Bachelor thesis submitted in partial fulfillment of the
requirements for the degree

Bachelor of Science

at Technische Universität München

From Monthly to Daily: Analyzing the Effect of Data Frequency on Portfolio Strategy Performances

Reviewer:	Prof. Dr. Christoph Kaserer Department of Financial Management and Capital Markets TUM School of Management Technische Universität München
Advisor:	Stefan Baumann
Study program:	TUM-BWL

Composed by: Yannick Tobias Wiest
Am Waldeck 5
D-86574 Axtbrunn
Tel.: +49 (0)174/6576872
Matr. Nr.: 03747866

Submitted on: July 11th, 2024

Abstract

This thesis investigates the effect of data and rebalancing frequency on the performance of the minimum- and mean-variance portfolios using equities from Norway and Belgium listed between 2003 and 2023. The results are first compared to the findings of Demiguel et al. (2009), who used monthly data for estimation, before extending the evaluation to daily data. I find that none of the portfolios can consistently outperform the 1/N portfolio in terms of Sharpe ratio, Certainty Equivalent, and turnover using monthly data. However, daily data increases stability and improves performance, allowing the portfolios to beat the 1/N portfolio. Additionally, the portfolios are tested for estimation periods shorter than six months and daily rebalancing. The models perform worse in this setting compared to previous observations and strategies designed for shorter timeframes. This suggests that further research is necessary to make better use of the portfolio theory by Harry Markowitz (1952) in the context of algorithmic trading environments.

Contents

List of Figures

List of Tables

List of Abbreviations

General

HFT	High-frequency trading

Portfolio strategies

1/N	Equally weighted portfolio
min	Minimum-variance portfolio
mv	Mean-variance portfolio
c-min	Constrained minimum-variance portfolio
c-mv	Constrained mean-variance portfolio
mom	Momentum portfolio
bol	Bollinger Bands portfolio

Performance metrics

SR	Sharpe ratio
TO	Turnover
CEQ	Certainty equivalent

Samples

CO-M	Combined monthly data
CO-D	Combined daily data
NO-M	Norway monthly data
NO-D	Norway daily data
BE-M	Belgium monthly data
BE-D	Belgium daily data

Used for formulas

SMA	Simple moving average
WS	Estimation window length
Close	Close price of an equity
P	Price of an equity
RI	Return index
PI	Price index
DY	Dividend yield
BB^{up}	Upper Bollinger Band
BB^{low}	Lower Bollinger Band

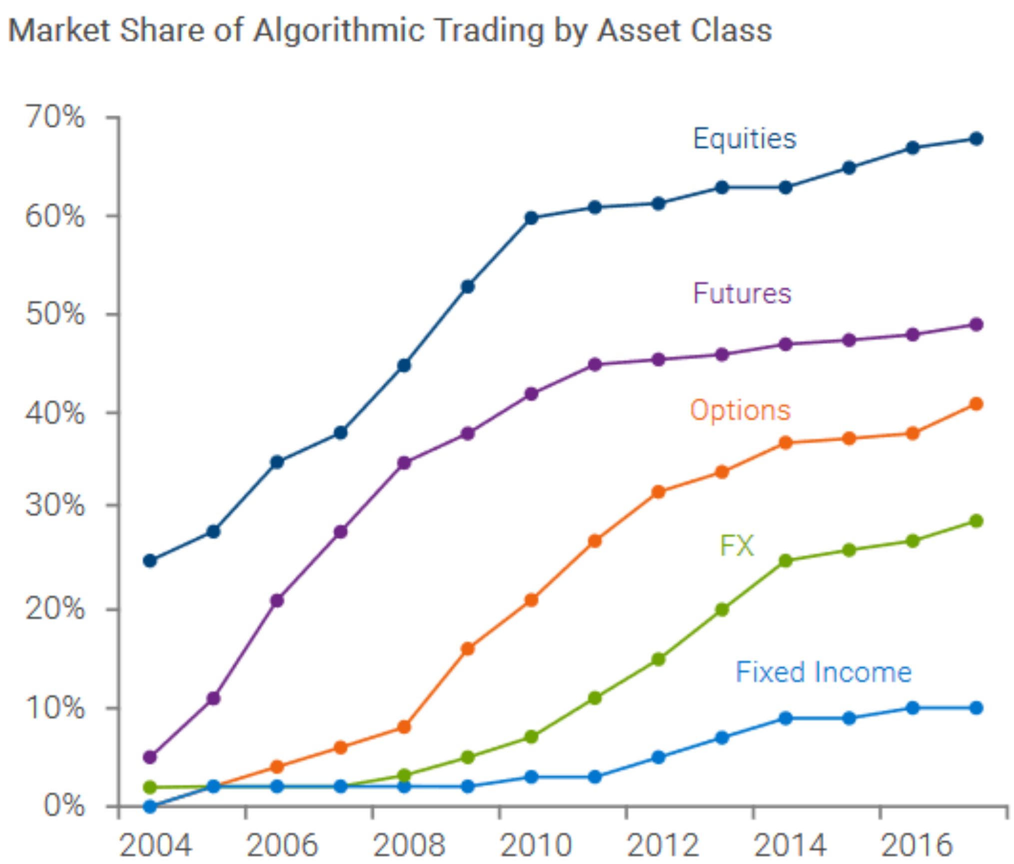
List of Symbols

N	Number of equities
w_i	N-vector of stock weights in iteration t
μ	In-sample excess returns
$\hat{\mu}$	Out-of-sample excess returns
Σ	$N \times N$ covariance matrix of excess returns
$\hat{\Sigma}$	Out-of-sample covariance matrix
σ^2	Variance of excess returns
σ	Standard deviation of excess returns
1_N	N-vector of ones
γ	Risk aversion of an investor
H_0	Null hypothesis
$\hat{z}_{JK}$	Test statistic
α	Significance level

1 Introduction

For the last 20 years, algorithmic trading has been taking over the markets. Vynckier et al. (2021) found that this phenomenon exists across all big sectors, including equities, futures, options, FX, and fixed income. According to Figure 1, the highest share of algorithmic trading can be found in the equities market. While only 25% of all trades on equities were executed by an algorithm in 2004, in 2017, almost 70% were done by computers. The concept of algorithmic trading and its variety is not easy to grasp as it expresses itself in many different facets, for example, trading speeds. On the one hand, there is high-frequency trading (HFT). This

Figure 1: Market share of algorithmic trading



This graph plots the market share of algorithmic trading by asset class. Time progression is plotted on the x-axis. Share in percent is plotted on the y-axis. The graph has been taken from the Dell Technologies report on algorithmic trading by Vynckier et al. (2021).

term does not describe a strategy itself but rather a tool to implement strategies at high speeds according to Gomber et al. (2011). The importance of speed for these strategies has been analyzed by Scholtus and van Dijk (2012), who found that only 50 milliseconds can already significantly disturb performance. These are speeds that humans cannot operate at and therefore cannot compete with. A common HFT strategy that Gomber et al. (2011) describe is market making. The strategy executor opens a buy and sell limit order simultaneously, with a spread between the ask and bid price. The idea behind this is to provide liquidity to the market by both buying and selling the same asset in exchange for a fee in the form of the price difference. As prices fluctuate, these orders are only kept open for a short period as HFT infers. On the other end of the spectrum, there are algorithms and portfolio strategies that determine portfolio constellations that are then held for longer periods, for example, days or months. In contrast to HFT, positions are generally not closed at the end of a trading day. This pool of strategies is referred to as Quant Portfolio Management by Gomber et al. (2011). Since the holding time for the assets increases, human intervention becomes possible, which is a key factor in detecting deviations from the model's predictions. However, math is still a dominant factor when determining the asset distribution.

One, if not the most prominent example, of a hedge fund that is leveraging mathematical models in combination with algorithmic trading, is the Medallion Fund by James Simons that is managed by his company, Renaissance Capital. Gergaud and Ziemba (2012) portray James Simons as a pioneer for algorithmic trading and praise him for his resilience in facing his competition. Cornell (2020) reports a 63.3% compound return for the fund from 1988 until 2018, massively outperforming the market. As mentioned, the base of the advanced strategies employed by hedge funds around the world are mathematical models stemming, for example, from portfolio theory. As they achieve such outstanding results with them, it is worth examining the different strategies that can be used.

In this thesis, I focus on the Quant Portfolio Management division of algorithmic trading and the underlying portfolio theory. One important contribution to this field was made by Harry Markowitz (1952) in 1952. He came up with a new way of portfolio estimation that disrupted the industry and led to many studies across the literature. Harry Markowitz (1952) proposed that investors should split their assets among stocks based solely on the consideration of their historical mean excess returns and variances. The model, known as the mean-variance portfolio, constructs a portfolio that maximizes the investor's expected return by considering the trade-off between risk (variance) and return (mean) relative to the investor's level of risk aversion. Soon, it was discovered that the Markowitz portfolio is prone to estimation error. This issue leads to a suboptimal distribution of assets in the portfolio. The symptoms of this are decreased out-of-sample performance because weights cannot be accurately estimated by the model. Early literature that discusses this problem has been provided by Michaud (1989) and Best and Grauer (1991). Since then, many portfolio strategies have emerged that take different measures to combat this issue or use different approaches to estimate the weights of a portfolio. A prominent example that is discussed by Demiguel et al. (2009) is the Bayesian diffuse-prior portfolio outlined by Christopher B. Barry (1974). Another approach is the Black-Litterman model presented by Black and Litterman (1992), which incorporates investor perspectives with market equilibrium, resulting in a more stable and optimized portfolio. An example of an idea that optimizes the model proposed by Harry Markowitz (1952) can be found in the paper by Jagannathan and Ma (2003), which advocates the use of constraints to achieve this optimization.

The goal of my thesis is to explore the performance of the minimum- and mean-variance portfolios in relation to the 1/N portfolio. Specifically, I examine the impact of varying the frequency of estimation data and rebalancing intervals on the performance of the models. My research addresses whether the frequency and granularity of data influence the outcomes of these strategies. To do this, I follow the approach of Demiguel et al. (2009) and primarily evaluate the out-of-sample performance of the aforementioned portfolios in comparison to the 1/N portfolio. The 1/N portfolio is composed by following the naive rule of distributing wealth equally across all available assets in each iteration. According to Demiguel et al. (2009), there are two reasons to choose this as the benchmark. First, the strategy is easy to implement, and furthermore, the results are stable as there is no estimation involved. Second, even though many advanced concepts in portfolio theory have emerged over the years, Benartzi and Thaler (2001) found that investors still tend to invest their funds according to the 1/N rule. On top of that, more recent literature by Bessler et al. (2021) still advocates the use of the 1/N portfolio for this purpose, indicating its prevalence as a useful benchmark. To extend the comparison, I also include dedicated strategies for daily rebalancing as such are not covered by Demiguel et al. (2009). These are the simple momentum strategy by Jegadeesh and Titman (1993) and Bollinger Bands as implemented by Williams (2006).

