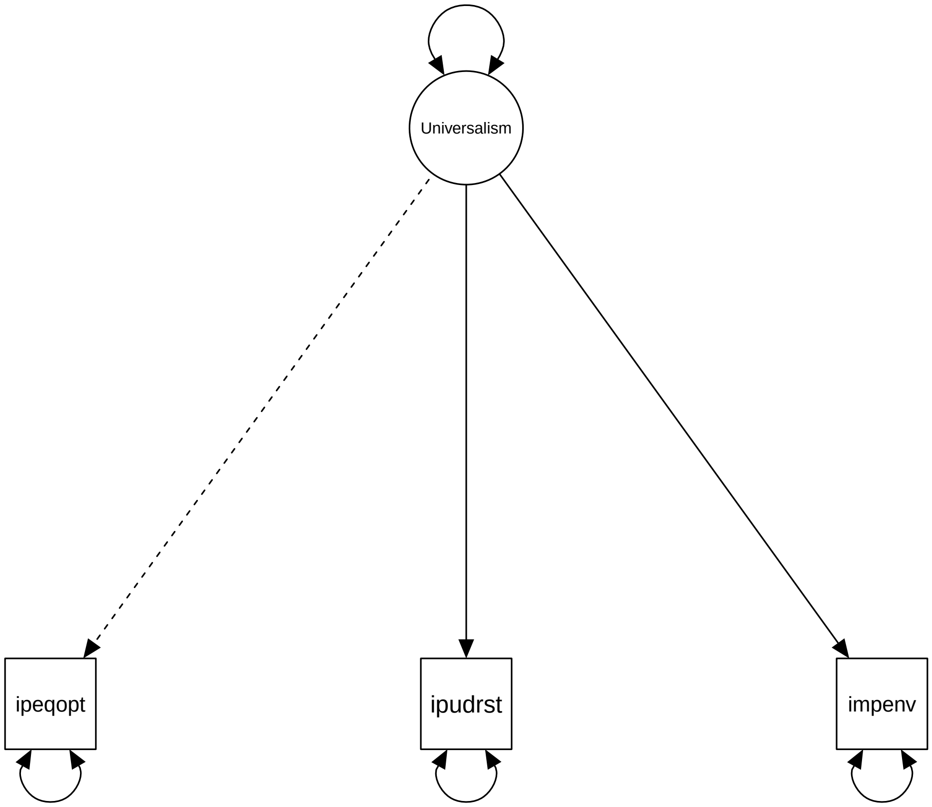
Tolerance: Acceptance and understanding of those who are different from oneself

In order to measure this attitude construct, three questions (items) were created in ESS 7 (2014) and queried in various countries (e.g. Czech Republic, Germany, Great Britain).

| Name | Label | Construct |
|---|---|---|
| ipeqopt | Important that people are treated equally and have equal opportunities | Universalism |
| ipudrst | Important to understand different people | Universalism |
| impenv | Important to care for nature and environment | Universalism |

### Step 2

Indicators are “effects” of the latent variable and therefore we specify a reflective measurement model




**Step 3**

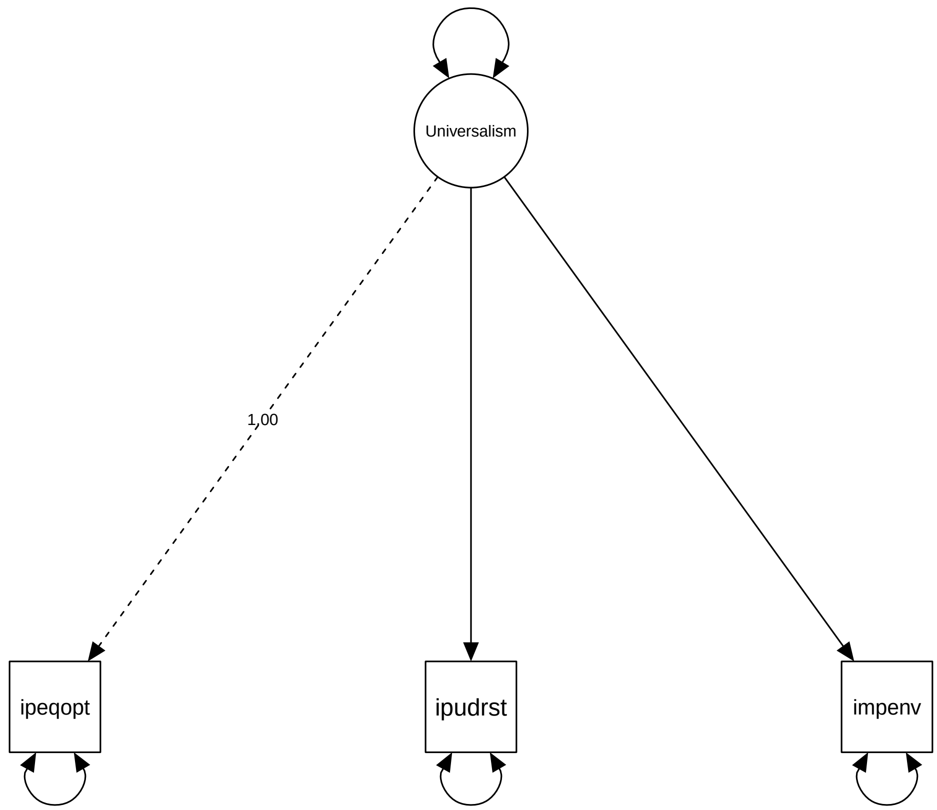
The syntax commands for specifying a lavaan model:

