Resonances

Yannick Wunderlich

HISKP, University of Bonn

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General outline of talk(s)

▷ Resonance phenomena in classical physics

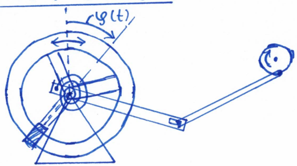
▷ Resonances in non-relativistic Quantum Mechanics

▷ Resonances in relativistic Quantum Field Theory

↳ Some remarks on resonances in Partial Wave Analysis Models

Classical systems that can resonate

"Pohl's wheel":



Resonance (R-I-C) circuit:

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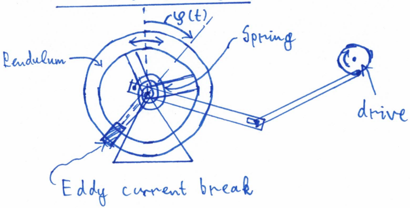

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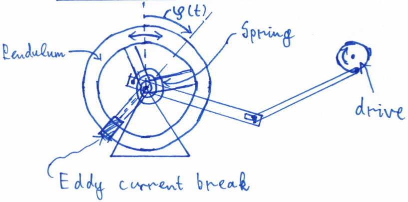
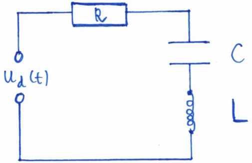


Resonance (R-L-C) circuit:



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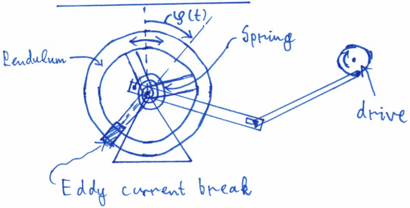

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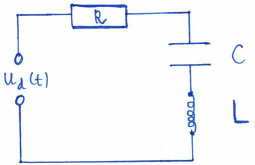

▷ Torques: $M_I = I\,\ddot{\varphi}(t)$; $M_F = -D\,\varphi(t)$

$M_D = -\beta\,\dot{\varphi}(t)$; $M_d = M_0 \cos(\omega t)$

▷ Voltages: $U_R(t) = R\,I(t)$; $U_C(t) = \frac{Q(t)}{C}$

$U_I(t) = (-)\,L\,\dot{I}(t)$; $U_d(t) = U_0 \sin(\omega t)$

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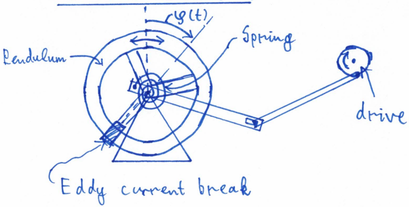

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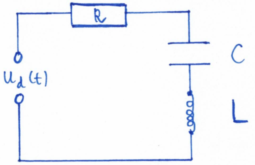

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▷ Torque equation of motion:

$$I\,\ddot{\varphi}(t) \equiv -\beta\,\dot{\varphi}(t) - D\,\varphi(t) + M_0 \cos(\omega t)$$

$$\Leftrightarrow \ddot{\varphi}(t) + \frac{\beta}{I}\dot{\varphi}(t) + \frac{D}{I}\varphi(t) = \frac{M_0}{I}\cos(\omega t)$$

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$U_I(t) = (-)L\,\dot{I}(t)$; $U_d(t) = U_0 \sin(\omega t)$

▷ "Kirchhoff's loop-law":

$$U_d(t) + U_I(t) \overset{!}{=} U_R(t) + U_C(t)$$

$$\Rightarrow L\,\dot{I} + R\,I + \frac{1}{C}Q = U_0 \sin(\omega t)$$

$$\overset{d/dt(\ldots)}{\Rightarrow} \ddot{I}(t) + \frac{R}{L}\dot{I}(t) + \frac{1}{LC}I(t) = \frac{\omega U_0}{L}\cos(\omega t)$$

Classical systems that can resonate

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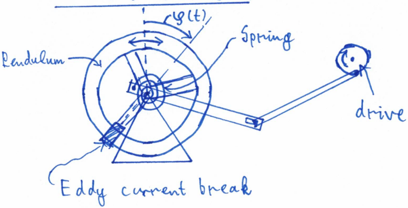

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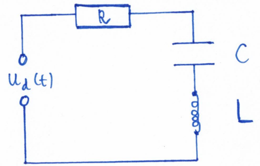

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▷ "Kirchhoff's loop-law":

$$U_d(t) + U_I(t) \stackrel{!}{=} U_R(t) + U_C(t)$$

$$\Rightarrow L\dot{I} + RI + \frac{1}{C}Q = U_0\sin(\omega t)$$

$$\stackrel{d/dt(\dots)}{\Rightarrow} \ddot{I}(t) + \frac{R}{L}\dot{I}(t) + \frac{1}{LC}I(t) = \frac{\omega U_0}{L}\cos(\omega t)$$

↳ Both problems reduced to "Standard Form": $\ddot{x}(t) + 2\gamma\,\dot{x}(t) + \omega_0^2\,x(t) = a\cos(\omega t)$

Solution of the Standard Form DEQ I

$$\ddot{x}(t) + 2\gamma\,\dot{x}(t) + \omega_0^2\, x(t) = a\cos(\omega t)$$

Trick: Switch to complex function $X(t)$, then:

$$x(t) \equiv \mathrm{Re}[X(t)] \;;\; \cos(\omega t) = \mathrm{Re}[e^{i\omega t}]$$

↳ The real part of the solution of

$$\ddot{X}(t) + 2\gamma\,\dot{X}(t) + \omega_0^2\, X(t) = a\, e^{i\omega t}$$

is the solution of the problem!