I compare the performance of seven different portfolio strategies across six samples[1]. As mentioned, the 1/N portfolio serves as a reference point for performance. Therefore, I always compare all metrics in relation to the benchmark. The three main metrics that I use are the out-of-sample Sharpe ratio, the Certainty Equivalent (CEQ), and the portfolio turnover, as proposed by Demiguel et al. (2009). Additionally, I look at the total return and the percentage of winning trades for each strategy. The idea for the latter is taken from Wiecki et al. (2016). My intention behind this is to investigate the real-world performance of the models. The strategies are listed in Table 1 and explained in Section 2. The samples are listed in Table 2 and their composition and preparation is discussed in Section 4.

My first contribution to the literature is proving that the findings of Demiguel et al. (2009) hold for my samples that consist of equities from Norway and Belgium. This means that none of the strategies mentioned in Table 1 can consistently outperform the benchmark. My analysis adds to the literature by showing that the empirical results of Demiguel et al. (2009) also hold for larger samples, as I use

[1]I construct a total of three samples that can be distinguished by contained equities, but since I use daily and monthly data for the analysis, each sample exists twice, leaving me with a total of six sets.

single equities rather than portfolios as chosen by Demiguel et al. (2009). This is significant because it demonstrates the robustness and applicability of their findings across different markets. I observe that the unconstrained optimization models produce even more extreme weights than outlined by Demiguel et al. (2009), yet they achieve a similar performance in terms of the Sharpe ratio. For example, weights for the mean-variance portfolio reach up to multiple million percent in my analysis. The short-sale constrained minimum-variance portfolio consistently performs well, often delivering the best results in my analysis for samples with monthly data and monthly rebalancing. However, its performance still falls short of the benchmark.

For my second contribution, I show that using daily instead of monthly data for estimation (while still rebalancing monthly) drastically improves the stability of the minimum- and mean-variance portfolios. Making the theoretical approach of Harry Markowitz (1952) more robust is a challenging task that has been extensively taken on by many researchers. The common goal is to derive a strategy that outperforms the 1/N benchmark, yields stable results, and has minimal estimation error. Tu and Zhou (2011), for example, proposed that the 1/N rule can be combined with sophisticated models that emerged from portfolio theory throughout the years to beat the benchmark itself. While they found that this is possible and leads to results superior to the benchmark, it requires multiple steps and strategies and therefore is more of a new creation rather than a small improvement. Another attempt at lowering the estimation error of the Markowitz portfolio is provided by Alla Petukhina et al. (2024). They suggest that the fluctuations in weights are caused by the heavy-tail characteristics of financial time series. Hence, their approach is to deviate from the classical and simple optimization problem. They propose to totally avoid the estimation of the covariance operator. In turn, they advocate a new toolbox for the calculation that uses modern statistics to come up with an efficient estimation. Again, this is neither a simple nor a trivial improvement of the theory provided by Harry Markowitz (1952). An idea that is in theory easy to implement is provided by Demiguel et al. (2009). They found that increasing the length of the estimation window to 3000 months on a 25-asset sample is an effective measure against estimation error. Following this idea, I increase the estimation window length to 1750 iterations by using daily instead of monthly data. This, of course, keeps the time span that is covered by the estimation constant and therefore is not exactly the same approach as advocated by Demiguel et al. (2009). However, this in turn makes the idea feasible in comparison to what Demiguel et al. (2009) propose. 3000 months are roughly 250 years and thus a time frame for which there will never be enough available data. 1750 trading days equal seven years for which it is in fact possible to obtain the relevant data. My analysis provides evidence that this measure increases stability and performance of the models, which is the common target of the studies by Tu and Zhou (2011), Alla Petukhina et al. (2024), and Demiguel et al. (2009).

My third and last contribution to the literature is that the classical portfolio strategies highlighted by Demiguel et al. (2009) cannot compete with strategies specifically designed for short estimation periods and daily rebalancing. This has not been researched much in prior literature, as it is rather counter intuitive to test the models on daily data. Rightfully so, I find that they do not work as well as they do for monthly data. However, I discuss this in a short section at the end of my thesis for two reasons. First, as mentioned at the beginning of the introduction, big hedge funds like Renaissance Capital use mathematical models as a foundation for their trading algorithms. While these strategies can involve longer holding periods, they are often focused on short-term trades, as discussed by Gergaud and Ziemba (2012). This inevitably leads to the question of how extensively the portfolio theory of Harry Markowitz (1952) can be used for short-term estimations without modifications. Second, considering the recent growth in algorithmic trading, I aim to broaden the reader's perspective on trading and portfolio theory by presenting an alternative, more present view that has been dominating the industry in recent years. This broader perspective is particularly relevant given the increasing market share of algorithmic trading as displayed in Figure 1.

The rest of this paper is structured as follows: Sections 2 and 3 describe the different portfolio strategies and the performance metrics that are used for the evaluation. Section 4 briefly describe the process of the data collection and how it is cleaned. After this, I present my findings in Sections 5 and 6 and discuss the limitations of my study and the corresponding results in Section 7. Finally, I end the thesis with a conclusion in Section 8.

2 Description of Considered Strategies

In the following pages, I briefly present the seven different strategies that are compared in this analysis. Note, that throughout this thesis I use the words "strategy" and "model" as synonyms. A comprehensive list of the strategies can be found in Table 1.

As a baseline, I use the 1/N strategy as suggested by Demiguel et al. (2009). In relation to this benchmark, the performance of the minimum-variance, as well as the performance of the mean-variance portfolio are discussed. For these two, I initially adopt the unconstrained models presented by Demiguel et al. (2009). Then, I impose an additional short sale constraint for both to potentially increase performance and stability of returns. To extend the thesis beyond the research of Demiguel et al. (2009), I also incorporate the simple momentum strategy described by Jegadeesh and Titman (1993) and the Bollinger Bands model as suggested by Williams (2006). I choose the latter two for the following two reasons. Since I extend the analysis to daily data and short estimation periods, they provide a useful reference point because both models were originally evaluated for short periods. However, even in this context, it is important to differentiate based on the length of the estimation period. Bollinger Bands work best for timeframes of up to 200 days according to Williams (2006), but are mostly used for periods with a length of around 40 days. Even though the momentum strategy by Jegadeesh and Titman (1993) was also tested on short estimation periods, it uses an estimation period between three months and one year and therefore is more on the upper end of the mentioned interval. This provides a practical context for the performance evaluation of the mean- and min-variance portfolio in Section 6 where I increase the data and rebalancing frequency. Moreover, while the strategies of Demiguel et al. (2009) all

Table 1: Portfolio strategies

#	Strategy	Abbreviation	Citation
Naive			
1.	Equaly weighted (benchmark)	1/N	Demiguel et al. (2009)
Variance			
2.	Minimum-variance	min	Demiguel et al. (2009)
3.	Mean-variance	mv	Demiguel et al. (2009)
Constrained			
4.	Constrained minimum-variance	c-min	Demiguel et al. (2009)
5.	Constrained mean-variance	c-mv	Demiguel et al. (2009)
Momentum			
6.	Momentum	mom	Jegadeesh and Titman (1993)
7.	Bollinger Bands	bol	Williams (2006)

This table lists the various portfolio strategies that are analyzed. They are categorized into naive, variance-based, constrained optimization, and momentum strategies. Each strategy is identified by a model name, an abbreviation, and the corresponding citation for further reference.

focus on balancing variance, both additional strategies follow different approaches, allowing for a comparison of two different aspects of investment strategies.

The following variables are defined uniformly across all equations according to the definitions used by Demiguel et al. (2009). Here, N represents the number of stocks that are available. w_t denotes the N-vector of stock weights in iteration t, with positive (negative) weights indicating a long (short) position. Zero weights indicate that there is no open position for this asset. In this analysis, the weights of a portfolio always sum up to one to ensure comparability. The estimation period length of a strategy is denoted by WS (window size). The WS is always measured in iterations.[2] Furthermore, the vector μ_t contains the expected excess returns and Σ_t denotes the associated $N \times N$ covariance matrix of returns, respectively. Finally, 1_N is the vector of length N with all entries equal to one.

[2]One needs to be careful when distinguishing WS in this thesis, as I analyze both daily and monthly observations. Therefore, the meaning of the term ”iteration” changes throughout the thesis. This will, of course, be clarified as needed when the definition changes.

2.1 Naive portfolio

The naive (also "1/N") portfolio is the first and most important strategy that I include. As proposed by Demiguel et al. (2009), the naive portfolio will be used as a baseline for strategy performances. The strategy is called naive because it is not based on any estimations or complex rules, and is therefore the simplest form of investing in the strategy pool. In each period, funds are equally distributed across all available stocks, deliberately ignoring past returns and other factors. Given this, the 1/N portfolio's performance reflects the average movement of all equities in the sample. Please note that even though no historic data is needed for the construction, I still require a stock to be valid throughout the estimation period of the other strategies to be considered for the 1/N portfolio. This ensures comparability by avoiding the bias that would arise from the returns of stocks that have not been listed long enough to be considered for other strategies.

2.2 Minimum-variance

The minimum-variance strategy (min) assumes that the one and only objective of an investor should be to minimize the variance of returns and therefore risk. Hence, the objective function follows as:

$$\begin{aligned} \min_{w_t} \quad & w_t^T \Sigma_t w_t \\ \text{s.t.} \quad & \mathbf{1}_N^T w = 1 \end{aligned} \tag{1}$$

The only constraint to this and the following mean-variance portfolio is that all weights need to sum up to one. This enforces that there is no cash position left and all assets of an investor are invested. This optimization problem can then be solved by using any quadratic problem solver. I use Gurobi Optimization (2024) because of its performance.

2.3 Mean-variance

The mean-variance portfolio (mv) strongly relates to the minimum-variance approach. It was first discussed by Harry Markowitz (1952), but for this paper, I refer to Demiguel et al. (2009) for the formulas. It expands the optimization problem by additionally considering expected returns. Now, an investor no longer only considers risk, but also wants to maximize their returns. This leads to a trade-off that is described by Equation (2).

$$\begin{aligned} \max_{w} \quad & \mu^T w_t - \frac{\gamma}{2} w_t^T \Sigma w_t \\ \text{s.t.} \quad & \mathbf{1}_N^T w = 1 \end{aligned} \tag{2}$$

A new variable γ is introduced here that denotes the risk aversion of a potential investor. If γ is low, returns and therefore risk are favored. Conversely, if γ increases, the penalty induced by variance increases, meaning investors value stable returns over higher but uncertain returns. I set $\gamma = 1$ for the evaluation part.

2.4 Constrained minimum-variance

Similar to Demiguel et al. (2009), I try to improve the minimum- and mean-variance portfolios by adding a short sale constraint. The intention behind this is to enhance the performance of the estimation and especially to combat the issue of extreme weights that arise from the unconstrained model.[3] This is backed by Jagannathan and Ma (2003), who show that restraining the model to only allow long positions leads to reduced overall risk. The optimization problem for the constrained minimum-variance strategy (c-min) is therefore given by Equation (1) and the following constraint:

$$w_i \geq 0 \quad \forall i \in 1, ..n \tag{3}$$

As advocated by Demiguel et al. (2009), this constraint disallows short positions as mentioned above by ensuring that every weight w_i is positive. This also indirectly prohibits leverage for the portfolio.