https://lavaan.ugent.be/tutorial/syntax1.html

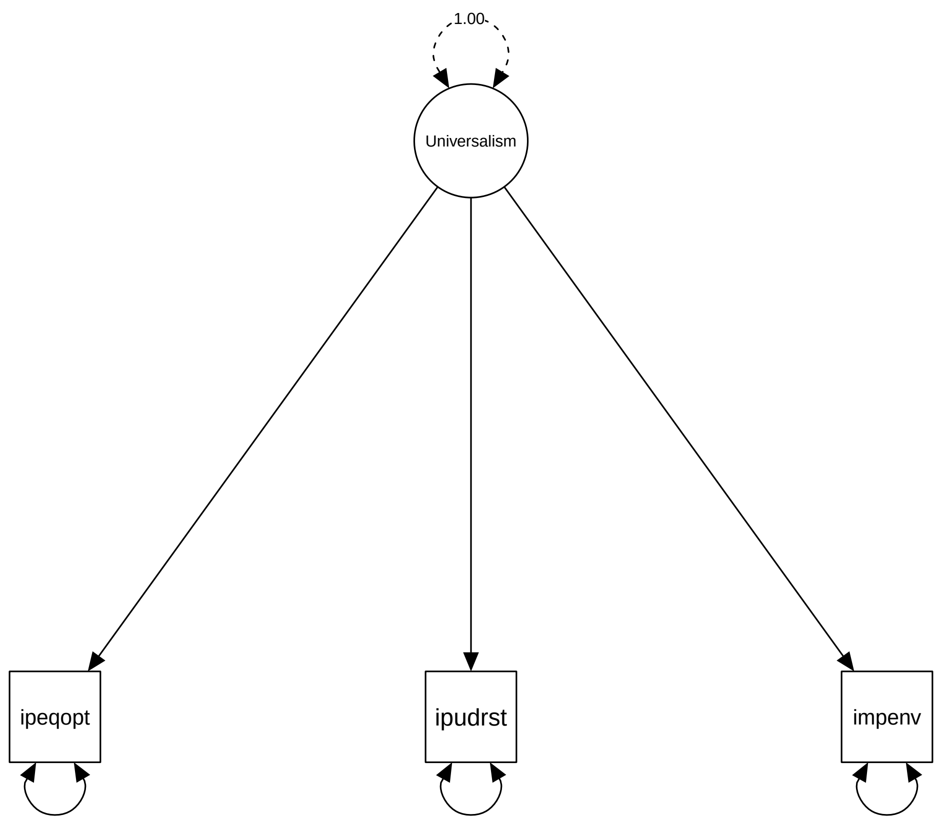
| Formula type | Operator | Mnemonic |
|---|---|---|
| latent variable definition | =~ | is measured by |
| regression | ~ | is regressed on |
| (residual) (co)variance | ~~ | is correlated with |
| intercept | ~1 | intercept |

```
universalism <- "uni =~ ipeqopt + ipudrst + impenv"
```

Lavaan automatically uses the reference indicator method, so one indicator's factual loading is fixed at 1. Accordingly, the variance of the latent factor is estimated.




In Lavaan, however, it is also very easy to use the fixed factor method, i.e. to fix the variance of the latent factor to 1. This allows all factor loadings to be freely estimated.



This requires the use of labels, i.e. the labeling of individual parameters in order to be able to control them directly. The syntax commands for labeling a lavaan model can be found here: https://lavaan.ugent.be/tutorial/syntax2.html. For this step, we only need to understand how individual parameters can be fixed and freely calculated. As soon as it comes to the calculation of measurement invariance, we will work with labels again. But more on that later.

Reference indicator method:

```
universalism <- "uni =~ 1*ipeqopt + ipudrst + impenv"
```

Fixed factor method:

```
universalism <- "
# specification of the measurement model
uni =~ NA*ipeqopt + ipudrst + impenv

# specification of the variance
uni ~~ 1*uni
"
```

Freely estimate the factor loading of the first indicator by NA*ipeqopt and fix the variance of the latent factor by 1*uni. The variance of a variable is specified as covariance with itself, hence uni ~~ uni. In the end we get uni ~~ 1*uni

### Step 4

Now let's get an overview of the properties of our variables. First, let's look at the descriptive statistics of our items to gain knowledge of missing values, central tendency properties, variability, and distribution.

```
ESS07 %>%
  select(ipeqopt, ipudrst, impenv) %>%
  mvn() %>%
  # optional for better display
  export_table(table_width = 1, digits = 3)
```

```
Test          |      HZ | p value | MVN
---------------------------------------
Henze-Zirkler | 133.042 |       0 |  NO

Test             |  Variable | Statistic |   p value | Normality
----------------------------------------------------------------
Anderson-Darling |  ipeqopt  |  362.6880 |  <0.001   |    NO
Anderson-Darling |  ipudrst  |  332.9049 |  <0.001   |    NO
Anderson-Darling |  impenv   |  322.7889 |  <0.001   |    NO

   n |  Mean | Std.Dev | Median | Min | Max | 25th | 75th |   Skew | Kurtosis
-----------------------------------------------------------------------------
5844 | 4.945 |   1.040 |      5 |   1 |   6 |    4 |    6 | -1.142 |    1.295
5844 | 4.717 |   1.027 |      5 |   1 |   6 |    4 |    5 | -0.937 |    0.892
5844 | 4.878 |   1.025 |      5 |   1 |   6 |    4 |    6 | -0.933 |    0.732
```

This shows that the individual variables are not normally distributed and that there is no multivariate normal distribution (Henze-Zirkler value). So, we don't have any missing values in our example dataset and a more than sufficient sample size. In addition, the variables show a right skewed distribution and a moderate variance.

Due to the missing multivariate normal distribution, we use the MLR estimator. If we were dealing with missing values, we would use Full Information Maximum Likelihood (FIML) as the estimation strategy.

**Step 5**

Now let's estimate the model using Lavaan. For this we use the cfa() function.

```
universalism_fit <- cfa(
  model = "Universalism =~ ipeqopt + ipudrst + impenv",
  data = ESS07,
  estimator = "MLR",
  # optional if there are missing values
  missing = "fiml.x"
)
```

Now we can look at the estimated model. To do this, we use the summary() function.