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▷ Theorem from Calculus II:

The general solution of an inhomogeneous ordinary DEQ is given by:

$$X(t) = X_{hom}(t) + X_{part}(t)$$

general sol. of homogeneous DEQ

particular sol. of inhomogeneous DEQ

Solution of the Standard Form DEQ II

For the solution of the homogeneous equation:

Ansatz: $\quad X_{hom}(t) := C_1 \, e^{i\nu t} \quad , \quad C_1 \in \mathbb{C} \quad ,$

has to solve: $\quad \ddot{X}_{hom}(t) + 2\gamma \dot{X}_{hom}(t) + \omega_0^2 X_{hom}(t) = 0$

$$\Leftrightarrow \quad \nu^2 - 2i\gamma\nu - \omega_0^2 = 0$$

$$\hookrightarrow \quad \underline{\nu_{1,2} = i\gamma \pm \sqrt{\omega_0^2 - \gamma^2} \equiv \pm W_0 + i\gamma \, ,}$$

$$W_0 := \sqrt{\omega_0^2 - \gamma^2} \, .$$

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$$w_0 := \sqrt{\omega_0^2 - \gamma^2} \, .$$

$\triangleright$ This solution of the above given quadratic equations generates functions $x_{hom}(t) = \mathrm{Re}[X_{hom}(t)]$ for the <u>different cases</u>:

(i) <u>damped oscillation</u>
$\omega_0 > \gamma$
$\hookrightarrow \nu_{1,2} = \pm w_0 + i\gamma$
w_0 is real

(ii) <u>aperiodic limit</u>
$\omega_0 = \gamma$
$\nu_1 = \nu_2 = i\gamma$
degeneracy

(iii) <u>overdamped case</u>
$\omega_0 < \gamma$
$\nu_1 =: i\gamma_1 \,,\ \nu_2 =: i\gamma_2$
imaginary solutions

Solution of the Standard Form DEQ III

(i) damped oscillation

$$x_{hom}(t) = A\, e^{-\gamma t} \cos(w_0 t + \alpha)$$

(ii) aperiodic limit

$$x_{hom}(t) = (A + B t)\, e^{-\gamma t}$$

2 int.- constants needed!!

(iii) overdamped case

$$x_{hom}(t) = A\, e^{-\gamma_1 t} + B\, e^{-\gamma_2 t}$$

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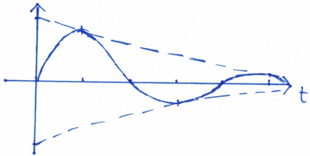

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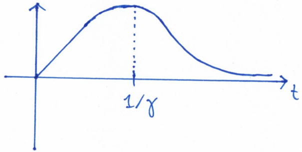

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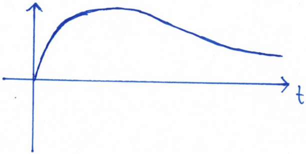


Solution of the Standard Form DEQ IV

For the inhomogenous equation, a "trained eye" / divine intervention motivates the Ansatz:

$$\underline{X_{part}(t) = a\,\chi(\omega)\,e^{i\omega t}}$$

complex function $\chi(\omega)$: "dynamic susceptibility"

Solution of the Standard Form DEQ IV

For the inhomogenous equation, a "trained eye" / divine intervention motivates the Ansatz:

$$\underline{X_{part}(t) = a\, \chi(\omega)\, e^{i\omega t}}$$

complex function $\chi(\omega)$: "dynamic susceptibility"

This fct. has to solve: $\ddot{X}_{part}(t) + 2\gamma \dot{X}_{part}(t) + \omega_0^2 X_{part}(t) = a\, e^{i\omega t}$

$$\hookrightarrow (-\omega^2)\, a\, \chi(\omega)\, e^{i\omega t} + 2i\gamma\omega\, a\, \chi(\omega)\, e^{i\omega t} + \omega_0^2\, a\, \chi(\omega)\, e^{i\omega t}$$

$$\stackrel{!}{=} a\, e^{i\omega t}$$

$$\hookrightarrow \boxed{\chi(\omega) = \frac{1}{\omega_0^2 - \omega^2 + 2i\gamma\omega}}$$

- Most important formula of this presentation
- Resonance phenomena encoded in a function that has a <u>pole in the complex ω^2- (or ω-) plane!</u>

Solution of the Standard Form DEQ V

▷ Disregard the cases (ii) $\gamma = \omega_0$ & (iii) $\gamma > \omega_0$ in the following

▷ The general solution for (i) $\gamma < \omega_0$ becomes:

$$x(t) = \mathrm{Re}[X_{hom}(t)] + \mathrm{Re}[X_{part}(t)]$$

$$= A\, e^{-\gamma t} \cos(\omega_0 t + \alpha) + \mathrm{Re}[a\, \chi(\omega)\, e^{i\omega t}]$$

we have: $\chi(\omega) = |\chi(\omega)|\, e^{i\delta(\omega)}$, with:

$$|\chi(\omega)| = \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}} \quad ; \quad \tan\delta(\omega) \equiv \frac{-|\mathrm{Im}[\chi(\omega)]|}{\mathrm{Re}[\chi(\omega)]} = \frac{2\gamma\omega}{\omega^2 - \omega_0^2}$$

▷ The solution of the Standard Form DEQ becomes:

$$x(t) = A\, e^{-\gamma t} \cos(\omega_0 t + \alpha) + a\, |\chi(\omega)| \cos(\omega t + \delta(\omega))$$

▷ For large times, $t \gg 1/\gamma$, one has:

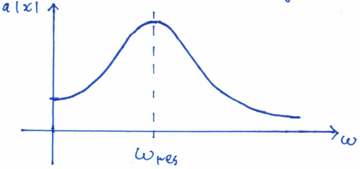
$$x(t) = a\, |\chi(\omega)| \cos(\omega t + \delta(\omega))$$, "Stationary state", investigate further...

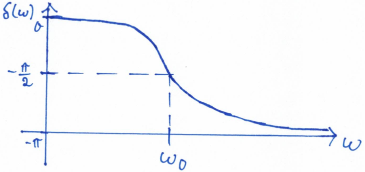
Characteristic "look" of resonance-curve

▷ "Amplitude" $a\,|x(\omega)| = \dfrac{a}{\sqrt{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}$; Phase $\delta(\omega) = \arctan\left[\dfrac{2\gamma\omega}{\omega^2-\omega_0^2}\right]$

peaks at $\omega_{res} \equiv \sqrt{\omega_0^2-2\gamma^2}$

shows characteristic "phase-motion"





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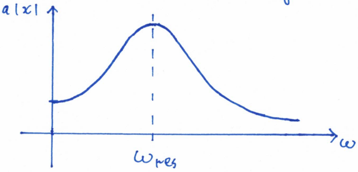
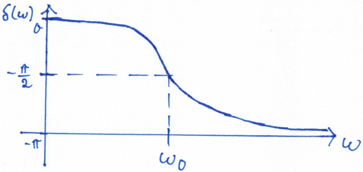
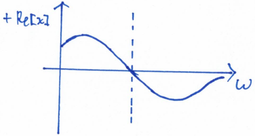


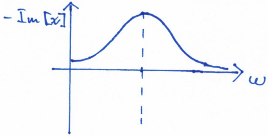


▷ For weak damping : $\gamma \ll \omega_0$, Taylor-exp. denominator of $\chi(\omega)$ around ω_0

↳ $\chi(\omega) \simeq (-)\dfrac{1}{2}\dfrac{1}{(\omega-\omega_0)-i\gamma}\dfrac{1}{\omega_0} = (-)\dfrac{1}{2\omega_0}\left\{\dfrac{(\omega-\omega_0)}{(\omega-\omega_0)^2+\gamma^2} + i\dfrac{\gamma}{(\omega-\omega_0)^2+\gamma^2}\right\}$

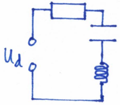
"Breit-Wigner"-form with ω-independent width





Intermediate summary I

▷ **Last time:** Considered classical systems (e.g. R-L-C circuit) described by standard-form diff. eq.