2.5 Constrained mean-variance

The constrained mean-variance portfolio (c-mv) is the last optimization model that is evaluated in this paper. It adopts the previously mentioned short sale constraint and therefore is the counterpart to the c-min portfolio for the mean-variance portfolio. Formally, Equation (2) is optimized with constraint (3).

2.6 Momentum

To extend the analysis beyond the strategies discussed by Demiguel et al. (2009), I also incorporate the simple momentum strategy (mom) proposed by Jegadeesh and Titman (1993). The Idea is to buy past "winners" and sell past "losers". A "winner" ("loser") is characterized by high (low) expected returns. The research of Jegadeesh and Titman (1993) suggests that following these trends is a viable approach for the construction of a portfolio. Therefore, the following rule is applied: In each iteration t, all equities are ranked according to their past performance. Based on this, the next portfolio is constructed by selling the bottom ten and buying the top ten percent of

[3] The unconstrained models tend to lead to unrealistically high leverages from time to time. This results in unreasonable gains and losses. This needs to be accounted for.

the created ranking. Note that this strategy is normally used with an estimation period of three to twelve months. It will therefore serve as a benchmark in the last part of the analysis. Nevertheless, I also test it against the strategies proposed by Demiguel et al. (2009) for five- and seven-year estimation periods.

2.7 Bollinger Bands

Originally introduced by John Bollinger, Bollinger Bands are a different way of approaching momentum. In this thesis, I follow the implementation of Williams (2006). Contrary to the momentum strategy presented by Jegadeesh and Titman (1993), the Bollinger Bands model assumes price reversion to the Simple Moving Average (SMA) of a stock, which is calculated with Equation (4). It is defined as the sum of the close prices in the estimation window divided by its length. In this analysis, I will use the return index instead of stock prices to also include dividends and other cash flows.

$$SMA = \frac{\sum_{j=t-WS}^{t} Close_i}{WS} \tag{4}$$

Next, an upper and a lower band are constructed using Equation (5). The upper band is defined as the SMA plus a constant D times the standard deviation of past returns of a stock. Recall that WS represents the length of the estimation period. For this paper, I set D to two, since this is the most commonly used factor and also the one chosen by Williams (2006).

$$Upper/LowerBand = SMA \pm \left(D \times \sqrt{\frac{\sum_{j=t-WS}^{t} Close_j - SMA}{WS}} \right) \tag{5}$$

Finally, buy signals, and sell signals, respectively, are generated with the following rule:

$$\begin{aligned} \text{Buy if:} \quad & P_i < BB_i^{low} \\ \text{Sell if:} \quad & P_i > BB_i^{up} \end{aligned} \tag{6}$$

In the above, P represents the current price and BB the respective Bollinger Band. A signal is generated whenever a stock's price moves outside the Bands, either falling below the lower band or rising above the upper band. If prices fall drastically, the model suggests that they will rise up to their SMA again. Therefore, a long position is opened. Vice versa, if prices rise disproportionately, a short position is taken. Positions are then held until a close signal is generated. As Williams (2006) does not specify any rules for taking profits, I propose closing a position whenever the price re-crosses the SMA. Since this is the only strategy in this thesis that keeps positions open for more than one iteration, a rebalancing rule is introduced. First, at the start of the next period, all new signals are calculated. From there, weights are rebalanced so they have equal absolute values, similar to the 1/N portfolio but with short positions.

3 Methodology and Performance Metrics

The goal of this thesis is to first recreate the results of Demiguel et al. (2009) for the given samples. Furthermore, I extend the research and evaluate whether an increase in data frequency from monthly to daily data leads to a noticeable difference in performance. For this, I examine the following three combinations: first, the return from monthly rebalancing using monthly data for estimation; second, the return from monthly rebalancing using daily data for estimation; and third, the return from daily rebalancing using daily data.

3.1 Methodology

The evaluation process is structured as follows: For each of the models described in Table 1, a portfolio is created and updated with the respective frequency for the respective sample from Table 2. All strategies except the 1/N portfolio use historical data to estimate stock weights. For this, one can now choose between a ”rolling” and an ”expanding” estimation window. An ”expanding” window would grow over time, since it includes all observations starting from a fixed date. Since Demiguel et al. (2009) opted for the ”rolling window”, I also adopt this technique. This means that in each iteration t the data from the period $t - WS$ to t, where WS denotes the length of the estimation period measured in iterations, is used to calculate future weightings. To also capture the effect of window size on performance, I evaluate two different lengths. For the monthly rebalancing part of this thesis, I consider $WS = 60$ (5 years) and $WS = 84$ (7 years). Contrary to Demiguel et al. (2009), I choose not to use $WS = 120$ (10 years), since my Dataframe is only 21 years long. On top of that, for better comparability, the start year for all evaluations will be 2010. To ensure fair and equal starting points, the start of the five-year window evaluation needs to be postponed until there is enough data for the longer period. For the combination of monthly rebalancing and daily data, I expand WS to 1250 and 1750. These are the equivalents to five and seven years of daily data.[4] Finally, as previously mentioned, I evaluate daily rebalancing with daily data. For this, I choose an estimation period of $WS = 40$ (days) and $WS = 125$ (days) in accordance with the analyses done by Williams (2006) and Jegadeesh and Titman (1993) and continue as presented for the monthly rebalancing.

[4] These numbers are calculated by multiplying the number of years by 250, which is the average number of trading days per year in my samples.

3.2 Performance Metrics

To compare the results from above, I implement the three main performance metrics used by Demiguel et al. (2009). These are the Sharpe ratio (SR), the turnover (TO), and the Certainty Equivalent (CEQ). To calculate them, I use the series of out-of-sample excess returns and weights that are calculated as described above.

The SR, which was first introduced by William F. Sharpe (1966), is calculated using Equation (7), where k denotes the strategy, $\hat{\mu}_k$ the excess return, and $\hat{\sigma}_k$ the standard deviation of $\hat{\mu}_k$.

$$\widehat{SR}_k = \frac{\hat{\mu}_k}{\hat{\sigma}_k} \tag{7}$$

The Sharpe ratio is one of the most popular measures for portfolio performance. It expresses the relation between additional excess return and volatility. A higher Sharpe ratio indicates better performance. To test whether a result is statistically different from the benchmark, I employ the test statistic proposed by J. D. Jobson and Bob M. Korkie (1981) as implemented by Demiguel et al. (2009). Given two portfolios i and n with their respective means, variances, and covariance of out-of-sample excess returns as declared above, the null hypothesis is defined as:

$$H_0 : \frac{\hat{\mu}_i}{\hat{\sigma}_i} - \frac{\hat{\mu}_n}{\hat{\sigma}_n} = 0 \tag{8}$$

The null hypothesis assumes that the Sharpe ratios of two portfolios are not different. Furthermore, the test statistic $\hat{z}_{JK}$, which is asymptotically distributed as a standard normal, is calculated as:

$$\hat{z}_{JK} = \frac{\hat{\sigma}_n\hat{\mu}_i - \hat{\sigma}_i\hat{\mu}_n}{\sqrt{\hat{\vartheta}}} \tag{9}$$

$$\hat{\vartheta} = \frac{1}{T-M}\left(2\hat{\sigma}_i^2\hat{\sigma}_n^2 - 2\hat{\sigma}_i\hat{\sigma}_n\hat{\sigma}_{i,n} + \frac{1}{2}\hat{\mu}_i^2\hat{\sigma}_n^2 + \frac{1}{2}\hat{\mu}_n^2\hat{\sigma}_i^2 - \frac{\hat{\mu}_i\hat{\mu}_n}{\hat{\sigma}_i\hat{\sigma}_n}\hat{\sigma}_{i,n}^2\right) \tag{10}$$

From this, I then calculate the p-values and compare them to a significance level of $\alpha = 0.2$. A p-value smaller than α means that the null hypothesis is rejected and therefore that the Sharpe ratios of the two portfolios are statistically different. As pointed out by Demiguel et al. (2009), this test is only valid if the returns are independently and identically distributed. This most often does not hold for data. With this in mind, the test results need to be considered with care and only serve as an indicator.

Next, the portfolio turnover between two periods is measured. It describes the absolute value of the assets that are involved in trading. This is important, especially when comparing monthly and daily rebalancing, since each transaction introduces additional monetary cost. Even though Gerig (2012) suggests that high-frequency trading is steadily reducing these in comparison to historical cost, the impact of transaction fees and spread will remain relevant if turnover is high. Since transaction costs are not included in the model's expected and realized returns and therefore not captured by the SR, it is essential to additionally compare turnovers. To make the formula easier to read and understand, I deviate from the one used by Demiguel et al. (2009) and use the representation of Hanauer and Windmüller (2023) instead. This, of course, does not change the outcome and is solely a cosmetic choice.

$$Turnover_t = 0.5 \times \sum_{i}^{N} |w_{i,t} - \tilde{w}_{i,t-1}| \tag{11}$$

Here, $w_{i,t}$ represents the newly calculated weight of asset i in iteration t and $\tilde{w}_{i,t-1}$ the return-adjusted weight of the respective asset before rebalancing. To be more specific, $\tilde{w}_{i,t-1}$ is the momentary weight of a stock position with initial weight $w_{i,t-1}$ at the end of $t-1$ after gaining or losing value during the period. It is important to realize that the formula implicitly assumes that the sum of weights is equal to one. Contrary to the strategies by Demiguel et al. (2009), the momentum strategy and the Bollinger Bands model do not inherently enforce this. As a solution, Hanauer and Windmüller (2023) propose splitting the portfolio into long and short positions. This, however, leads to separate turnovers for each, which are then not comparable to the turnovers of Demiguel et al. (2009). To solve this issue, I propose that a dummy stock, which represents the leftover cash, is inserted into the portfolio instead. This dummy stock will always have a weight so the total positions sum up to one. The weight is calculated with Equation (12).

$$w_{cash,t} = 1 - \sum_{i}^{N} w_{i,t} \tag{12}$$

By the construction of the models, this weight is never negative, since the maximum absolute value of the short positions for both momentum portfolios is always ≤ 1. For simplicity, I assume that this cash position does not generate or lose value during the period. This can then easily be plugged into Equation (11).

Last but not least, the Certainty Equivalent is calculated as follows:

$$\widehat{CEQ}_k = \hat{\mu}_k - \frac{\gamma}{2}\hat{\sigma}_k^2 \tag{13}$$

Here, again, $\hat{\mu}_k$ and $\hat{\sigma}_k^2$ denote the mean of excess returns and their respective variance. Furthermore, γ represents the investor's risk aversion level as previously defined for the mean-variance portfolio. Hence, it is set to one, respectively. Thus, the CEQ is interpreted as the certain, risk-free return an investor would choose over the uncertain, possibly higher or lower return of a strategy.

In addition to these three main metrics, I also examine the total return and the rate of winning trades for all strategies. With the help of these, I explore whether the models can be applied to a real-world portfolio or not. I define the total return as the percentage gain in value over the total evaluation period, and the rate of winning trades as the ratio of the number of positive returns to the number of iterations.