Fit.measures = T shows us the most important fit measurements of our model and thus the fit to the data. Standardized = T shows us standardized values in the output and rsquare shows us the explained variance of the dependent variable.

```
summary(
  object = universalism_fit,
  fit.measures = T,
  standardized = T,
  rsquare = T
)
```

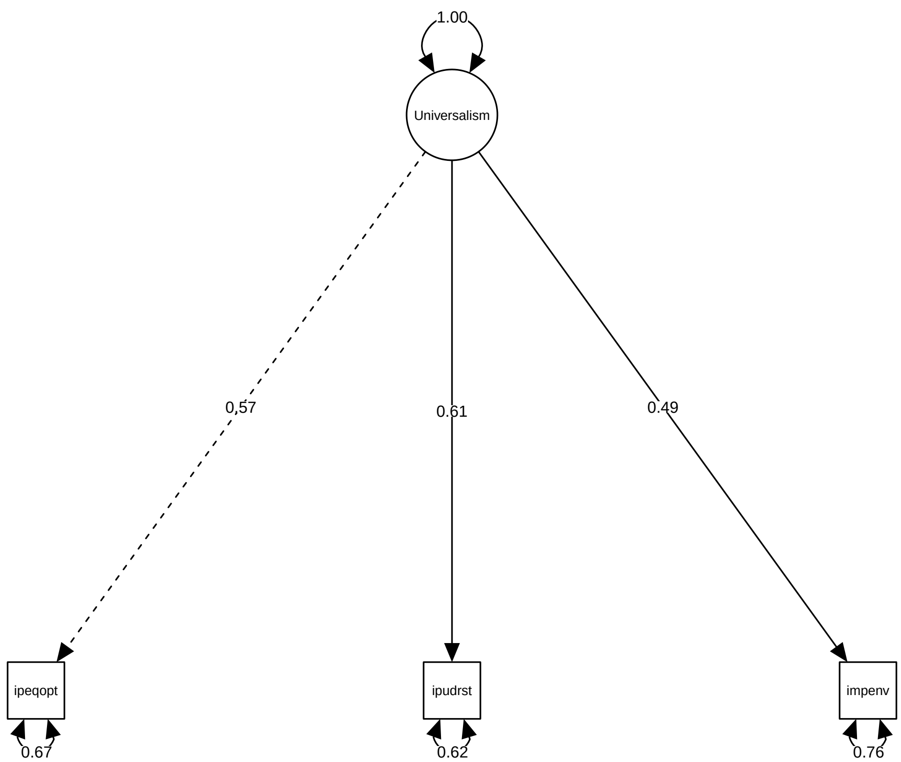
```
lavaan 0.6-12 ended normally after 24 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         9

  Number of observations                          5844
  Number of missing patterns                         1

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                 0.000       0.000
  Degrees of freedom                                 0           0

Model Test Baseline Model:

  Test statistic                              1532.717    1023.100
  Degrees of freedom                                 3           3
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.498

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    1.000       1.000
  Tucker-Lewis Index (TLI)                       1.000       1.000

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -24640.784  -24640.784
```

```
  Loglikelihood unrestricted model (H1)     -24640.784  -24640.784

  Akaike (AIC)                               49299.567   49299.567
  Bayesian (BIC)                             49359.626   49359.626
  Sample-size adjusted Bayesian (BIC)        49331.026   49331.026

Root Mean Square Error of Approximation:

  RMSEA                                          0.000       0.000
  90 Percent confidence interval - lower         0.000       0.000
  90 Percent confidence interval - upper         0.000       0.000
  P-value RMSEA <= 0.05                             NA          NA

  Robust RMSEA                                               0.000
  90 Percent confidence interval - lower                     0.000
  90 Percent confidence interval - upper                     0.000

Standardized Root Mean Square Residual:

  SRMR                                           0.000       0.000

Parameter Estimates:

  Standard errors                             Sandwich
  Information bread                           Observed
  Observed information based on                Hessian

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Universalism =~
    ipeqopt           1.000                               0.595    0.572
    ipudrst           1.059    0.063   16.852    0.000    0.630    0.613
    impenv            0.837    0.046   18.010    0.000    0.498    0.486

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .ipeqopt           4.945    0.014  363.381    0.000    4.945    4.753
   .ipudrst           4.717    0.013  351.024    0.000    4.717    4.592
   .impenv            4.878    0.013  363.981    0.000    4.878    4.761
    Universalism      0.000                               0.000    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .ipeqopt           0.729    0.030   24.070    0.000    0.729    0.673
   .ipudrst           0.659    0.029   22.371    0.000    0.659    0.624
   .impenv            0.802    0.023   34.294    0.000    0.802    0.764
    Universalism      0.354    0.027   13.341    0.000    1.000    1.000

R-Square:
                   Estimate
    ipeqopt           0.327
    ipudrst           0.376
    impenv            0.236
```

Since we estimated a model with three indicators, it is a saturated model, i.e. a model identified with 0 degrees of freedom.