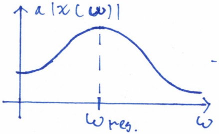
$$\underline{\ddot{x}(t) + 2\gamma \dot{x}(t) + \omega_0^2 x(t) = a \cos(\omega t)}$$

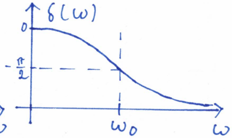
↳ DEQ has classical solution

$$x(t) = A e^{-\gamma t} \cos(W_0 t + \alpha) + a\,|x(\omega)| \cos(\omega t + \delta(\omega))$$

capable of showing <u>resonance behavior</u>

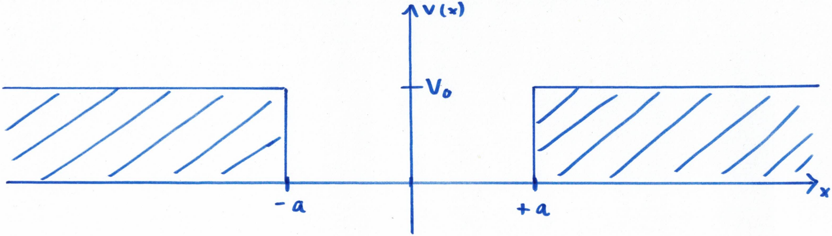
for $t \gg 1/\gamma$





▷ **Today:** Let's look at some Quantum Mechanics!

Q. M. potential - well in 1 - dim.



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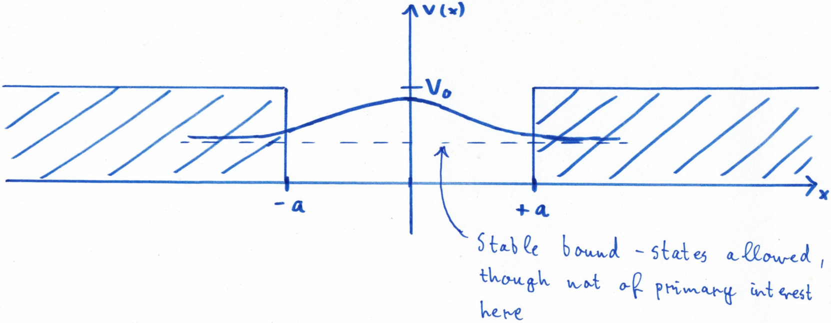

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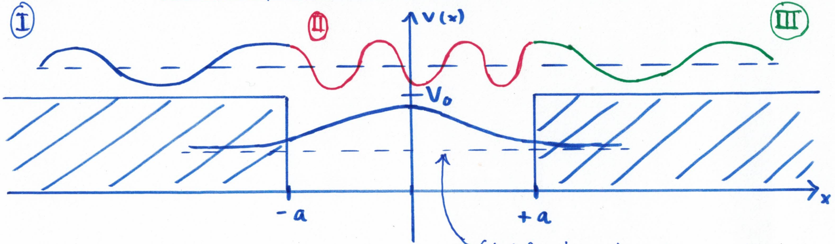

Stable bound-states allowed, though not of primary interest here

Ansatz for scattering-states:

I: $\psi_{in}(x) = A\, e^{ikx} + B\, e^{-ikx}$

II: $\psi_{II}(x) = C\, e^{iqx} + D\, e^{-iqx}$

III: $\psi_{out}(x) = E\, e^{ikx} + F\, e^{-ikx}$

1-dim. Schrödinger-eq.:

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + V(x)\,\psi(x) = E\,\psi(x)$$

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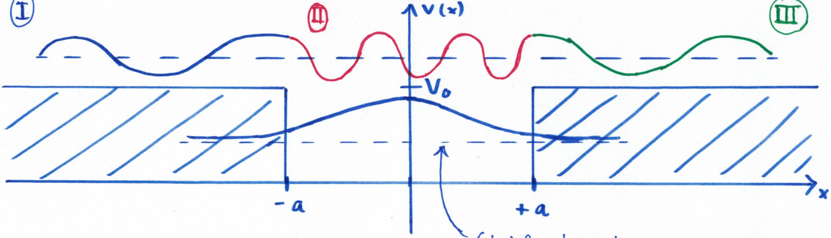

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F = 0 later ; only right - moving transmitted waves !

1 - dim. Schrödinger - eq. :

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + V(x)\,\psi(x) = E\,\psi(x)$$

▷ $q = \frac{\sqrt{2mE}}{\hbar}$ & $k = \frac{\sqrt{2m(E-V_0)}}{\hbar}$ define solutions !

Boundary (connecting) conditions for 1-dim. potential well

▷ Demand continuity of $\psi(x)$
and $\psi'(x)$ @ $x = \pm a$
(motivated by Schrödinger-eq.)

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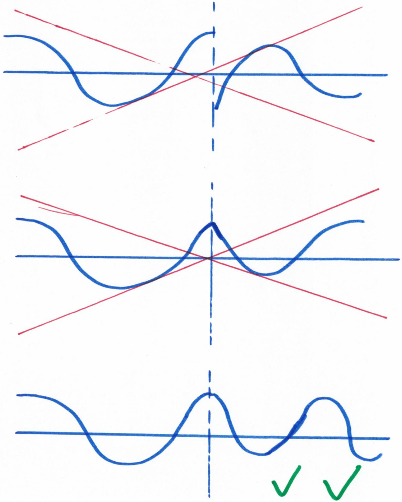