4 Data

The data used for the analysis is taken exclusively from Refinitiv Datastream. I use equities from both Norway and Belgium, with data ranging from January 2003 to December 2023. In total, there are 580 stocks in this dataset, along with their respective return index[5], market values,[6] and turnovers by volume for the time series. I choose market value and turnover by volume over market capitalization and turnover by value because both the data quality and frequency of those are better than for their counterparts on Refinitiv Datastream. For turnover, the use of volume rather than value does not make a difference, since I only care if there was turnover at all. From this, I then construct three samples, with the main sample containing all 562 stocks. Additionally, to increase the number of observations, I evaluate the data from each country separately. This leads to a sample for Norway with a total of 347 equities and one for Belgium with 215 equities. Since the goal is to examine the effect of increased data frequency on performance, monthly and daily data are used. Therefore, the three samples are split accordingly, resulting in a final six sets. A concise list of these can be found in Table 2. Finally, I add risk-free rates in the form of 3-month government bond yields to each sample.[7]

Datastream has several known issues relating to data quality that were studied in prior literature. Hence, to enhance sample quality, both the static and dynamic

[5] The return index on Datastream is calculated with the following formula, where RI is the return index, PI the price index, and DY the dividend yield in %:

$$RI_t = RI_{t-1} \times \frac{PI_t}{PI_{t-1}} \times (1 + \frac{DY_t}{100} \times \frac{1}{N})$$

[6] Note that, due to limited availability, the monthly market value is stretched and also used for the daily evaluation. Even though this reduces data quality for the daily sample, it can be justified for two reasons. First, market value will only be used to eliminate small companies from the evaluation and therefore has no significant direct impact. Second, huge fluctuations in market value are generally not to be expected.

[7] For the combined sample, I assume that an investor seeks to diversify as optimally as possible. Hence, I use the average of the Norwegian and Belgian risk-free rates as a combined risk-free rate. This implies a 50/50 split of funds.

Table 2: Samples

#	Name	Abbreviation	Frequency	Size
Combined data from Norway and Belgium				
1.	Combined Monthly	CO-M	monthly	253, 562
2.	Combined Daily	CO-D	daily	5379, 562
Norway only				
3.	Norway Monthly	NO-M	monthly	253, 347
4.	Norway Daily	NO-D	daily	5276, 347
Belgium only				
5.	Belgium Monthly	BE-M	monthly	253, 215
6.	Belgium Daily	BE-D	daily	5379, 215

This table lists the different samples that are used for the analysis. The table includes their name, abbreviation, frequency, and size. The size column displays the dimensions of the data matrix, with the first value representing the number of observations and the second indicating the number of equities in the sample.

screens suggested by Hanauer and Windmüller (2023) are applied. A table with descriptions for the static screens can be found in Table A1.[8] The following paragraph will briefly present all dynamic screens selected by Hanauer and Windmüller (2023). Furthermore, I discuss additions that I find necessary for my particular dataset to enhance data and result quality. Since the sample size in this thesis is smaller compared to that in Hanauer and Windmüller (2023),[9] I cannot be as lenient and must apply stricter criteria to mitigate data errors. A summary of all applied dynamic screens can be found in Table 3.

Hanauer and Windmüller (2023) suggest the use of several dynamic screens, which they collected from Ince and Porter (2006), Schmidt et al. (2011), and Griffin et al. (2010). First, Ince and Porter (2006) found that due to the way Datastream handles data, zero returns are continuously posted after a stock is delisted. These need to be subsequently deleted. Then, following Schmidt et al. (2011), all return spikes with returns greater than 990% in the monthly data need to be removed. Similarly, Ince and Porter (2006) propose that returns with strong return reversals have to be deleted. The exact formula for this can be found in Table 3. For the daily samples, the same methods as for the monthly data are applied with adjusted parameters. These can also be found in Table 3.

As mentioned, I impose further constraints since my samples are significantly smaller than the ones used by Hanauer and Windmüller (2023). First, I require valid nonzero trading volumes for a stock for at least 95% of the observations used for the estimation. This prevents stocks from being selected for the portfolio if they are

[8]The static screens were already applied to the dataset when I received it from the chair. As there are no additions to the static screens described by Hanauer and Windmüller (2023) from my side, I deem it sufficient to point to their paper for a detailed explanation.

[9]Hanauer and Windmüller (2023) use 66,905 equities whereas I only use 580.

Table 3: Dynamic screens

#	Description	Citation
General		
1.	Delete all zero returns after a company was delisted. This needs to be done, since Datastream continues to provide the last valid data point for all following iterations.	Ince and Porter (2006)
2.	In each iteration, exclude all stocks with a market value in the lower 25th percentile.	Self-imposed
3.	In each iteration, exclude all stocks that have not been traded in $> 95\%$ of month (days) in the estimation period. Stocks are considered not traded if the respective entry in the turnover data is zero.	Self-imposed
Monthly data only		
4m.	Delete all data points associated with monthly returns $> 990\%$.	Schmidt et al. (2011)
5m.	Delete all data points associated with monthly returns that fulfill the equation: RI_{t-1} or $RI_t > 300\%$ and $(1+RI_{t-1})(1+RI_t)-1 < 50\%$	Ince and Porter (2006)
Daily data only		
4d.	Delete all data points associated with daily returns $> 200\%$	Griffin et al. (2010)
5d.	Delete all data points associated with daily returns that fulfill the equation: RI_{t-1} or $RI_t > 100\%$ and $(1+RI_{t-1})(1+RI_t)-1 < 20\%$	Ince and Porter (2006), Griffin et al. (2010)

This table lists the different dynamic screens that are applied to the data. This table includes and extends the rules composed by Hanauer and Windmüller (2023) and refers to their original sources. Note that all rules annotated with an m (d) are solely applied to monthly (daily) data.

not frequently traded, because selecting those would lead to questionable results. The reason for that is simple. When looking at the return index data, it is easy to observe that equities with low trading volume often register price jumps if and only if they are traded. This implies that the price moved because of the trade and therefore could not be realized as a return in a real-world scenario. Hence, such trades are prohibited to prevent unrealistic returns that bias the results. Second, in each iteration t, stocks are sorted by their market value. The lower 25th percentile is then excluded from the selection process. Companies with low market value tend to fall into the same category as mentioned above. Therefore, this constraint reassures the effectiveness of the first.

5 Evaluation Against DeMiguel

In this section, I present the evaluation of the strategies listed in Table 1 for monthly data and monthly rebalancing on the samples listed in Table 2. I first compare them to the results of the 1/N strategy, which serves as a benchmark. Then, I relate my findings to the results of Demiguel et al. (2009) and briefly discuss their similarity.

5.1 Presentation of the Findings for Monthly Rebalancing

I compute all ratios in this section for estimation windows of 60 and 84 iterations. Here, one iteration is equal to one month. To enhance the readability of the following paragraphs, all mentioned values will be followed by a tuple in brackets consisting of window size and sample name.[10]

5.1.1 Sharpe ratio

The first performance indicator for this analysis is the Sharpe ratio. The results for this are displayed in Table 4. Each cell shows the specific Sharpe ratio, with the p-value provided in parentheses.

Table 4: Sharpe ratios for monthly data & monthly rebalancing

	$WS = 60$			$WS = 84$		
Strategy	CO-M	NO-M	BE-M	CO-M	NO-M	BE-M
1/N	0.1886	0.1710	0.1850	0.2055	0.1861	0.1997
min	0.4266	0.3398	0.2238	0.4234	0.0722	0.0907
	(0.00)	(0.05)	(0.68)	(0.01)	(0.27)	(0.29)
mv	0.3520	0.1205	0.0358	0.1832	-0.1058	0.0714
	(0.17)	(0.68)	(0.20)	(0.85)	(0.01)	(0.26)
c-min	0.4491	0.3913	0.1957	0.3718	0.4366	0.2232
	(0.00)	(0.00)	(0.87)	(0.01)	(0.00)	(0.74)
c-mv	0.0463	0.0507	0.1661	0.0221	0.0457	0.1058
	(0.05)	(0.10)	(0.82)	(0.01)	(0.05)	(0.22)
mom	0.2941	0.0796	0.1948	0.2607	0.1217	0.2041
	(0.44)	(0.49)	(0.94)	(0.67)	(0.61)	(0.97)
bol	-0.3938	-0.3151	-0.2444	-0.3746	-0.3753	-0.2204
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

This table reports Sharpe ratios for the strategies from Table 1 for the monthly samples from Table 2 for monthly rebalancing. The evaluation period ranges from January 2010 to December 2023. The first three columns report results for a 60-iteration (month) window. The last three columns report results for an 84-iteration (month) window. In parentheses, the p-value relative to the 1/N portfolio is reported.

To start, I examine the first row, which contains the benchmark portfolio. My first observation is that the naive strategy leads to the most stable result across all six columns, as there is no error-prone estimation involved. The average Sharpe ratio of all samples is 0.1893. Interestingly, I find that increasing the estimation period also results in an increased Sharpe ratio. More precisely, the Sharpe ratio for the CO-M sample jumped from 0.1886 (60, CO-M) to 0.2055 (84, CO-M), which is an increase of 9.0%. Similar results can be observed for the other two. This is

[10]Example: (60, CO-M) refers to a value from the analysis of the CO-M sample where the estimation period was set to 60.

due to the dynamic screens I apply. Recall that for the 1/N strategy, companies need to have valid returns for the entire estimation period to be considered. Hence, by increasing the window size, younger and potentially more unstable companies are automatically filtered out.

Demiguel et al. (2009) provides strong evidence for estimation error in their paper by comparing in-sample and out-of-sample performance of the optimization models. My analysis adds to this evidence by comparing the Sharpe ratios of the min and mv portfolio across estimation periods. Notice that there are significant fluctuations not only between the samples, but also between the estimation period sizes. While the minimum-variance portfolio is able to post a maximum Sharpe ratio of 0.4266 (60, CO-M), it also has results as weak as 0.0722 (84, NO-M). Note that the superior result is statistically different with a p-value of 0.00, whereas the weak result is not with a p-value of 0.27. I reason that inaccurate estimation of stock weights is the direct cause of these unstable results. The additional constraints that are added to the min- and mean-variance prove to be an effective countermeasure for dealing with those fluctuations. Especially, the c-min portfolio stabilizes at Sharpe ratios that are above the SR of the 1/N portfolio and also mostly statistically different. This shows prove for the findings of Demiguel et al. (2009) and Jagannathan and Ma (2003) that the short sale constraint indeed reduces risk and improves results.

The third observation I made is that the momentum strategy proposed by Jegadeesh and Titman (1993) works well on the dataset, although it experiences a decline in Sharpe ratio for all three samples when the estimation period is increased. This result is interesting because the strategy was not designed to work on such long estimation periods. Jegadeesh and Titman (1993) only tested it for three- to twelve-month estimation windows. Regardless, none of the results are statistically different from the benchmark, meaning that the performance is not superior to the 1/N portfolio. As mentioned, this has to be taken with a grain of salt, as Jegadeesh and Titman (1993) advocate shorter estimation windows. A more meaningful result will thus be provided in Section 6 where I consider daily data and daily rebalancing.