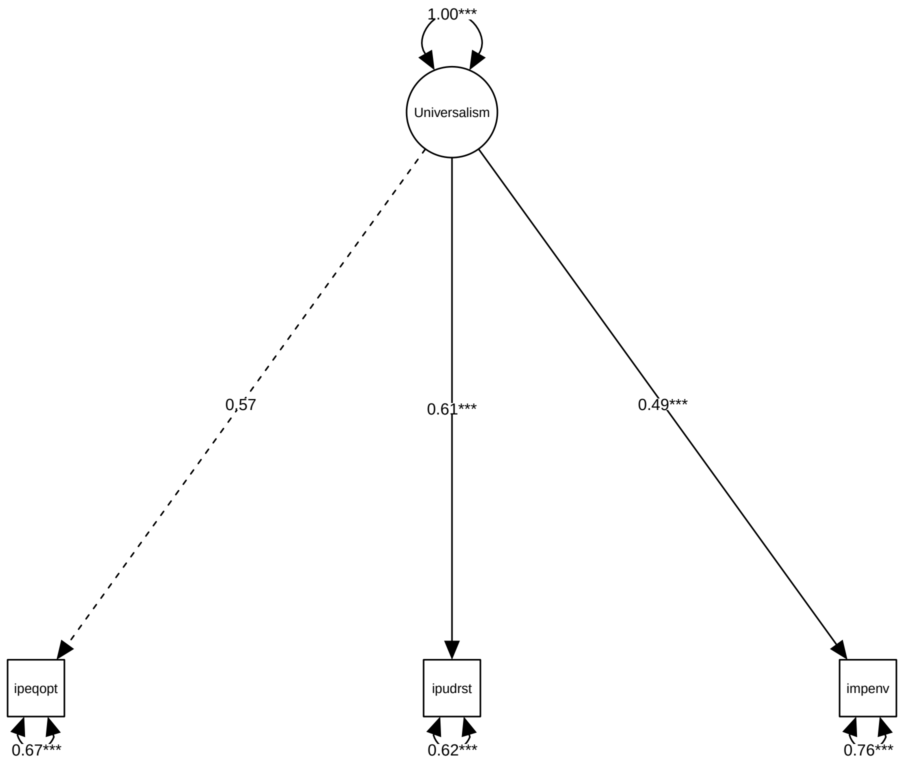
This allows the following graphic to be created with standardized factor loadings.

```
universalism_plot <- universalism_fit %>%
  semPaths(
    whatLabels = "std",
    style = "mx",
    layout = "tree",
    edge.color = "black",
    intercepts = F,
    nCharNodes = 50
  )
```




The semptools package also allows us to easily add additional information to our graphics ( https://cran.r-project.org/web/packages/semptools/vignettes/semptools.html). Here I have also included the degree of statistical significance.

```
universalism_plot %>%
  mark_sig(universalism_fit) %>%
  plot()
```




**Step 6:**

A look at the fit measures shows us that the model is saturated due to the 0 degrees of freedom and thus achieves a perfect data fit.

**Step 7:**

In the following we want to check whether the selected items represent a factor. It can be seen that the factor loadings all reach the cut off of >= 0.4 and thus, in combination with the fit indices, there is an appropriate measurement model.

If there are recognizable misfits, we can access the modification indices.

Modification indices (MIs) = Approximation of the reduction of X2 if a single constrained parameter is freely estimated

Constrained parameters in CFA: - Factor cross-loadings - Residual correlations

–> Test of local fit –> One parameter at a time

```
universalism_fit %>%
  modificationindices(standardized = T, sort. = T)
```

| lhs | op | rhs | mi | epc | sepc.lv | sepc.all | sepc.nox |
|---|---|---|---|---|---|---|---|

Accordingly, there are no modification options, since the model has just been identified and no degrees of freedom are available.

So let’s look at two more models of Theory of Basic Human Values from the ESS07.

## CFA model of Tradition/Conformity

### Step 1:

| Name | Label | Construct |
|---|---|---|
| ipmodst | Important to be humble and modest, not draw attention | Tradition |
| imptrad | Important to follow traditions and customs | Tradition |
| ipfrule | Important to care for nature and environment | Conformity |
| ipbhprp | Important to behave properly | Conformity |

### Step 2:

Indicators are "effects" of the latent variable and therefore we specify a reflective measurement model.

### Step 3:

We use the reference indicator method again to calculate the model.

### Step 4:

```
ESS07 %>%
  select(ipmodst, imptrad, ipfrule, ipbhprp) %>%
  mvn() %>%
  # optional
  export_table(table_width = 1, digits = 3)
```

```
Test          |     HZ | p value | MVN
--------------------------------------
Henze-Zirkler | 20.393 |       0 |  NO

Test             |  Variable | Statistic |   p value | Normality
----------------------------------------------------------------
Anderson-Darling |  ipmodst  |  242.6587 |  <0.001   |     NO
Anderson-Darling |  imptrad  |  202.8157 |  <0.001   |     NO
Anderson-Darling |  ipfrule  |  173.6286 |  <0.001   |     NO
Anderson-Darling |  ipbhprp  |  227.9644 |  <0.001   |     NO

   n |  Mean | Std.Dev | Median | Min | Max | 25th | 75th |   Skew | Kurtosis
-----------------------------------------------------------------------------
5844 | 4.344 |   1.208 |      5 |   1 |   6 |    4 |    5 | -0.643 |   -0.191
5844 | 4.155 |   1.380 |      4 |   1 |   6 |    3 |    5 | -0.529 |   -0.601
5844 | 3.700 |   1.393 |      4 |   1 |   6 |    2 |    5 | -0.161 |   -0.984
5844 | 4.258 |   1.281 |      5 |   1 |   6 |    3 |    5 | -0.554 |   -0.514
```

The variables are scaled metrically on a scale from 0 to 6 and distributed skewed to the right. In addition, there is neither a univariate nor a multivariate normal distribution.

