1 i) hermitian matrix A

let $(\vec{x}, \lambda)$ be an eigenpair

$$A\vec{x} = \lambda \vec{x}$$

$$\vec{x}^* A \vec{x} = \vec{x}^* \lambda \vec{x} = \lambda \vec{x}^* \vec{x} = \lambda \|\vec{x}\|_2$$

$$\vec{U} \cdot \vec{V} = \vec{U}^T \vec{V} = \vec{V}^T \vec{U} = \vec{V} \cdot \vec{U}$$

$$= (A\vec{x})^* \vec{x} = \vec{x}^* A^* \vec{x}$$

$$\Rightarrow \quad \vec{x}^* A^* \vec{x} = \lambda \|\vec{x}\|_2$$

$$x^* \lambda^* \vec{x}$$

$$\lambda^* \|\vec{x}\|_2 = \lambda \|\vec{x}\|_2 \iff \lambda^* = \lambda \qquad \square$$

ii) A positive definit matrix

$$0 < \varepsilon \leq \vec{v}^T A \vec{v}$$

$$\varepsilon \leq \lambda \vec{v}^T \vec{v} = \lambda \|\vec{v}\|_2$$

$$0 < \frac{\varepsilon}{\|\vec{v}\|_2} \leq \lambda \qquad \square$$

iii) A, B are simmilar if there exists a matrix P (invertible) with $B = P^{-1} A P$

$$\chi_A(\lambda) = \det(A - \lambda I)$$

$$\chi_B(\lambda) = \det(B - \lambda I) = \det(\underbrace{P^{-1} A P - \lambda I}_{})$$

$$P^{-1} A P - P^{-1} \lambda I P = P^{-1}(A - \lambda I) P$$

$$= \det(P^{-1}) \det(A - \lambda I) \det(P)$$

$$= \frac{1}{\det(P)} \det(P) \det(A - \lambda I)$$

$$= \det(A - \lambda I) = \chi_A(\lambda)$$

$\square$