Finally, I want to point out that the Bollinger Bands do not work as an indicator on any of the monthly datasets. This can be concluded since a negative Sharpe ratio is observed for all samples, with the worst being -0.3938 (60, CO-M), which indicates the total failure of the model. All Sharpe ratios for this strategy are also statistically different to the benchmark with a p-value of 0.00 which strengthens the conclusion that this is in fact a worse performance. This result, however, is not surprising, as the literature provided by Williams (2006) suggests the use of daily data in the first place. Because of this, Bollinger Bands will not be the focus of the further analysis until I reach daily data and daily rebalancing.

5.1.2 Certainty Equivalent

The second metric discussed by Demiguel et al. (2009) is the Certainty Equivalent (CEQ). It confirms the results that were already observed for the Sharpe ratio. None

Table 5: Certainty Equivalents for monthly data & monthly rebalancing

	$WS = 60$			$WS = 84$		
Strategy	CO-M	NO-M	BE-M	CO-M	NO-M	BE-M
1/N	0.0064	0.0067	0.0058	0.0068	0.0072	0.0063
min	0.0095	0.0125	0.006	0.0100	0.0023	-0.0230
mv	-8.79E+13	-3.46E+13	-1.3E+13	-5.84E+13	-6.82E+12	-3.72E+12
c-min	0.0081	0.0089	0.0042	0.0074	0.0102	0.0052
c-mv	0.0005	0.0006	0.0083	-0.0017	0.0001	0.0046
mom	0.0160	0.0031	0.0112	0.0135	0.0068	0.0118
bol	-0.0114	-0.0135	-0.0095	-0.0113	-0.0180	-0.0078

This table reports Certainty Equivalents (CEQ) for the strategies from Table 1 for the monthly samples from Table 2 for monthly rebalancing. The evaluation period ranges from January 2010 to December 2023. The first three columns report results for a 60-iteration (month) window. The last three columns report results for an 84-iteration (month) window.

of the strategies mentioned are able to consistently outperform the 1/N portfolio, with the c-min portfolio coming the closest.

First, and most trivial to observe, is that both unconstrained optimization models lead to unstable results for the datasets. The CEQ of the min portfolio fluctuates heavily between 0.0125 (60, NO-M) and -0.0230 (84, BE-M). For the mv strategy, the CEQ is a large negative number, indicating that an investor would never choose this model.

The second observation is that the c-min portfolio is able to beat the benchmark in four out of six cases, only falling behind for the Belgium sample for both estimation periods. For example, the CEQ for the 1/N is 0.0058 (60, BE-M) compared to 0.0042 (60, BE-M) for the c-min strategy. The c-mv model also manages to stabilize and now posts Certainty Equivalents around zero for all combinations. These are both significant improvements compared to the unconstrained strategies.

Also notable, the momentum portfolio by Jegadeesh and Titman (1993) is able to outperform the benchmark by a factor of two for the 60-month estimation period in all samples, but not for the 84-month window. This underlines the prior observation that momentum cannot be preserved forever. I want to remind the reader that this result has to be interpreted with caution since the momentum strategy is usually not used with long estimation periods.

5.1.3 Turnover

Table 6 reports monthly turnovers. This is the third and final metric proposed by Demiguel et al. (2009). Similarly to their analysis, the first line of Table 6 contains the absolute turnovers for the 1/N strategy. All other turnovers are reported as relative values to the benchmark. For each observation, it also reports the absolute turnover of the strategy in parentheses.

Table 6: Turnovers for monthly data & monthly rebalancing

	$WS = 60$			$WS = 84$		
Strategy	CO-M	NO-M	BE-M	CO-M	NO-M	BE-M
1/N	0.0420	0.0506	0.033	0.0406	0.0498	0.0325
min	5.66 (0.2376)	10.50 (0.5315)	13.27 (0.4379)	8.08 (0.3282)	57.72 (2.8747)	308.25 (10.018)
mv	2.1E+10 (8.63E+08)	7.24E+09 (3.66E+08)	9.6E+09 (3.17E+08)	1.73E+10 (7.03E+08)	3.35E+09 (1.67E+08)	4.47E+09 (1.45E+08)
c-min	3.20 (0.1343)	2.46 (0.1245)	3.18 (0.1050)	2.54 (0.1032)	2.29 (0.1142)	2.38 (0.0775)
c-mv	4.47 (0.1877)	3.59 (0.1816)	6.04 (0.1993)	4.12 (0.1672)	3.08 (0.1535)	5.62 (0.1825)
mom	8.32 (0.3496)	8.54 (0.4320)	11.13 (0.3674)	7.98 (0.3240)	8.10 (0.4034)	9.53 (0.3096)
bol	8.40 (0.3526)	7.59 (0.3843)	11.45 (0.3778)	6.84 (0.2776)	6.28 (0.3126)	9.62 (0.3125)

This table reports mean monthly turnovers for the strategies from Table 1 for the monthly samples from Table 2 for monthly rebalancing. The evaluation period ranges from January 2010 to December 2023. The first three columns report results for a 60-iteration (month) window. The last three columns report results for an 84-iteration (month) window. For the 1/N strategy, the absolute value is reported. For all other strategies, the value relative to the 1/N strategy is reported, with the absolute value shown in brackets.

First, and not surprisingly, I observe that the 1/N portfolio has by far the lowest turnovers across all samples, ranging from 0.0325 (84, BE-M) to 0.0506 (60, NO-M). This result is expected by construction since every available stock is always considered in the portfolio with equal weights. Therefore, there are no big changes in the composition and thus no high turnover. Especially, there are never zero weights for an available equity, which leads to a well-balanced distribution of assets.

Comparing the turnovers of the unconstrained portfolios, I find that the mean-variance portfolio results in exorbitantly higher turnovers than the min portfolio. This can be attributed to the characteristics of the optimization problems. Since Equation (1) is a minimization problem that is bounded by zero and Equation (2) is a maximization problem that is not, the mean-variance portfolio sometimes leads to unbounded leverage and therefore unbounded weights. This is evidently displayed in the turnovers. The instability of the unconstrained models is also reflected in these results, as the turnovers vary wildly across the six samples, even though they should converge as the samples contain similar stocks by construction. The minimum-variance model, despite its advantage over the mv strategy, has turnovers that are at least 5.66 (60, CO-M) times higher than those of the benchmark.

Now, the question arises if the constrained versions of these two models mitigate the problem of high turnovers. It turns out that constraining both portfolios lowers turnovers significantly. Most importantly, I observe that the constraints help to stabilize results, as already shown for the Sharpe ratios. While the unconstrained minimum-variance portfolio reported turnovers with a gap of 5346% from lowest to highest, this gap narrows down to a mere 39% for the c-min portfolio. Also, the minimum relative turnover is halved from 5.66 (60, CO-M) to 2.29 (NO-M). Moreover, Table 6 shows that the results for the c-mv portfolio converge to those of the c-min portfolio, although they remain at a slightly higher level. From this, I conclude that the short sale constraint is fulfilling its purpose in this category.

Finally, I observe that the momentum portfolio results in turnovers around eight times higher than those of the 1/N portfolio across all samples. The lowest turnover is recorded for the CO-M sample and the 84-month estimation period, where the mom strategy has a turnover that is 6.84 times higher than the benchmark.

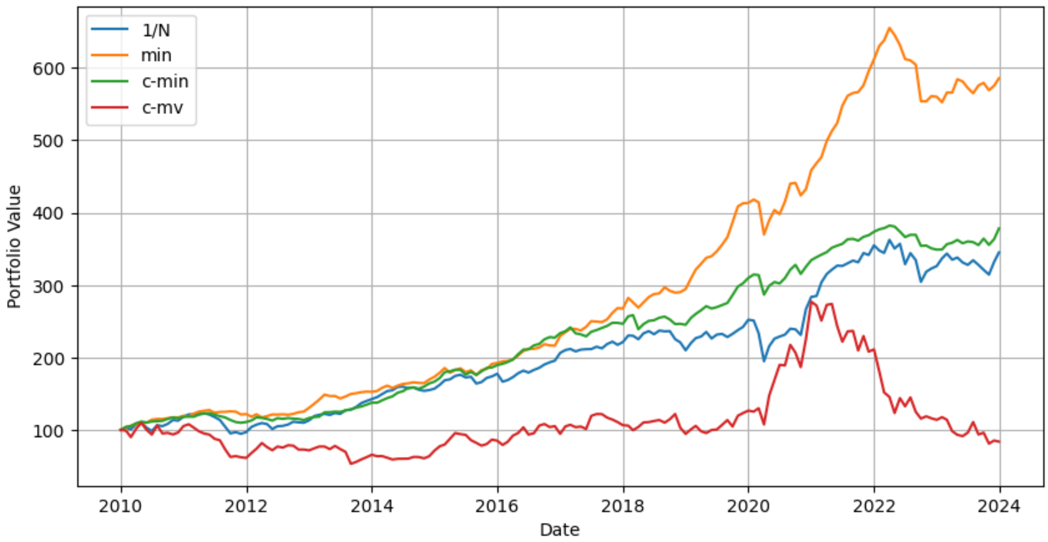
When split by estimation period, one can also observe that a longer estimation period lowers the turnover for all strategies (excluding min and mv) across all observations, without exceptions. Note that besides better estimation because of increased data availability, one cause for this is decreased portfolio size since more companies are filtered out due to the dynamic screens presented in Table 3.

5.1.4 Winning Trades and Total Return

As mentioned, I also calculate and compare the percentage of winning trades and the total returns of the strategies. The motivation for this is to show the real-world performance of the strategies in Table 1. As discussed by Cascon and Shadwick (2006) and Mistry and Shah (2013), there are limitations to the meaningfulness of the Sharpe ratio in terms of hedge fund performance, for example when returns are not normally distributed. While the Sharpe ratio is a good theoretical indicator, it does not capture the realized return and can also be positive by construction, even if the portfolio defaults during the iterations. Therefore, in the following paragraph, I investigate which strategy could be applied to a real-world portfolio using its total return. I also show the correlation between Table 7 and Figure 2 whenever possible.

Table 7 contains the percentage of winning trades for each strategy. Figure 2 exemplarily illustrates the portfolio value over time starting from a base value of 100 for the CO-M sample with an estimation period of 84 iterations. In the following analysis, I refer to these values only because the visualization simplifies the discus-

Figure 2: Stock returns over time



This figure plots returns for the 1/N, min, c-min and c-mv portfolios for the monthly samples from Table 2 for monthly rebalancing. The evaluation period ranges from January 2010 to December 2023. The value of a portfolio is plotted on the y-axis. The timeframe is plotted on the x-axis.

sion of the portfolio's value evolution's. This part should thus not be considered a comprehensive analysis, but rather a concise and complementary mention.

Before continuing, please notice that the unconstrained mean-variance portfolio is missing in this graph, as well as Table 7. The reason for this was already foreshadowed in Tables 5 and 6. The calculated weights (and therefore leverage) for this strategy are sometimes so high that there inevitably are periods where the loss exceeds 100%. This is displayed not only in the return for this period, but also in meaningless CEQs and turnovers. Therefore, no useful portfolio value progression can be graphed out, and I exclude the strategy from further analysis in this section. Also, I decide to not show the performance of the momentum strategy and the Bollinger Bands indicator, as these strategies are not meant for the current evaluation window size.