### Step 5:

```
tradition_conformity_fit <- cfa(
  model = "TraditionConformity =~ ipmodst + imptrad + ipfrule + ipbhprp",
  data = ESS07,
  estimator = "MLR",
  # optional if there are missing values
```

```r
  missing = "fiml.x"
)
```

```r
summary(
  object = tradition_conformity_fit,
  fit.measures = T,
  standardized = T,
  rsquare = T
)
```

```
lavaan 0.6-12 ended normally after 31 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        12

  Number of observations                          5844
  Number of missing patterns                         1

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                17.439      14.378
  Degrees of freedom                                 2           2
  P-value (Chi-square)                           0.000       0.001
  Scaling correction factor                                  1.213
    Yuan-Bentler correction (Mplus variant)

Model Test Baseline Model:

  Test statistic                              2339.880    1848.011
  Degrees of freedom                                 6           6
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.266

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.993       0.993
  Tucker-Lewis Index (TLI)                       0.980       0.980

  Robust Comparative Fit Index (CFI)                         0.994
  Robust Tucker-Lewis Index (TLI)                            0.981

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -38377.968  -38377.968
  Scaling correction factor                                  1.002
      for the MLR correction
  Loglikelihood unrestricted model (H1)     -38369.248  -38369.248
  Scaling correction factor                                  1.032
      for the MLR correction

  Akaike (AIC)                               76779.936   76779.936
  Bayesian (BIC)                             76860.014   76860.014
  Sample-size adjusted Bayesian (BIC)        76821.881   76821.881
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.036       0.033
  90 Percent confidence interval - lower         0.022       0.019
  90 Percent confidence interval - upper         0.053       0.048
  P-value RMSEA <= 0.05                          0.909       0.973

  Robust RMSEA                                               0.036
  90 Percent confidence interval - lower                     0.020
  90 Percent confidence interval - upper                     0.054

Standardized Root Mean Square Residual:

  SRMR                                           0.011       0.011

Parameter Estimates:

  Standard errors                             Sandwich
  Information bread                           Observed
  Observed information based on                Hessian

Latent Variables:
                         Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  TraditionConformity =~
    ipmodst                 1.000                                0.426    0.353
    imptrad                 1.546    0.095   16.327    0.000    0.659    0.477
    ipfrule                 1.875    0.112   16.689    0.000    0.799    0.574
    ipbhprp                 2.084    0.110   18.900    0.000    0.888    0.693

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .ipmodst           4.344    0.016  274.875    0.000    4.344    3.596
   .imptrad           4.155    0.018  230.189    0.000    4.155    3.011
   .ipfrule           3.700    0.018  203.091    0.000    3.700    2.657
   .ipbhprp           4.258    0.017  254.054    0.000    4.258    3.323
    TraditnCnfrmty    0.000                               0.000    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .ipmodst           1.278    0.027   48.093    0.000    1.278    0.876
   .imptrad           1.470    0.036   41.297    0.000    1.470    0.772
   .ipfrule           1.301    0.038   34.612    0.000    1.301    0.671
   .ipbhprp           0.853    0.039   21.855    0.000    0.853    0.519
    TraditnCnfrmty    0.182    0.018   10.217    0.000    1.000    1.000

R-Square:
                   Estimate
    ipmodst           0.124
    imptrad           0.228
    ipfrule           0.329
    ipbhprp           0.481
```

The model is identified with two degrees of freedom.

**Step 6:**

The fit indices show a good fit to the data. All fit indices are below their cut offs.

**Step 7:**

In this model, the factor loading of the item ipmodst is clearly misfit, since the factor loading cut-off of 0.4 is not reached.

Here it has to be considered whether the item has a value for the construct or whether the item has to be removed from the measurement model. We decide to take it out.

Further modifications are not necessary since the model shows a very good fit to the data.

```
tradition_conformity_fit2 <- cfa(
  model = "TraditionConformity =~ imptrad + ipfrule + ipbhprp",
  data = ESS07,
  estimator = "MLR",
  # optional if there are missing values
  missing = "fiml.x"
)
```

```
summary(
  object = tradition_conformity_fit2,
  fit.measures = T,
  standardized = T,
  rsquare = T
)
```

```
lavaan 0.6-12 ended normally after 28 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         9

  Number of observations                          5844
  Number of missing patterns                         1

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                 0.000       0.000
  Degrees of freedom                                 0           0

Model Test Baseline Model:

  Test statistic                              1849.694    1427.618
  Degrees of freedom                                 3           3
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.296

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    1.000       1.000
  Tucker-Lewis Index (TLI)                       1.000       1.000

  Robust Comparative Fit Index (CFI)                         1.000
  Robust Tucker-Lewis Index (TLI)                            1.000
```

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -29217.708  -29217.708
  Loglikelihood unrestricted model (H1)     -29217.708  -29217.708

  Akaike (AIC)                               58453.417   58453.417
  Bayesian (BIC)                             58513.475   58513.475
  Sample-size adjusted Bayesian (BIC)        58484.876   58484.876

Root Mean Square Error of Approximation:

  RMSEA                                          0.000       0.000
  90 Percent confidence interval - lower         0.000       0.000
  90 Percent confidence interval - upper         0.000       0.000
  P-value RMSEA <= 0.05                             NA          NA

  Robust RMSEA                                               0.000
  90 Percent confidence interval - lower                     0.000
  90 Percent confidence interval - upper                     0.000

Standardized Root Mean Square Residual:

  SRMR                                           0.000       0.000

Parameter Estimates:

  Standard errors                             Sandwich
  Information bread                           Observed
  Observed information based on                Hessian

Latent Variables:
                         Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  TraditionConformity =~
    imptrad                 1.000                                0.673    0.488
    ipfrule                 1.247    0.059   21.170    0.000    0.839    0.602
    ipbhprp                 1.252    0.061   20.475    0.000    0.842    0.658

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .imptrad           4.155    0.018  230.189    0.000    4.155    3.011
   .ipfrule           3.700    0.018  203.091    0.000    3.700    2.657
   .ipbhprp           4.258    0.017  254.054    0.000    4.258    3.323
    TraditnCnfrmty    0.000                               0.000    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .imptrad           1.452    0.036   39.882    0.000    1.452    0.762
   .ipfrule           1.236    0.043   29.051    0.000    1.236    0.637
   .ipbhprp           0.932    0.040   23.156    0.000    0.932    0.568
    TraditnCnfrmty    0.453    0.032   14.009    0.000    1.000    1.000

R-Square:
                   Estimate
```

```
imptrad          0.238
ipfrule          0.363
ipbhprp          0.432
```

This makes the measurement model suitable for further use.

## CFA model Perceived Threat

### Step 1:

| Name | Label | Construct |
|---|---|---|
| imbgeco | Immigration bad or good for country's economy | Perceived threat |
| imueclt | Country's cultural life undermined or enriched by immigrants | Perceived threat |
| imtcjob | Immigrants take jobs away in country or create new jobs | Perceived threat |
| rlgueim | Religious beliefs and practices undermined or enriched by immigrants | Perceived threat |

### Step 2:

Indicators are "effects" of the latent variable and therefore we specify a reflective measurement model

### Step 3:

We use the reference indicator method again to calculate the model.

### Step 4:

```
ESS07 %>%
  select(imbgeco, imueclt, imtcjob, rlgueim) %>%
  mvn() %>%
  # optional for better display
  export_table(table_width = 1, digits = 3)
```

```
Test          |     HZ | p value | MVN
--------------------------------------
Henze-Zirkler | 17.085 |       0 |  NO

Test             |  Variable | Statistic |   p value | Normality
----------------------------------------------------------------
Anderson-Darling |  imbgeco  |   61.6271 |  <0.001   |    NO
Anderson-Darling |  imueclt  |   53.8325 |  <0.001   |    NO
Anderson-Darling |  imtcjob  |  103.8354 |  <0.001   |    NO
Anderson-Darling |  rlgueim  |  108.6560 |  <0.001   |    NO

   n |  Mean | Std.Dev | Median | Min | Max | 25th | 75th |  Skew | Kurtosis
-----------------------------------------------------------------------------
5844 | 4.979 |   2.466 |      5 |   0 |  10 |    3 |    7 | 0.186 |   -0.521
5844 | 4.696 |   2.577 |      5 |   0 |  10 |    3 |    6 | 0.174 |   -0.623
5844 | 5.187 |   2.301 |      5 |   0 |  10 |    4 |    7 | 0.228 |   -0.165
5844 | 5.318 |   2.188 |      5 |   0 |  10 |    4 |    7 | 0.041 |    0.053
```

The variables are scaled metrically on a scale from 0 to 10. Nevertheless, there is neither a univariate nor a multivariate normal distribution.

**Step 5:**

```r
perceived_threat_fit <- cfa(
  model = "PerceivedThreat =~ imbgeco + imueclt + imtcjob + rlgueim",
  data = ESS07,
  estimator = "MLR",
  # optional if there are missing values
  missing = "fiml.x"
)
```

```r
summary(
  object = perceived_threat_fit,
  fit.measures = T,
  standardized = T,
  rsquare = T
)
```

```
lavaan 0.6-12 ended normally after 27 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        12

  Number of observations                          5844
  Number of missing patterns                         1

Model Test User Model:
                                              Standard      Robust
  Test Statistic                               279.766     184.727
  Degrees of freedom                                 2           2
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.514
    Yuan-Bentler correction (Mplus variant)

Model Test Baseline Model:

  Test statistic                              8797.948    5161.721
  Degrees of freedom                                 6           6
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.704

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.968       0.965
  Tucker-Lewis Index (TLI)                       0.905       0.894

  Robust Comparative Fit Index (CFI)                         0.969
  Robust Tucker-Lewis Index (TLI)                            0.906

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -49161.345  -49161.345
  Scaling correction factor                                  1.214
      for the MLR correction
  Loglikelihood unrestricted model (H1)     -49021.462  -49021.462
```

```
  Scaling correction factor                                  1.257
      for the MLR correction

  Akaike (AIC)                               98346.690   98346.690
  Bayesian (BIC)                             98426.768   98426.768
  Sample-size adjusted Bayesian (BIC)        98388.636   98388.636

Root Mean Square Error of Approximation:

  RMSEA                                          0.154       0.125
  90 Percent confidence interval - lower         0.139       0.113
  90 Percent confidence interval - upper         0.170       0.138
  P-value RMSEA <= 0.05                          0.000       0.000

  Robust RMSEA                                               0.154
  90 Percent confidence interval - lower                     0.136
  90 Percent confidence interval - upper                     0.173

Standardized Root Mean Square Residual:

  SRMR                                           0.026       0.026

Parameter Estimates:

  Standard errors                             Sandwich
  Information bread                           Observed
  Observed information based on                Hessian

Latent Variables:
                     Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  PerceivedThreat =~
    imbgeco             1.000                               1.967    0.798
    imueclt             1.105    0.022   51.221    0.000    2.174    0.844
    imtcjob             0.792    0.015   52.565    0.000    1.559    0.677
    rlgueim             0.692    0.020   34.946    0.000    1.362    0.622

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .imbgeco           4.979    0.032  154.352    0.000    4.979    2.019
   .imueclt           4.696    0.034  139.327    0.000    4.696    1.823
   .imtcjob           5.187    0.030  172.337    0.000    5.187    2.254
   .rlgueim           5.318    0.029  185.807    0.000    5.318    2.431
    PerceivedThret    0.000                               0.000    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .imbgeco           2.211    0.085   26.148    0.000    2.211    0.364
   .imueclt           1.913    0.094   20.355    0.000    1.913    0.288
   .imtcjob           2.865    0.087   33.077    0.000    2.865    0.541
   .rlgueim           2.933    0.080   36.474    0.000    2.933    0.613
    PerceivedThret    3.870    0.119   32.528    0.000    1.000    1.000

R-Square:
                   Estimate
```

```
imbgeco           0.636
imueclt           0.712
imtcjob           0.459
rlgueim           0.387
```

The model is identified with two degrees of freedom.

**Step 6:**

The model shows a clear misspecification of the model for the RMSEA indice.

**Step 7:**

The factor loadings all show sufficient loading. So we can immediately devote ourselves to the modification indices in order to understand the misspecification evident in the RMSEA.

```
perceived_threat_fit %>%
  modificationindices(standardized = T, sort. = T)
```

| | lhs | op | rhs | mi | epc | sepc.lv | sepc.all | sepc.nox |
|---|---|---|---|---|---|---|---|---|
| 19 | imueclt | ~~ | rlgueim | 238.993104 | 0.8771423 | 0.8771423 | 0.3702775 | 0.3702775 |
| 16 | imbgeco | ~~ | imtcjob | 238.993083 | 0.9085458 | 0.9085458 | 0.3609752 | 0.3609752 |
| 17 | imbgeco | ~~ | rlgueim | 207.424644 | -0.7466824 | -0.7466824 | -0.2931758 | -0.2931758 |
| 18 | imueclt | ~~ | imtcjob | 207.424451 | -0.9445400 | -0.9445400 | -0.4034743 | -0.4034743 |
| 20 | imtcjob | ~~ | rlgueim | 1.848284 | -0.0628233 | -0.0628233 | -0.0216717 | -0.0216717 |
| 15 | imbgeco | ~~ | imueclt | 1.848267 | -0.1265846 | -0.1265846 | -0.0615456 | -0.0615456 |

The modification indices tell us that adjusting the residual covariance between items imueclt and rlgueim would have the largest effect on model fitting (in terms of the chi-square test statistic).

This is because some of the covariation of imueclt and rlgueim is due to sources other than the common factor.

Reasons: method effects, item wording, acquiescence, social desirability, unaccounted theoretical common causes.

We then modify our initial model and remove the residual covariance constraint between the two items, which is fixed at 0, and let it calculate freely.

