Overview

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1 First

The ideal gas Helmholtz free energy as a function of c_p

$$a(\rho, T) = \int_{T_0}^{T} c_p \mathrm{d}s - RT - T\int_{T_0}^{T} \frac{c_p}{s}\mathrm{d}s + R\log(T) + TR\log(\rho/\rho_0)$$

hence α as a function of δ and τ, without keeping track of the constants,is

$$\begin{aligned}
\alpha(\delta, \tau) &= \frac{a(\delta, \tau)}{RT} \\
&= \tau \int_{\tau}^{\tau_{end}} \frac{c_p'}{s^2}\mathrm{d}s - \int_{\tau}^{\tau_{end}} \frac{c_p'}{s}\mathrm{d}s \frac{\tau(\log(\tau) - \log(T_c))}{T_c} + \log(\delta) \\
&= -\int_{\tau}^{\tau_{end}} c_p' \frac{\tau - s}{s^2}\mathrm{d}s - \frac{\tau(\log(\tau) - \log(T_c))}{T_c} + \log(\delta)
\end{aligned}$$

where $c_p' = c_p/R$. It follows that the derivatives are

$$\alpha_\delta = \frac{1}{\delta} \tag{1}$$

$$\alpha_{\delta\delta} = -\frac{1}{\delta^2} \tag{2}$$

$$\alpha_\tau = -\int_{\tau}^{\tau_{end}} c_p'{}_\tau \frac{\tau - s}{s^2} + c_p' \frac{1}{s^2}\mathrm{d}s - \frac{\log(\tau) - \log(T_c) + 1}{T_c} \tag{3}$$

$$\alpha_{\tau\tau} = -\int_{\tau}^{\tau_{end}} c_p'{}_{\tau\tau} \frac{\tau - s}{s^2} + c_p'{}_\tau (\frac{2}{s^2} - \frac{1}{\tau T_c} \tag{4}$$