In terms of total return, the unconstrained min portfolio takes the top spot with a return of 485.68%. This is also displayed in the high winning rate of 70.83%. Note that the winning rate for this strategy is not stable, as it falls to 57.14% for the 84-month NO-M sample. The constrained version shows enhanced stability in terms of winning rate in Table 7, but this leads to a trade-off for returns. The c-min portfolio is only able to post a return of 278.19%, which is only on par with the 245.09% of the 1/N benchmark. Recall that the Sharpe ratio in Table 4 for this strategy is almost twice as high. This can be attributed to the value evolution over time. While moving equally until 2018, one can observe a gap that opens from there on and only closes again at the start of 2022. This shows that the returns of the c-min portfolio are more stable and consistent throughout time compared to the 1/N portfolio. This might be preferred by risk-averse investors. The constrained mean-

Table 7: Winning trades (in %) for monthly data & monthly rebalancing

	$WS = 60$			$WS = 84$		
Strategy	CO-M	NO-M	BE-M	CO-M	NO-M	BE-M
1/N	66.07	63.10	64.29	66.67	63.69	64.29
min	71.43	66.07	59.52	70.83	57.14	57.74
c-min	70.24	73.81	65.48	70.83	74.40	69.64
c-mv	48.21	47.62	56.55	50.60	50.60	54.76

This table reports the percentage of winning trades for the strategies from Table 1 for the monthly samples from Table 2 for monthly rebalancing. It is calculated by dividing the number of trades with positive returns by the number of iterations. The values are displayed as percentages. The evaluation period ranges from January 2010 to December 2023. The first three columns report results for a 60-iteration (month) window. The last three columns report results for an 84-iteration (month) window.

variance portfolio cannot achieve the same results and reports a negative return in the end, even though it peaked at around 180% profit in 2021.

Finally, I want to remind the reader that these returns do not consider transaction costs. In the previous section on turnover, it already became evident that the 1/N strategy has the lowest turnover by far. In a real-world scenario, this would most probably put the 1/N strategy in the top spot. To be certain about that, more calculations would be necessary that do not fit in the scope of this thesis. If properly done, this would add to the evidence of Demiguel et al. (2009) that none of the mentioned strategies is able to beat the 1/N portfolio consistently.

5.2 Discussion of the Results in Relation to DeMiguels Findings

To answer the first research question, I now compare my results to the findings of Demiguel et al. (2009). In total, my analysis provides strong evidence that underlines the findings of Demiguel et al. (2009). This means that their conclusions also hold for the Norwegian and Belgian stock markets and for samples with up to 562 equities. First and most importantly, none of the strategies can consistently outperform the benchmark 1/N portfolio throughout all observations. While some strategies managed to beat the benchmark in one of the three main categories, no strategy achieved better results overall. Furthermore, I come to the conclusion that constraining the minimum- and mean-variance portfolios enhances their performance and stability drastically. I also find that there is a severe estimation error and instability of results involved when using the unconstrained portfolio methods. For my analysis, that results in huge fluctuations across the samples, even though they should yield similar results because they are constructed from the same dataset and contain the same equities. Also in accordance with the results of Demiguel et al. (2009), the minimum-variance and the respective constrained version display the best results across Sharpe ratio, CEQ, and turnover.

6 Increasing Data and Rebalancing Frequencies

In this section, I investigate the impact of increased data and rebalancing frequencies on the strategies from Table 1. The goal is to determine whether more granular data and rebalancing can mitigate the effects of estimation errors. Also, I test whether the Markowitz portfolio is a feasible approach for shorter estimation periods. I do this in two steps. First, I only increase the frequency of the estimation data, and then, in the second step, I also increase the rebalancing frequency. This way, possible over-engineering will become evident.

6.1 Increasing the Frequency of Estimation Data

For step one, I increase the frequency of the estimation data while sticking to monthly rebalancing. The size of the estimation window is therefore increased to 1250 and 1750 trading days. These numbers are calculated by multiplying the number of years by 250, which is the average number of trading days per year in my samples. In summary, this still implies a five- and seven-year estimation period. To keep the evaluations similar, only the frequencies of the daily return index and risk-free rate are adjusted. Other than that, I use monthly data and, therefore, the dynamic screens for monthly data, most importantly for turnovers. I exclude the momentum and the Bollinger Bands strategy from this part, as they are not generally meant for such long estimation periods as mentioned earlier.

6.1.1 Sharpe ratio

Table 8 displays the Sharpe ratios of the strategies for daily estimation data. Since I want to show the potential improvements of increasing the data frequency relative to the previous section, I add the results from Table 4 beneath each observation to facilitate the direct comparison.

First, I need to clarify why the 1/N Sharpe ratios are slightly different, since I initially would have expected them to be the same as in the prior Section. The reason for this is that stocks are not always listed on the first day of a month. This means that in some cases, they are already used on the first of a month in the monthly data approach when, in fact, they were only available at the end of the month. This is now accounted for with daily data, and therefore the results show a small difference.

Recall that the min and mv portfolios resulted in enormous fluctuations for the monthly data in Table 4. I observe that increasing the data frequency helps to mitigate this problem. While there was a 490.86% gap between the lowest and highest SR for the min strategy in Table 4, this gap has now closed to a mere 44.72%. The same holds for the mv portfolio, where the gap narrowed down to 121.91% compared to the initial 432.70%. This indicates more stable predictions, as ini-

Table 8: Sharpe ratios for daily data & monthly rebalancing

	$WS = 1250$			$WS = 1750$		
Strategy	CO-M	NO-M	BE-M	CO-M	NO-M	BE-M
1/N	0.1914	0.1675	0.1793	0.2111	0.1835	0.1916
	(0.1886)	(0.1710)	(0.1850)	(0.2055)	(0.1861)	(0.1997)
min	0.4016	0.3699	0.2775	0.404	0.3995	0.2646
	(0.4266)	(0.3398)	(0.2238)	(0.4234)	(0.0722)	(0.0907)
mv	0.3717	0.2448	0.2556	0.2984	0.1675	0.2388
	(0.3520)	(0.1205)	(0.0358)	(0.1832)	(-0.1058)	(0.0714)
c-min	0.3281	0.3354	0.2435	0.3577	0.3573	0.2597
	(0.4491)	(0.3913)	(0.1957)	(0.3718)	(0.4366)	(0.2232)
c-mv	0.1116	0.0689	0.1822	0.0741	0.0714	0.1873
	(0.0463)	(0.0507)	(0.1661)	(0.0221)	(0.0457)	(0.1058)

This table reports Sharpe ratios for the strategies from Table 1 for the daily samples from Table 2 for monthly rebalancing. The evaluation period ranges from January 2010 to December 2023. The first three columns report results for a 1250-iteration (trading days) window. The last three columns report results for a 1750-iteration (trading days) window.

tially expected for monthly data as well.[11] From this, I conclude that increasing the data frequency can help to reduce estimation error and instability of the optimization models. Surprisingly and fortunately for potential investors, the gap closed, mostly because the overall performance increased while still almost reaching the same peaks as recorded in Table 4. For example, the c-mv improved its worst performance from 0.0221 (84, CO-M) by threefold to now 0.0741 (1750, CO-M). For the min, mv, and c-min portfolios, the increased frequency for the estimation results in a performance that is significantly superior to the benchmark in all observations. The only exception to this is the mean-variance strategy, which only posts a Sharpe ratio of 0.1675 (1750, NO-M) compared to a SR of 0.1835 (1750, NO-M) for the 1/N portfolio.

The results for the CEQ improve similarly, thus I choose not to include a separate analysis for this to prevent repetitiveness.

6.1.2 Turnover

Before I present the results for daily rebalancing in the next section, I want to show how the turnover changed for daily data and monthly rebalancing. This will be an important metric for the evaluation in the next section, since I omit transaction costs in this analysis. Therefore, I try to indirectly involve them through a discussion of turnover. Table 9 displays the turnovers for each strategy, with the turnover for the monthly data in brackets, similar to Table 8.

[11] Once again, recall that by construction, the samples are all very similar and thus should lead to similar results.

Table 9: Turnovers for daily data & monthly rebalancing

	$WS = 1250$			$WS = 1750$		
Strategy	CO-M	NO-M	BE-M	CO-M	NO-M	BE-M
1/N	0.0441	0.0542	0.0341	0.0427	0.0526	0.0338
	(0.0420)	(0.0506)	(0.0330)	(0.0406)	(0.0498)	(0.0325)
min	3.20	2.22	2.62	2.50	1.88	2.16
	(5.66)	(10.50)	(13.27)	(8.08)	(57.72)	(308.25)
mv	5334.63	2281.90	2886.05	3757.28	1718.12	2228.25
	(-)	(-)	(-)	(-)	(-)	(-)
c-min	1.32	1.35	1.47	1.18	1.27	1.30
	(3.20)	(2.46)	(3.18)	(2.54)	(2.29)	(2.38)
c-mv	4.34	2.99	5.59	3.83	2.74	4.67
	(4.47)	(3.59)	(6.04)	(4.12)	(3.08)	(5.62)

This table reports turnovers for the strategies from Table 1 for the daily samples from Table 2 for monthly rebalancing. The evaluation period ranges from January 2010 to December 2023. The first three columns report results for a 1250-iteration (trading days) window. The last three columns report results for a 1750-iteration (trading days) window.

My first observation is that the increased data frequency helps with weight estimation in the mean-variance model. Using monthly data resulted in turnovers that were so high that they were barely comparable to the other results. With daily data, the model yields turnovers that are still huge compared to the other models, but the difference has decreased drastically.

The second observation is that the unconstrained minimum-variance portfolio now only has turnovers that are on average twice as high as those of the 1/N strategy. The smallest recorded ratio is 1.88 (1750, NO-M). Also, there are no more outliers. For the monthly data, I observed a turnover factor of 308.25 for the BE-M sample and an 84-month estimation period. This is reduced to only 2.16 when using daily data for the estimation. For the c-min portfolio, these results are even better, with the factors decreasing to around 1.30 on average. This means that the increase in Sharpe ratio that we have seen in Table 8 is ”for free” as we do not incur additional stress due to turnover.

6.1.3 Total Return

While having a visual representation for returns is certainly pleasant to look at and easy to understand, it lacks explanatory power across all samples at once. Hence, contrary to the previous total return section, I present the results for daily estimation and monthly rebalancing in Table 10.