```
perceived_threat_fit2 <- cfa(
  model = "
  PerceivedThreat =~ imbgeco + imueclt + imtcjob + rlgueim

  imueclt   ~~  rlgueim
  ",
  data = ESS07,
  estimator = "MLR",
  # optional if there are missing values
  missing = "fiml.x"
)
```

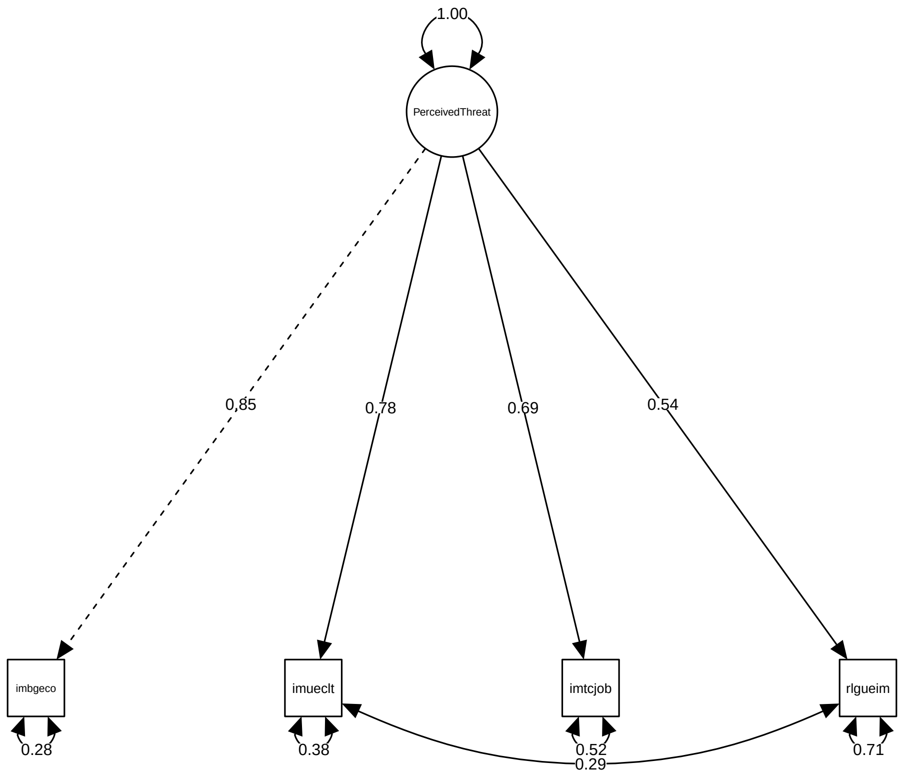
```
summary(
  object = perceived_threat_fit2,
  fit.measures = T,
  standardized = T,
  rsquare = T
)
```

```
lavaan 0.6-12 ended normally after 31 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        13

  Number of observations                          5844
  Number of missing patterns                         1

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                47.224      31.440
  Degrees of freedom                                 1           1
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.502
    Yuan-Bentler correction (Mplus variant)

Model Test Baseline Model:

  Test statistic                              8797.948    5161.721
  Degrees of freedom                                 6           6
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.704

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.995       0.994
  Tucker-Lewis Index (TLI)                       0.968       0.965

  Robust Comparative Fit Index (CFI)                         0.995
  Robust Tucker-Lewis Index (TLI)                            0.969

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -49045.074  -49045.074
  Scaling correction factor                                  1.238
      for the MLR correction
  Loglikelihood unrestricted model (H1)     -49021.462  -49021.462
  Scaling correction factor                                  1.257
      for the MLR correction

  Akaike (AIC)                               98116.148   98116.148
  Bayesian (BIC)                             98202.900   98202.900
  Sample-size adjusted Bayesian (BIC)        98161.589   98161.589

Root Mean Square Error of Approximation:

  RMSEA                                          0.089       0.072
  90 Percent confidence interval - lower         0.068       0.055
  90 Percent confidence interval - upper         0.111       0.091
  P-value RMSEA <= 0.05                          0.001       0.016

  Robust RMSEA                                               0.088
  90 Percent confidence interval - lower                     0.064
```

```
  90 Percent confidence interval - upper                         0.116

Standardized Root Mean Square Residual:

  SRMR                                           0.012       0.012

Parameter Estimates:

  Standard errors                             Sandwich
  Information bread                           Observed
  Observed information based on                Hessian

Latent Variables:
                     Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  PerceivedThreat =~
    imbgeco             1.000                                2.100    0.851
    imueclt             0.963    0.019   50.389    0.000    2.022    0.785
    imtcjob             0.760    0.017   46.031    0.000    1.596    0.694
    rlgueim             0.559    0.018   30.307    0.000    1.174    0.537

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .imueclt ~~
   .rlgueim           0.848    0.072   11.715    0.000    0.848    0.287

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .imbgeco           4.979    0.032  154.352    0.000    4.979    2.019
   .imueclt           4.696    0.034  139.327    0.000    4.696    1.823
   .imtcjob           5.187    0.030  172.337    0.000    5.187    2.254
   .rlgueim           5.318    0.029  185.807    0.000    5.318    2.431
    PerceivedThret    0.000                               0.000    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .imbgeco           1.673    0.091   18.481    0.000    1.673    0.275
   .imueclt           2.551    0.097   26.176    0.000    2.551    0.384
   .imtcjob           2.747    0.084   32.790    0.000    2.747    0.519
   .rlgueim           3.409    0.086   39.542    0.000    3.409    0.712
    PerceivedThret    4.408    0.126   34.979    0.000    1.000    1.000

R-Square:
                   Estimate
    imbgeco           0.725
    imueclt           0.616
    imtcjob           0.481
    rlgueim           0.288
```

```r
perceived_threat_fit2 %>%
  semPaths(
    whatLabels = "std",
    style = "mx",
    layout = "tree",
    edge.color = "black",
```

```
    intercepts = F,
    nCharNodes = 50
  )
```




The RMSEA is then back in our anticipated cut-off area and we can continue working with the measurement model.

**Step 8:**

Finally, we want to display the mean structure of our model.

In practice, the only reason why a user would add intercept-formulas in the model syntax is because some constraints must be specified on them.