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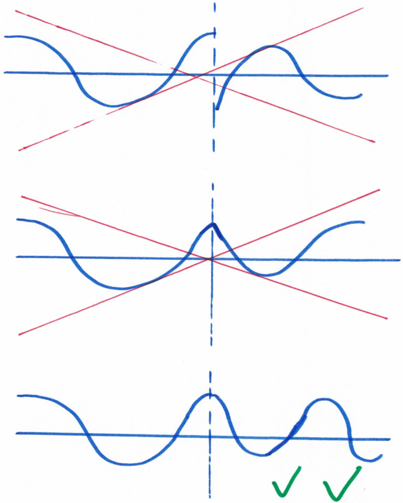
▷ In practice, for potential-well:



<u>$x=-a$:</u> $\psi_{in}(-a) \overset{!}{=} \psi_{II}(-a)$ and

$\psi'_{in}(-a) \overset{!}{=} \psi'_{II}(-a)$;

<u>$x=+a$:</u> $\psi_{II}(a) \overset{!}{=} \psi_{out}(a)$ and

$\psi'_{II}(a) \overset{!}{=} \psi'_{out}(a)$

Boundary (connecting) conditions for 1-dim. potential well

- Demand continuity of $\psi(x)$ and $\psi'(x)$ @ $x = \pm a$ (motivated by Schrödinger-eq.)

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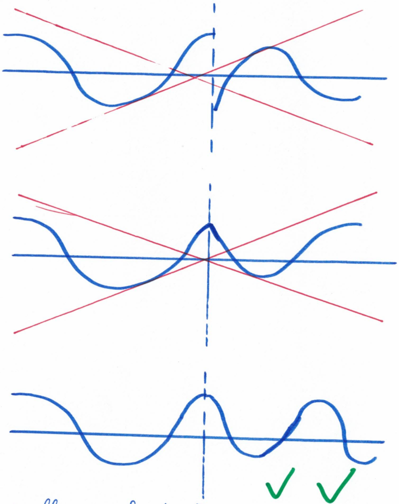

- Boundary-conditions unearth relations among $\{A, B, C, D, E, F\}$

$\rightsquigarrow$ Solution of Schrödinger-eq. up to overall normalization!

A lot of tedious algebra

▷ Invoke both boundary-conditions to connect $\{A, B\} \to \{E, F\}$

▷ E.g.: $x = -a$:

$$\psi_{in}(-a) \overset{!}{=} \psi_{II}(-a) \iff A e^{-ika} + B e^{ika} \overset{!}{=} C e^{-iqa} + D e^{iqa}$$

$$\psi'_{in}(-a) \overset{!}{=} \psi'_{II}(-a) \iff A ik e^{-ika} - B ik e^{ika} \overset{!}{=} C iq e^{-iqa} - D iq e^{iqa}$$

$$\Updownarrow$$

$$\begin{bmatrix} e^{-ika} & e^{ika} \\ ik e^{-ika} & (-) ik e^{ika} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} \overset{!}{=} \begin{bmatrix} e^{-iqa} & e^{iqa} \\ iq e^{-iqa} & (-) iq e^{iqa} \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix}$$

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$$\psi_{in}(-a) \stackrel{!}{=} \psi_{II}(-a) \iff A e^{-ika} + B e^{ika} \stackrel{!}{=} C e^{-iqa} + D e^{iqa}$$

$$\psi'_{in}(-a) \stackrel{!}{=} \psi'_{II}(-a) \iff Aik\, e^{-ika} - Bik\, e^{ika} \stackrel{!}{=} Ciq\, e^{-iqa} - Diq\, e^{iqa}$$

$$\Updownarrow$$

$$\begin{bmatrix} e^{-ika} & e^{ika} \\ ik\, e^{-ika} & (-)ik\, e^{ika} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} \stackrel{!}{=} \begin{bmatrix} e^{-iqa} & e^{iqa} \\ iq\, e^{-iqa} & (-)iq\, e^{iqa} \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix}$$

$$\Updownarrow$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \underbrace{\frac{1}{2} \begin{bmatrix} \left(1 + \frac{q}{k}\right) e^{i(k-q)a} & \left(1 - \frac{q}{k}\right) e^{i(k+q)a} \\ \left(1 - \frac{q}{k}\right) e^{-i(k+q)a} & \left(1 + \frac{q}{k}\right) e^{-i(k-q)a} \end{bmatrix}}_{=:\, M(a)} \begin{bmatrix} C \\ D \end{bmatrix}$$

A lot of tedious algebra

- Try to get $\{A, B\} \leftrightarrow \{E, F\}$
- $\begin{bmatrix} A \\ B \end{bmatrix} = M(a) \begin{bmatrix} C \\ D \end{bmatrix}$ already obtained.

  $\underline{x = +a}$ - boundary conditions yield in much the same way:

$$\begin{bmatrix} E \\ F \end{bmatrix} = M(-a) \begin{bmatrix} C \\ D \end{bmatrix} . \quad \Rightarrow \begin{bmatrix} A \\ B \end{bmatrix} = M(a) \begin{bmatrix} C \\ D \end{bmatrix} = \underline{M(a)\, M(-a)^{-1} \begin{bmatrix} E \\ F \end{bmatrix}} .$$

A lot of tedious algebra

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▷ One obtains:

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} e^{2ika} \left\{ \cos(2qa) - \frac{i}{2}\left(\frac{k}{q} + \frac{q}{k}\right) \sin(2qa) \right\} , & \frac{i}{2qk} (k-q)(k+q) \sin(2qa) \\ \frac{(-)i}{2qk} (k-q)(k+q) \sin(2qa) , & e^{-2ika} \left\{ \cos(2qa) + \frac{i}{2}\left(\frac{k}{q} + \frac{q}{k}\right) \sin(2qa) \right\} \end{bmatrix}$$

$$\times \begin{bmatrix} E \\ F \end{bmatrix}$$

▷ As mentioned before: $F = 0$ can be set!

Reward of tedious algebra!

$\triangleright\ F = 0 \implies$

$$A = e^{2ika}\left\{\cos(2qa) - \frac{i}{2}\left(\frac{k}{q} + \frac{q}{k}\right)\sin(2qa)\right\} E$$

$$B = \frac{(-)\,i}{2qk}\,(k-q)(k+q)\,\sin(2qa)\; E$$

Reward of tedious algebra!

▷ $F = 0 \Rightarrow$

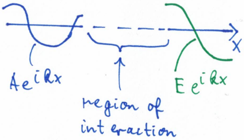
$$A = e^{2ika}\left\{\cos(2qa) - \frac{i}{2}\left(\frac{k}{q} + \frac{q}{k}\right)\sin(2qa)\right\} E$$

$$B = \frac{(-)\,i}{2qk}\,(k-q)(k+q)\,\sin(2qa)\;E$$

▷ Define Transmission- (or Scattering-) amplitude

≡ fraction of amplitudes of in- & outgoing waves:

$$S(E) := \frac{E}{A} = \frac{e^{-2ika}}{\cos(2qa) - \frac{i}{2}\left(\frac{k}{q} + \frac{q}{k}\right)\sin(2qa)}$$



Reward of tedious algebra!