My first observation for total return is that the mean-variance portfolio still defaults in all samples during the evaluation period, despite its increased Sharpe ratio in Table 8. This, however, is not surprising, as the strategy outputs abnormal

Table 10: Total return for daily data & monthly rebalancing

	$WS = 1250$			$WS = 1750$		
Strategy	CO-M	NO-M	BE-M	CO-M	NO-M	BE-M
1/N	227.73	270.38	156.89	258.57	308.46	174.50
	(220.65)	(267.69)	(166.58)	(245.09)	(298.80)	(188.98)
min	301.68	409.91	166.60	281.31	466.37	153.74
	(436.62)	(866.84)	(174.17)	(485.68)	(55.72)	(-44.88)
mv	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00
	(-100.00)	(-100.00)	(-100.00)	(-100.00)	(-100.00)	(-100.00)
c-min	233.48	394.27	148.39	259.81	455.74	160.88
	(326.43)	(431.82)	(102.95)	(278.23)	(558.96)	(140.19)
c-mv	222.63	78.08	677.23	78.19	87.02	655.70
	(22.97)	(35.34)	(304.32)	(-16.19)	(25.91)	(119.80)

This table reports total return for the strategies from Table 1 for the daily samples from Table 2 for monthly rebalancing. The evaluation period ranges from January 2010 to December 2023. The first three columns report results for a 1250-iterations (trading days) window. The last three columns report results for a 1750-iteration (trading days) window.

leverage and therefore bears immense risk by design.

The second thing I want to point out is that the returns of the minimum-variance portfolio are much more stable with the increased estimation data frequency. For the monthly data, it put up both extremely high returns with 866.84% (60, NO-M) and low, even negative returns with -44.88% (84, BE-M). These spikes decreased drastically to 466.37% (1750, NO-M) and 166.60% (1250, BE-M), which makes the output more reliable. The average return across all samples, in turn, remained relatively constant, only decreasing from 329.02% to 307.66%.

Third and finally, I find that the stability of the results of both constrained models increased significantly too. For the c-min portfolio, the peak return reduces from 558.96% (84, NO-M) to 455.74% (1750, NO-M). The low also increased from 102.95% (60, BE-M) to 148.39% (1250, BE-M). For the constrained mean-variance strategy, this effect is even stronger. I record no decrease in peak return here. Instead, all six returns increased for the daily estimation data in comparison to the monthly estimation data.

6.1.4 Results for Daily Data and Monthly Rebalancing

Overall, I observe a huge improvement in the results for the optimization models across all samples when using daily instead of monthly data for the estimation. Table 8 shows that the min, mv and c-min portfolios are now able to consistently beat the 1/N strategy in terms of Sharpe ratio. It is also not to be neglected that there is no case where the daily data leads to significantly worse results across all samples. This means that daily data beats monthly data as a basis for estimation and should be used whenever available. I observe that increasing the data frequency helps to not only improve but also stabilize the unstable results of the portfolio strategies by Harry Markowitz (1952).

6.2 A Glimpse into Shorter Estimation Periods

In the previous section, I have shown that an increase in data frequency leads to a drastic improvement of the results. Now I experiment with shorter estimation periods, combined with daily rebalancing. I show the effects of daily rebalancing on the optimization models and whether they also work if they are applied to more granular data at higher speeds. Therefore, in the next part, the data and rebalancing frequency are increased to daily. Also, the estimation periods are shortened drastically. The new estimation periods for the daily samples are 40 and 125 iterations, which are expressed in days. This is roughly equivalent to two and six months, as there are approximately 250 trading days in a year. I choose these because they overlap with the estimation periods chosen by Jegadeesh and Titman (1993) and Williams (2006). I reevaluate all strategies analogous to Section 5. These new results are then benchmarked against the results from the previous section. To facilitate the comparison with the monthly results, I still calculate the monthly Sharpe ratio, CEQ, and turnover for all models.[12]

6.2.1 Sharpe ratio

For the third and last time, I start by looking at the Sharpe ratios of the strategies which are displayed in Table 11. My first observation is that for the 1/N portfolio, there is no big difference compared to previous results, suggesting that neither estimation period length nor rebalancing frequency affects it. This, of course, is expected and mainly proves that no major error happened during the analysis. The SR remains constant at a level of around 0.18 for all samples.

The second observation concerns the momentum strategy by Jegadeesh and Titman (1993). Table 11 shows that rebalancing daily is not efficient and decreases performance drastically. For the two-month estimation periods, all Sharpe ratios are negative, with the worst being -0.4161 (40, BE-D). Thus, I also include a monthly rebalancing version. This version does substantially better across all samples, especially for the six-month estimation window. Here, it reaches a peak SR of 0.3168 (60, CO-D) for the combined sample. Nevertheless, it only outperforms the benchmark in three of the six observations, suggesting that it is not superior to the 1/N portfolio.

[12]This means that returns are accumulated each month and from this the Sharpe ratio and CEQ are calculated. Monthly turnovers are derived from summing up the turnovers from daily rebalancing month-wise.

Table 11: Sharpe ratios for daily data & daily rebalancing

	$WS = 40$			$WS = 125$		
Strategy	CO-D	NO-D	BE-D	CO-D	NO-D	BE-D
1/N	0.1958	0.1778	0.1916	0.1833	0.1606	0.1934
min	0.1264	0.1093	0.1100	0.2301	-0.0746	0.2583
mv	-0.0019	0.0586	0.0997	0.0917	-0.0765	0.0731
c-min	0.0764	0.1098	0.0934	0.2583	0.3059	0.2352
c-mv	-0.2526	-0.0437	-0.3702	0.0679	0.0929	0.1102
bol	0.2935	0.1498	0.3201	0.0483	-0.0322	0.1671
mom daily	-0.3175	-0.1658	-0.4161	0.0294	0.0201	-0.1074
mom monthly	0.1220	0.1324	-0.0586	0.3168	0.2257	0.2005

This table reports Sharpe ratios for the strategies from Table 1 for the daily samples from Table 2 for daily rebalancing. The evaluation period ranges from January 2010 to December 2023. The first three columns report results for a 40-iteration (trading days) window. The last three columns report results for an 125-iteration (trading days) window.

My third observation is that the minimum- and mean-variance portfolios both perform worse on this smaller timeframe compared to the results from Table 4 and Table 8. All results observed in Table 11 are substantially inferior to the prior results. While the min strategy manages to outperform the benchmark at least in some cases, for example, with an SR of 0.2582 (125, BE-D) for the BE-D sample, the mean-variance portfolio does not post any good results for this analysis. It even returns negative SRs in two out of six cases, with the worst being -0.0765 (125, NO-D). The c-min strategy is the only one of the optimization models that can compete with the benchmark in terms of SR, but only for the 125-day estimation period. It manages to outperform the benchmark three out of three times, with a peak SR of 0.3059 (125, NO-D). However, I also observe lower Sharpe ratios here compared to those for daily data and monthly rebalancing from the previous section. From this, I conclude that the models by Harry Markowitz (1952) suffer from shortening the timeframes without further adjustments.

Last but not least, I evaluate the Bollinger Bands, as these are the only dedicated day trading strategy with daily rebalancing in this selection. I observe that the strategy works well for the two-month estimation period, where it outperforms the benchmark twice. It achieves an SR of 0.2935 for the CO-D sample and an even higher SR of 0.3201 for the BE-D sample. However, increasing the length of the estimation window is not optimal for this strategy. The SR decreases across all three samples, with the lowest now being -0.0322 (125, NO-D).

6.2.2 Turnover

To determine if daily rebalancing is feasible in a real-world scenario, it is important to consider turnover, since transaction costs are not factored into this analysis.

Table 12: Turnovers for daily data & daily rebalancing

	$WS = 40$			$WS = 125$		
Strategy	CO-D	NO-D	BE-D	CO-D	NO-D	BE-D
1/N	0.3074	0.2652	0.1670	0.1929	0.2261	0.1485
min	10.21	16.70	27.84	35.92	222.81	42.36
mv	-	-	-	-	-	-
c-min	14.64	12.41	20.07	6.48	4.99	6.76
c-mv	15.26	16.91	29.17	11.65	10.45	15.53
bol	34.8	42.04	69.56	32.59	29.39	47.81
mom daily	23.89	27.54	46.17	21.91	20.04	30.82
mom monthly	4.32	5.08	8.12	4.18	3.80	5.45

This table reports monthly turnovers for the strategies from Table 1 for the daily samples from Table 2 for daily rebalancing. The evaluation period ranges from January 2010 to December 2023. The first three columns report results for a 40-iteration (trading days) window. The last three columns report results for an 125-iteration (trading days) window.

The first observation that I made is that the turnover for the 1/N portfolio rises significantly when switching to daily rebalancing. In the last section, it only required a monthly turnover of around 0.04, while now this has increased to an average of 0.2179 per month in Table 12. Since I already analyzed that the Sharpe ratio remains the same, it is easy to conclude that for the benchmark, daily rebalancing does not make any sense.

Above, I already found that the optimization models do not work on the evaluation timeframe. Hence, I skip the analysis of their turnovers. The only exception is the c-min portfolio, which performed well on the 125-day estimation period. The turnovers for this strategy are around six times higher in this setting compared to the benchmark. Turnover peaks at 6.76 (125, BE-D) for the longer estimation window. Further analysis would be needed to determine if this increase in transaction costs outweighs the surplus in SR. Similar results are observed for the monthly momentum strategy. It has a turnover that is around five times higher than that of the benchmark. Especially for the 125-day estimation period, this could thus be a viable strategy, as the Sharpe ratios were all higher than the benchmark.

My third observation is that the surplus performance for the Bollinger Bands strategy comes at the cost of high turnovers. For example, for the CO-D sample, it reports a turnover 34.8 times higher than the 1/N strategy for the two-month estimation window.

6.2.3 Results for Shorter Timeframes

The last research question that needs to be answered is whether the Markowitz portfolio can compete with strategies that are dedicated to short timeframes. In total, it becomes evident that the minimum- and mean-variance portfolios are not meant to be used for short estimation periods. My evaluation shows that almost all observed performances suffer from the increase in trading and estimation speed. The only exception to this is the constrained minimum-variance portfolio, which still yields decent results for the 125-day estimation period. I see the reason for this in the length of the estimation window. Two and six months are not enough to give a meaningful estimate of the long-term behavior of a stock. While momentum is evident in the short term, the variance and mean return cannot be estimated sufficiently, leading to wrong approximations and therefore substantially worse out-of-sample performance. Furthermore, I observe that the Bollinger Bands indicator works best on the 40-day estimation period, whereas the momentum strategy is most effective with a six-month estimation period. Notably, daily rebalancing is not optimal for the momentum strategy by Jegadeesh and Titman (1993). This implies that trends need time to develop and that short-term drawdowns can occur during a general uptrend, and vice versa. Thus, the answer to the last research question is that it is generally not feasible to use the Markowitz portfolio for short timeframes unless adjustments are made. If an investor wants to trade on such a frequency, it is advisable to choose the Bollinger Bands indicator or the momentum strategy over the optimization models. Further research in this area is needed to conclude whether there is a way to adjust the latter to work for this incentive too.

7 Limitations

Even though I try to include dynamic screens (by Hanauer and Windmüller (2023) and self-proposed) to enhance the data quality and results of the analysis, there are several factors that are not considered and therefore bias the final analysis. Hence, I now want to highlight some limitations of the analysis in this section.