```
perceived_threat_mean <- cfa(
  model = "
  PerceivedThreat =~ imbgeco + imueclt + imtcjob + rlgueim

  imueclt   ~~  rlgueim
  ",
  data = ESS07,
  estimator = "MLR",
  # optional if there are missing values
  missing = "fiml.x",
  meanstructure = T
)
```

```
summary(
  object = perceived_threat_mean,
  fit.measures = T,
  standardized = T,
```

```r
  rsquare = T
)
```

```
lavaan 0.6-12 ended normally after 31 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        13

  Number of observations                          5844
  Number of missing patterns                         1

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                47.224      31.440
  Degrees of freedom                                 1           1
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.502
    Yuan-Bentler correction (Mplus variant)

Model Test Baseline Model:

  Test statistic                              8797.948    5161.721
  Degrees of freedom                                 6           6
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.704

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  Comparative Fit Index (CFI)                    0.995       0.994
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      for the MLR correction
  Loglikelihood unrestricted model (H1)     -49021.462  -49021.462
  Scaling correction factor                                  1.257
      for the MLR correction

  Akaike (AIC)                               98116.148   98116.148
  Bayesian (BIC)                             98202.900   98202.900
  Sample-size adjusted Bayesian (BIC)        98161.589   98161.589

Root Mean Square Error of Approximation:

  RMSEA                                          0.089       0.072
  90 Percent confidence interval - lower         0.068       0.055
  90 Percent confidence interval - upper         0.111       0.091
  P-value RMSEA <= 0.05                          0.001       0.016
```

```
  Robust RMSEA                                                  0.088
  90 Percent confidence interval - lower                        0.064
  90 Percent confidence interval - upper                        0.116

Standardized Root Mean Square Residual:

  SRMR                                           0.012          0.012

Parameter Estimates:

  Standard errors                              Sandwich
  Information bread                            Observed
  Observed information based on                 Hessian

Latent Variables:
                     Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  PerceivedThreat =~
    imbgeco             1.000                               2.100    0.851
    imueclt             0.963    0.019   50.389    0.000    2.022    0.785
    imtcjob             0.760    0.017   46.031    0.000    1.596    0.694
    rlgueim             0.559    0.018   30.307    0.000    1.174    0.537

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .imueclt ~~
   .rlgueim           0.848    0.072   11.715    0.000    0.848    0.287

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .imbgeco           4.979    0.032  154.352    0.000    4.979    2.019
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   .imtcjob           5.187    0.030  172.337    0.000    5.187    2.254
   .rlgueim           5.318    0.029  185.807    0.000    5.318    2.431
    PerceivedThret    0.000                               0.000    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .imbgeco           1.673    0.091   18.481    0.000    1.673    0.275
   .imueclt           2.551    0.097   26.176    0.000    2.551    0.384
   .imtcjob           2.747    0.084   32.790    0.000    2.747    0.519
   .rlgueim           3.409    0.086   39.542    0.000    3.409    0.712
    PerceivedThret    4.408    0.126   34.979    0.000    1.000    1.000

R-Square:
                   Estimate
    imbgeco           0.725
    imueclt           0.616
    imtcjob           0.481
    rlgueim           0.288
```

# Measurement Invaraince

## Procedure

Are measurements actually comparable across groups/time?

Definition: Measurement invariance is a property of a measurement instrument, implying that the instrument measures the same concept in the same way across subgroups of respondents or time points of data assessment.

Measurement invariance (MI) can be assessed with multiple-group CFA (MGCFA). Impose equality constrains on measurement parameters across groups/time ("fix them to be equal").

Use a sequential testing strategy to test different levels of measurement invariance: - Configural MI - Metric (weak) MI - Scalar (strong) MI - Strict MI - ...

**Configural measurement invariance**

Equality constraints: Equal structure: pattern of factors and relationships of indicators and factors is equal across groups

Implications: Same construct exists across groups; no comparisons of estimates!

**Metric measurement invariance**

Equality constraints: Equal structure + equal factor loadings across groups

$\lambda^1 = \lambda^2 = \lambda^G$

**Scalar measurement invariance**

Equality constraints: Equal structure + equal factor loadings + equal intercepts across groups

$\lambda^1 = \lambda^2 = \lambda^G$

$\tau^1 = \tau^2 = \tau^G$

Implications: Respondents from different groups "use" the scale in a similar manner; comparison of latent means are valid

**Result**

- Configural invariance: same structure across groups
- Metric invariance: factor loadings set equal across groups
- Scalar invariance: factor loadings and intercepts set equal across groups

How decide which level of measurement invariance is supported by the data?

1. Consecutively estimate the configural, metric, and scalar invariance models
2. Inspect model fit as usual: $\chi^2$ (df), CFI, RMSEA
3. Inspect model fit differences between models:

- $\chi^2/\Delta\chi^2(df/\Delta df)$
- $CFI/\Delta CFI; RMSEA/\Delta RMSEA$

## References

Novick, M. R. (1966). The axioms and principal results of classical test theory. *Journal of Mathematical Psychology*, *3*(1), 1–18. Retrieved February 20, 2023, from https://linkinghub.elsevier.com/retrieve/pii/0022249666900022