▷ $F = 0 \implies A = e^{2ika} \left\{ \cos(2qa) - \frac{i}{2} \left(\frac{k}{q} + \frac{q}{k} \right) \sin(2qa) \right\} E$

$$B = \frac{(-)i}{2qk} (k-q)(k+q) \sin(2qa)\, E$$

▷ Define Transmission- (or Scattering-) amplitude $\equiv$ fraction of amplitudes of in- & outgoing waves:

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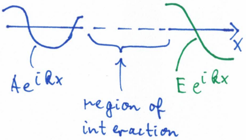
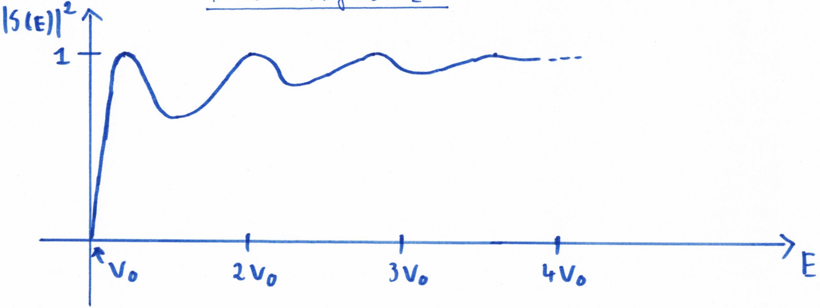


↳ master-formula of this talk

↳ Again, a complex fct. that has poles appears!

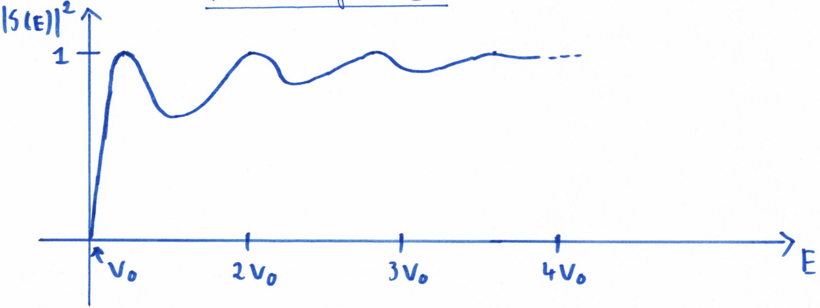
↳ Amazing: 1-dim. Q.M. problem yields resonances!

Transmission-amplitude $|S(E)|^2$ as a fct. of real energies E



▷ peaks $\Leftrightarrow$ resonances!

Transmission-amplitude $|S(E)|^2$ as a fct. of real energies E



▷ Peaks ⟷ resonances!

▷ $$|S(E)|^2 = \frac{1}{1 + \frac{1}{4}\left(\frac{k}{q} + \frac{q}{k}\right)^2 \sin^2(2qa)}$$ reaches max. value 1

for: $\sin^2(2qa) = 0 \iff 2qa = n\pi$.

▷ For full magic: consider $S(E)$ as an analytic function of $E \in \mathbb{C}$.

Interlude: branches of $\sqrt{z}$

▷ $S(E)$ depends on E through $q \equiv \frac{\sqrt{2mE}}{\hbar}$ and $k \equiv \frac{\sqrt{2m(E-V_0)}}{\hbar}$

↳ Look at behaviour of $\sqrt{z}$ for complex z.

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▷ Problem: define: $w := z^2 \Leftrightarrow z = \sqrt{w}$,

however: $w = (\pm z)^2 = (\pm\sqrt{w})^2$

↳ Square-root function is multi-valued!

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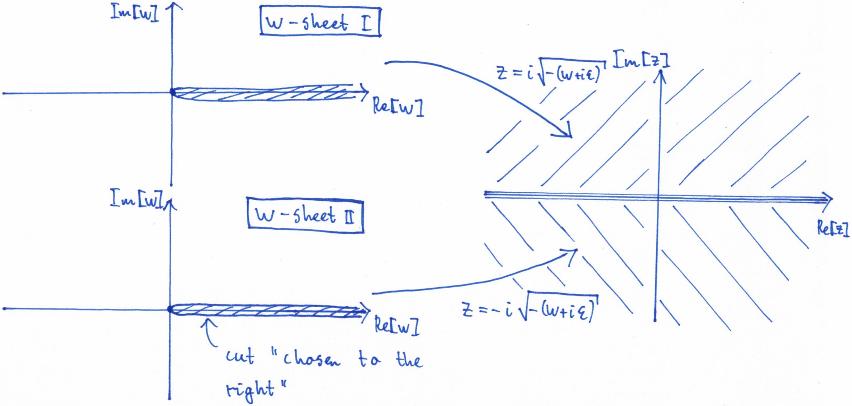
▷ Instead of battleing with one fct. taking multiple values on one copy of the full complex plane, enlarge the domain of $\sqrt{z}$ to two copies of the complex plane!

↳ "branches" $+\sqrt{w}$ and $-\sqrt{w}$ exist on one copy each!

↳ "Riemann-sheets"!

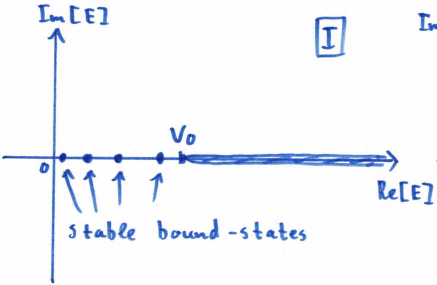
Branches of $\sqrt{z}$ – II

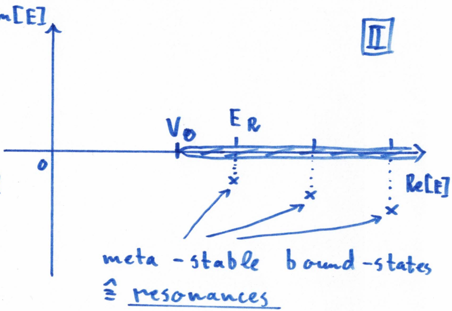
▷ Branches $+\sqrt{w}$ and $-\sqrt{w}$ on different sheets "glued together" continuously along a so-called "<u>branch-cut</u>":



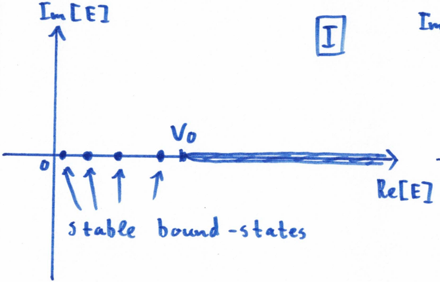
▷ $S(E)$ is a complicated function composed of square-roots!
↳ $S(E)$ inherits this "analytic structure".