First, the samples are very focused and strongly overlap by construction. This limits the analysis to very specific markets. It is important to note that there is effectively only one empirical dataset, which is split into two parts by country to demonstrate strategy instability and estimation error. In cases where both are nonexistent, the applied strategies converge to the same result. Therefore, this approach is not optimal for analyzing the performance of strategies across the broader market. A potential improvement for the current sample would be to segment it by industry, market capitalization, or other relevant metrics. This would give more insight while still working on the limited data. However, the only true solution to this issue is to gather more diverse data initially. This way, a more general result and conclusion can be obtained.

Second, the evaluation period only spans from 2010 to 2023. While thirteen years is not particularly short, it certainly is in relation to other papers. Most of the empirical datasets used by Demiguel et al. (2009) consist of 40 years of data. This poses two problems. First, a short timeframe only allows for a restricted analysis of the real-world performance, as few market conditions are covered. Second, there has only been one big crisis from 2010 to 2023 that impacted the stock market over a longer period. This was the COVID-19 pandemic. For a detailed study of its effects, I want to refer to Jabeen et al. (2022). Notably, there were also minor crises, such as the European debt crisis (Livio Stracca, 2015) and various geopolitical tensions, but none had a drastic impact on the stock market compared to major crises like the financial crisis of 2008 (Baily et al., 2008). The start of the war in Ukraine is also included in this dataset, but its effects on the stock market cannot be analyzed from an ex-post position yet. Of course, this leads to a small but not irrelevant bias for the dataset, since performance in good times is never an indicator for performance in bad times. To better understand the models' performances, the observation period should be extended to include more market conditions and crises.

Third, this study is limited to using return index data as price points. Both the momentum strategy by Jegadeesh and Titman (1993) and the Bollinger Bands model as implemented by Williams (2006) were and should be used on price data. Unfortunately, it was not possible for me to get my hands on price and return index data.[13] For more accurate results, I suggest rerunning this analysis using both price data and return index data. Even though this is an issue, my analysis shows that both strategies work on return index data too. I reason that using this is feasible and has minimal bias, since dividends and similar would move the price in a real-world scenario if they were not paid out.

By addressing these limitations in future research, the robustness and applicability of the findings can be significantly enhanced.

8 Conclusion

In this paper, I have explored the performance of various portfolio strategies within the context of algorithmic trading. The primary objective was to test whether the results from Demiguel et al. (2009) hold for the given empirical dataset of equities from Norway and Belgium. More precisely, the objective was to show that the minimum- and mean-variance portfolios are outperformed by the 1/N portfolio. I therefore compared the unconstrained and constrained versions of the models to the 1/N portfolio. I found that none of them were able to consistently yield results

[13]The Refinitiv Datastream account that was provided for this thesis had a query limit of 10,000,000 data points. This forced me to choose either.

that are superior to the 1/N strategy in terms of Sharpe ratio, certainty equivalent, and turnover. To better understand if this also means worse results in a real-world portfolio, I analyzed the total return and the percentage of winning trades. The results from this supported the prior ones. All strategies had similar returns to those of the 1/N portfolio, while taking on more risk or higher turnover.

Furthermore, I investigated how data and rebalancing frequency impact the performance of the chosen strategies to determine whether it is useful to use daily instead of monthly data for estimation. I found that increasing the frequency of the estimation data, while keeping the timeframe of estimation and rebalancing constant, leads to improved results for the minimum- and mean-variance portfolios. They were able to consistently beat the benchmark in terms of Sharpe ratio and also came close in terms of turnover. This result suggests that they may now be a viable option to choose from in such a setting.

Lastly, I analyzed whether the Markowitz portfolio also works for two- and six-month estimation periods and daily rebalancing. For this, I compared the results not only to the 1/N portfolio, but also to the momentum strategy described by Jegadeesh and Titman (1993) and Bollinger Bands as implemented by Williams (2006). I found that, besides the constrained minimum-variance portfolio, the results suffered compared to the previous sections.

While my findings show that data frequency positively impacts the performance of the models from Table 1, it is important to acknowledge the limitations discussed in Section 7. Future research should aim to address these limitations by utilizing more diverse and extended datasets. Also, more evaluation methods need to be incorporated to grasp the influence of transaction costs, resilience in crises, and overall performance.

In conclusion, my research supports the results presented by Demiguel et al. (2009). The 1/N portfolio is superior to the general portfolio theory and thus a viable benchmark for a first analysis. Additionally, it suggests that, whenever available, an investor should use more granular data to estimate the parameters to reduce fluctuation and estimation error. Also, I have shown that one cannot use the Markowitz portfolio for day trading without further adjustments. Future studies should continue to explore these dynamics, particularly in the context of evolving market conditions and advancements in algorithmic trading technologies.

Appendix

A.1 Static screens

The table below displays a summary of all static screens applied by Hanauer and Windmüller (2023). It is an almost exact copy of their work and only in this appendix for completeness. Also note, that I did not apply these screens on my own. I want to thank my supervisor Stefan Baumann for applying them before handing me the dataset for this thesis.

Table A1: Static screens

#	Description	Datastream item(s) involved
1.	For firms with more than one security, only the one with the biggest market capitalization and liquidity is used.	MAJOR = Y
2.	The type of security must be equity.	TYPE = EQ
3.	Only the primary quotations of a security are analyzed.	ISINID = P
4.	Firms are located in the respective domestic country.	GOGEN = country shortcut
5.	Securities are listed in the respective domestic country.	GEOLN = country shortcut
6.	Securities with quoted currency different from the one of the associated country are disregarded.	PCUR = currency shortcut of the country
7.	Securities with ISIN country code different from the one of the associated country are disregarded.	GGISN = country shortcut
8.	Securities whose name fields indicate non-common stock affiliation are disregarded	NAME, ENAME, ECNAME

This table lists the different static screens that are applied to the data. In Column 2, it briefly describes the static screen. Column 3 lists the involved Datastream items. It is directly taken from Hanauer and Windmüller (2023). No additional static screens were added by me.

References

Alla Petukhina, Yegor Klochkov, Wolfgang Karl Härdle, and Nikita Zhivotovskiy (2024). Robustifying markowitz. *Journal of Econometrics*, 239(2):105387.

Baily, M., Litan, R., and Johnson, M. (2008). The origins of the financial crisis. *Lessons from the Financial Crisis: Causes, Consequences, and Our Economic Future*.

Benartzi, S. and Thaler, R. H. (2001). Naive diversification strategies in defined contribution saving plans. *The American Economic Review*, 91(1):79–98.

Bessler, W., Taushanov, G., and Wolff, D. (2021). Factor investing and asset allocation strategies: a comparison of factor versus sector optimization. *Journal of Asset Management*, 22.

Best, M. and Grauer, R. (1991). On the sensitivity of mean-variance-efficient portfolios to changes in asset means: Some analytical and computational results. *Review of Financial Studies*, 4:315–342.

Black, F. and Litterman, R. (1992). Global portfolio optimization. *Financial Analysts Journal*, 48(5):28–43.

Cascon, A. and Shadwick, W. F. (2006). The cs character and limitations of the sharpe ratio. *Journal of Investment Consulting*, 8(1):36–52.

Christopher B. Barry (1974). Portfolio analysis under uncertain means, variances, and covariances. *The Journal of Finance*, 29(2):515–522.

Cornell, B. (2020). Medallion fund: The ultimate counterexample? *The Journal of Portfolio Management*, 46:jpm.2020.1.128.

Demiguel, V., Garlappi, L., and Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1-n portfolio strategy? *The Review of Financial Studies*, 22(5):1915–1953.

Gergaud, O. and Ziemba, W. (2012). Great investors: Their methods, results and evaluation. *Journal of Portfolio Management*, 38.

Gerig, A. (2012). High-frequency trading synchronizes prices in financial markets. *SSRN Electronic Journal*.

Gomber, P., Arndt, B., Lutat, M., and Uhle, T. (2011). High-frequency trading. *SSRN Electronic Journal*.

Griffin, J. M., Kelly, P. J., and Nardari, F. (2010). Do market efficiency measures yield correct inferences? a comparison of developed and emerging markets. *The Review of Financial Studies*, 23(8):3225–3277.

Gurobi Optimization (13/06/2024). The leader in decision intelligence technology - gurobi optimization. https://www.gurobi.com/.

Hanauer, M. X. and Windmüller, S. (2023). Enhanced momentum strategies. *Journal of Banking & Finance*, 148(C).

Harry Markowitz (1952). Portfolio selection. *The Journal of Finance*, 7(1):77–91.

Ince, O. S. and Porter, R. B. (2006). Individual equity return data from thomson datastream: Handle with care! *Journal of Financial Research*, 29(4):463–479.

J. D. Jobson and Bob M. Korkie (1981). Performance hypothesis testing with the sharpe and treynor measures. *The Journal of Finance*, 36(4):889–908.

Jabeen, S., Farhan, M., Zaka, M. A., Fiaz, M., and Farasat, M. (2022). Covid and world stock markets: A comprehensive discussion. *Frontiers in Psychology*, 12.

Jagannathan, R. and Ma, T. (2003). Risk reduction in large portfolios: Why imposing the wrong constraints helps. *The Journal of Finance*, 58(4):1651–1683.

Jegadeesh, N. and Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance*, 48(1):65–91.

Livio Stracca (2015). Our currency, your problem? the global effects of the euro debt crisis. *European Economic Review*, 74:1–13.

Michaud, R. (1989). The markowitz optimization enigma: Is 'optimized' optimal? *Financial Analysts Journal - FINANC ANAL J*, 45:31–42.

Mistry, J. and Shah, J. (2013). Dealing with the limitations of sharpe ratio for portfolio evaluation. *Journal of Commerce and Accounting Research*, 2:10–18.

Schmidt, P., von Arx, U., Schrimpf, A., Wagner, A., and Ziegler, A. (2011). On the construction of common size, value and momentum factors in international stock markets: A guide with applications. *SSRN Electronic Journal*.

Scholtus, M. L. and van Dijk, D. J. C. (2012). High-frequency technical trading: The importance of speed. *SSRN Electronic Journal*.

Tu, J. and Zhou, G. (2011). Markowitz meets talmud: A combination of sophisticated and naive diversification strategies. *Journal of Financial Economics*, 99(1):204–215.

Vynckier, E., Pirastru, G., Ashely, J., Manthey, K., and Miller, D. (2021). Algorithmic trading hpc & ai reference guide. *Dell Technologies*.

Wiecki, T., Campbell, A., Lent, J., and Stauth, J. (2016). All that glitters is not gold: Comparing backtest and out-of-sample performance on a large cohort of trading algorithms. *The Journal of Investing*, 25:69–80.

William F. Sharpe (1966). Mutual fund performance. *The Journal of Business*, 39(1):119–138.

Williams, O. D. (2006). Empirical optimization of bollinger bands for profitability. *SSRN Electronic Journal*.

Declaration of Academic Integrity

Hereby, I declare that I have composed the presented paper independently on my own and without any other resources than the ones indicated. All thoughts taken directly or indirectly from external sources are properly denoted as such.

This paper has neither been previously submitted to another authority nor has it been published yet.

86574 Axtbrunn, 11.07.2024

Place, Date

Yannick Wiest

Signature