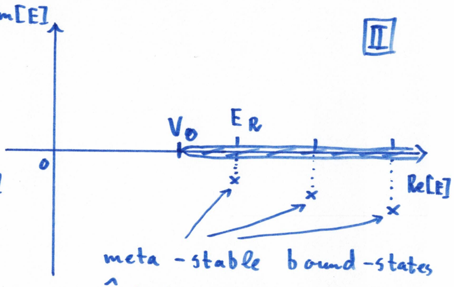
Poles of $S(E)$ in the complex E-plane





Poles of $S(E)$ in the complex E-plane

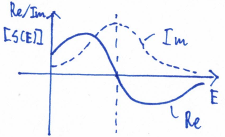


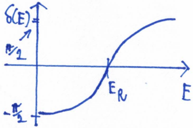


$\hat{=}$ resonances

▷ Breit-Wigner-parametrizations

$$S(E) \simeq (-)^n \frac{i\Gamma/2}{(E-E_R)+i\Gamma/2}$$





from expansion of $S(E)$'s denominator around a particular resonance-energy

↳ only valid locally !!

Scattering of wave-packet on the potential well

▷ Consider general 1-dim. Gaussian wave-packet:

$$\Psi_{in}(x,t) \equiv \int_0^\infty \frac{dp}{2\pi\hbar}\, g(p)\, \exp\left[\frac{i}{\hbar}\left(px - E(p)t\right)\right] \quad ; \quad E(p) \equiv \frac{p^2}{2m} \, ,$$

scattered to:

$$\Psi_{out}(x,t) = \int_0^\infty \frac{dp}{2\pi\hbar}\, g(p)\, \exp\left[\frac{i}{\hbar}\left(px - E(p)t - 2pa + \delta(E)\hbar\right)\right] |S(E)| \, .$$

Scattering of wave-packet on the potential well

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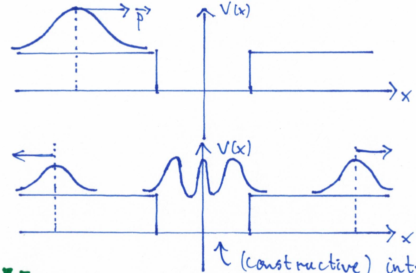
▷ Simple picture:



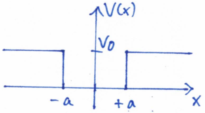
$\Rightarrow$

↑ (constructive) interference of Fourier-modes builds resonance

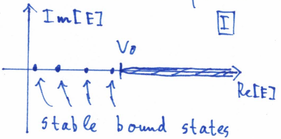
↳ better: movies !!

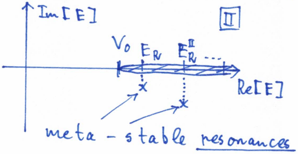
Intermediate summary II

▷ **Last time:** 1-dim. square-well potential yielded, in non-rel. QM:

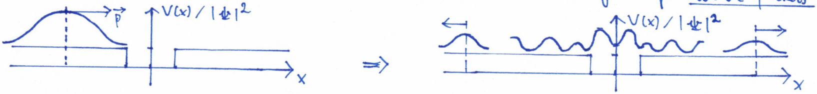


↳ Transition-amplitude $\underline{S(E)}$, having resonances as poles on the 2nd Riemann-sheet:





↳ Behavior reflected in t-dependent scattering of wave-packets



(↳ movies!!)

▷ **Today:** Complications in non-rel. QM / Relativistic reactions!

Complications - I : 3D space

▷ In the real world, we have (at least) 3 spatial dimensions, not just 1 !

↳ $\exists$ rotations as symmetry-trafo.'s generated by

$$\hat{\vec{L}} \equiv \hat{\vec{r}} \times \hat{\vec{p}} = (-i)\hbar\,(\vec{r} \times \vec{\nabla}) \quad : \quad U := \exp\left[\frac{i}{\hbar}\Delta\varphi\,\hat{\vec{n}}\cdot\hat{\vec{L}}\right]$$

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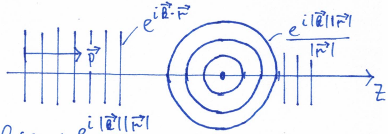
▷ Scattering on potential $V(\vec{r})$ $\{= V(|\vec{r}|)\}$ in 3 D still describe= able by stationary standard-solution !



For $|\vec{r}| \gg a$:
$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + \underbrace{f(\vartheta,\varphi)}_{\text{"scattering amplitude"}}\,\frac{e^{i|\vec{k}||\vec{r}|}}{|\vec{r}|}$$

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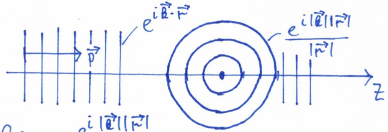

For $|\vec{r}| \gg a$: $\quad \psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + \underline{f_{k}(\vartheta,\varphi)}\,\dfrac{e^{i|\vec{k}||\vec{r}|}}{|\vec{r}|}$

"scattering amplitude"

▷ For $V = V(|\vec{r}|)$, i.e. rotationally symmetric potential :

$$[\hat{H}, \hat{\vec{L}}^2] = 0 \;, \quad [\hat{H}, \hat{L}_z] = 0 \;, \quad [\hat{\vec{L}}^2, \hat{L}_z] = 0 \,.$$ This leads to ...

Complications - I.2 : 3D space

D... the very (!) important concept of <u>partial - waves</u>!

Expand $f_{\vec{k}}(\vartheta, \varphi)$ into eigen-functions of $\hat{\vec{L}}^2$ & L_z:

$$f_{\vec{k}}(\vartheta, \varphi) = f_{\vec{k}}(\vartheta) = \underline{\sum_{\ell=0}^{\infty} (2\ell+1) f_\ell(k) P_\ell(\cos\vartheta)}$$

↑ azimuthal symmetry

Complications - I.2 : 3D space

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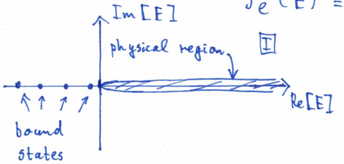
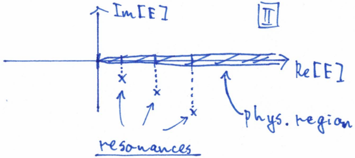
↑ azimuthal symmetry

▷ $\hat{\vec{L}}$ is conserved → scattering occurs essentially independently in each ℓ-wave → analytic structure very similar to $S(E)$ in 1-dim. square-well occurs in each partial wave!

$$S_\ell(E) \equiv 1 + 2i\,k[E]\, f_\ell(k[E])$$

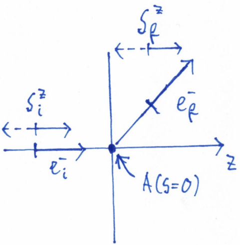




Complications – II : Spin & Statistic

▷ In reality, particles can have non-vanishing <u>spin</u>, e.g.

e^- : $S_{e^-} = \frac{1}{2}$; $S^z_{e^-} = \pm \frac{1}{2}$. Represented e.g. by Pauli-spinors $\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.



↳ Number of scatt.-amplitudes <u>increased</u>,

e.g. $e^- + A(S=0) \longrightarrow e^- + A(S=0)$
$(1/2) + (0) \longrightarrow (1/2) + (0)$

described by $f \equiv \begin{pmatrix} f_{++} & f_{+-} \\ f_{-+} & f_{--} \end{pmatrix}$ ← matrix in spinor-space

▷ <u>"Spin-statistics-theorem"</u> :

For <u>identical</u> particles in either $|i\rangle$ or $|f\rangle$, wave-fct.'s have to have correct symmetry-properties : $|\ldots, a, \ldots, b, \ldots\rangle \equiv (-)^{\alpha} |\ldots, b, \ldots, a, \ldots\rangle$.

Complications – III : multi-particle systems & channels

▷ For complications until now: resonance-poles still found in partial-waves of definite J^P (P ← parity, J ← total angular momentum)

▷ In reality, particles not just "scattered off a potential", instead e.g.

*) 2 → 2 scatt.: separate motion of $\vec{R} := \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$ & $\vec{r} := \vec{r}_1 - \vec{r}_2$.

↳ treat relative-motion the same way as potential-scattering → easy ✓

*) 3 → 3 scatt.: not so easily separable / treatable any more

▷ Major complication: existence of several "channels"

↳ same initial state can lead to different final states,

e.g., e^- scattered off Hydrogen-atom H :

$$e^- + H \longrightarrow \begin{cases} e^- + H & \text{elastic scattering} \\ e^- + H^* & \text{excitation} \\ e^- + e^- + p & \text{break-up} \end{cases}$$

Channels : non-relativistic vs. relativistic

▷ Common feature of non-rel. atomic and nuclear reactions:

number of particles assumed to be "elementary" is conserved!

(exception : photons γ)

E.g.: channels from previous slide

$$|e^-, (p\,e)\rangle \longrightarrow \begin{cases} |e^-, (p\,e)\rangle, & \text{elastic scattering} \\ |e^-, (p\,e)^*\rangle, & \text{excitation} \\ |e^-, e^-, p\rangle, & \text{break up} \end{cases}$$

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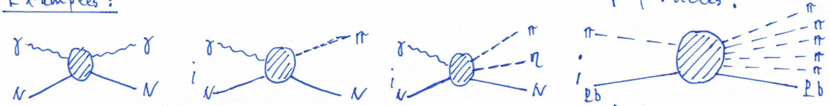
- New in relativistic reactions:

  (Quite) free interchange of mass $\longleftrightarrow$ energy: creation & annihilation of particles!

Examples:



Photoproduction

A COMPASS-reaction

Multi-channel S- & T-matrices

▷ Thinking in terms of ever-persisting "quasi-classical" particles <u>not helpful</u> any more in relativistic reactions!

Rather, things to <u>hold on to</u>:

*) 4-momentum conservation

*) Certain symmetries / quantum-numbers (baryon-#, el. charge, strangeness, ...)

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For given initial state $|i\rangle$

↳ look at PDG-tables of particles

↳ compare masses, quantum-#'s for different final states

↳ Become <u>creative</u> in <u>writing down channels</u> !

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▷ Further consistency-constraints obtained by so-called "<u>S-matrix</u>":

$$\underline{|f\rangle = \hat{S}\,|i\rangle}$$

In <u>Q.M.</u> / <u>QFT</u>:

$$\hat{S} = \hat{U}_I\,(t_f \to +\infty,\ t_i \to -\infty)$$

interaction-picture time-evolution operator

<u>Decompose:</u>

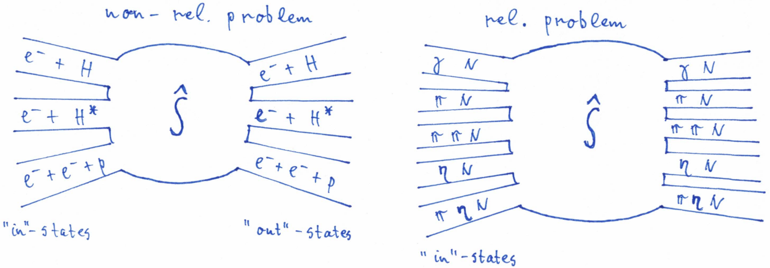
$$\hat{S} = \mathbb{1} + i\,\hat{T}$$

non-interactions ≙ "boring part"

"<u>T-matrix</u>"

Multi-channel $\hat{S}$- & $\hat{T}$-matrices II

▷ Useful picture from [J. R. Taylor: "Scattering theory"]



Multi-channel $\hat{S}$- & $\hat{T}$-matrices II

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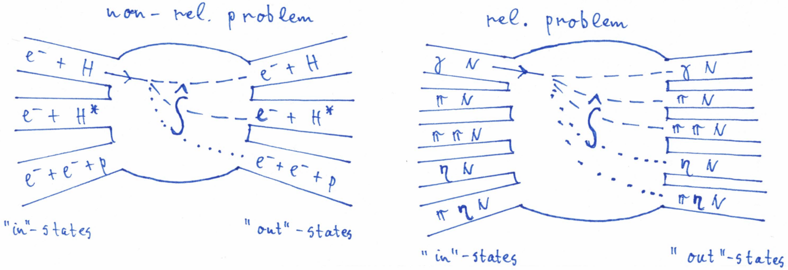

↳ $\hat{S}$-operator directs "probability-flow" from a specific "in"-state $|i\rangle$ to possibly different "out"-states.

*) Energy has to excess kinematic thresholds for channels to be open (schematically):

— — — — — — : $E \geq E^{(2)}_{thr.}$ $[E^{(3)}_{thr.}]$; . . . . : $E \geq E^{(3)}_{thr.}$ $[E^{(5)}_{thr.}]$

▷ Are there further guiding-principles in this mess?

Unitarity, Analyticity & Crossing

▷ For a given initial state $|i\rangle$, the sum of the probabilities to end up in one of the final states $\{|n\rangle\}$ has to be 1:

$$1 \overset{!}{=} \sum_n P_{n\leftarrow i} \overset{\text{Standard Q.M.}}{=} \sum_n |\langle n|\hat{S}|i\rangle|^2 = \sum_n \langle n|\hat{S}|i\rangle^* \langle n|\hat{S}|i\rangle$$

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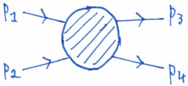
▷ Analyticity: Matrix elements $\langle m|\hat{S}|n\rangle$ have to be analytic (≡ "complex differentiable") functions of complex extensions of certain Lorentz-invariants!

E.g.: "Mandelstam-variables"



$s := (p_1+p_2)$, $t := (p_1-p_3)$, $u := (p_1-p_4)$

Unitarity, Analyticity & Crossing

▷ Analyticity has to hold except at certain **singularities** (≡ points @ which $\langle m | \hat{S} | n \rangle$ is not complex-differentiable)

Broadly speaking:

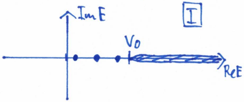
(i) bound states & resonances $\leftrightarrow$ 1st order poles

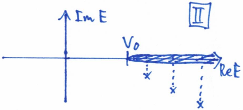
(ii) thresholds $\leftrightarrow$ branch-points

($\sqrt{\ldots}$ - branch-point or more complicated one's)

↳ "Physical" amplitudes are boundary-values of $\langle m | \hat{S} | n \rangle$ on the real axis!

Exactly as in our 1-dim. Q.M. problem!





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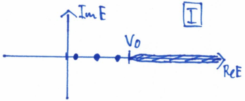

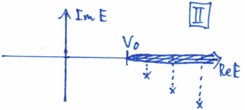

▷ "Crossing symmetry": Same analytic function $\langle m|\hat{S}|n\rangle$ has to describe the direct channel as well as "crossed channels".

E.g.: $\pi N \to \pi N$

"s-channel"

"t-channel"

A practical analysis-tool: the K-matrix

▷ Consider $\hat{T}$-matrix, for one species of scalar particles and only 2-body initial- and final states, defined by the infinite "bubble-sum" of re-scatterings:

$$\hat{T} = K + K\,i\varrho\,K + K\,i\varrho\,K\,i\varrho\,K + \ldots$$

$$= K \times \{1 + i\varrho K + i\varrho K\, i\varrho K + \ldots\}$$

$$= K \times \sum_{n=0}^{\infty} (i\varrho K)^n \underset{\uparrow}{=} \frac{K}{1 - i\varrho K}$$

"geometric series"

 $\equiv K$

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▷ In new convention to define $\hat{S}$-matrix / $\hat{T}$-matrix:

$$\hat{S} \equiv 1 + 2 i\rho \hat{T} = \frac{1 - i\rho K}{1 - i\rho K} + \frac{2 i\rho K}{1 - i\rho K} = \frac{1 + i\rho K}{1 - i\rho K}$$

The K-matrix - Part II

▷ $\hat{S}$ is unitary $\Leftrightarrow$ K is real $\{ K_{ab}$ real & symmetric in mult.-chn. case $\}$

E.g.:
$$\hat{S}^\dagger \hat{S} = \left(\frac{1+i\rho K}{1-i\rho K}\right)^\dagger \left(\frac{1+i\rho K}{1-i\rho K}\right) = \frac{(1-i\rho K^\dagger)(1+i\rho K)}{(1+i\rho K^\dagger)(1-i\rho K)} \underset{\substack{\uparrow \\ K=K^\dagger}}{=} 1 .$$

↳ Can make some real (& symmetric) Ansatz for K

$\Rightarrow$ Automatically get unitary $\hat{S}$!

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$\Rightarrow$ Automatically get unitary $\hat{S}$!

▷ E.g.: K is simple real pole in 1-channel example:

$$K = \frac{g^2}{s-m^2} \quad \Rightarrow \quad \hat{T} = K(1-i\rho K)^{-1} = \frac{g^2}{s-m^2}\left(1 - i\rho\frac{g^2}{s-m^2}\right)^{-1}$$

$$= \frac{g^2}{s-m^2-i\rho g^2} \quad \text{"relativistic Breit-Wigner"}$$

↳ Has resonance-poles on 2^{nd} Riemann-sheet due to the fact that

$$\rho(s) = \frac{1}{16\pi}\sqrt{1-\frac{4m^2}{s}} \quad \text{involves square-root!}$$

What makes resonance-physics difficult?

▷ Now that we have nice K-matrix-tool, could fit (e.g. like BnGa-group) things like

$T = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix}$; $K = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix}$

$\rho = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix}$; $g = \begin{bmatrix} \times & \times & \times \end{bmatrix}$

$$K_{ab} = \sum_{\alpha} \frac{g_a^{(\alpha)} g_b^{(\alpha)}}{s - m_\alpha^2} + f_{ab} \quad \Rightarrow \underline{\text{extract resonance parameters}}!$$

"background"

What are the issues ?? What makes this so damn hard ?

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What are the issues ?? What makes this so damn hard ?

(i) <u>Particle-zoo</u> ($p, n, \Delta, \Sigma, \dots ; \pi, \eta, K, \dots$)

↳ Existence of many channels / thresholds

↳ Maintaining correct unitarity & analyticity-properties is, over wide energy-regions very hard (almost impossible)

↳ Riemann-sheet structures of $\hat{S}$ become highly complicated!

(ii) <u>Background</u>, i.e. "non-pole"-contributions only poorly known in some channels!

(iii) <u>Programming-effort</u>: Somebody has to fit complicated $\hat{S}$-, $\hat{T}$, $\hat{K}$-matrices to large data-sets {≡ the "actual work"}.

Therefore, getting correct resonance (= pole -) parameters out of scattering - data is already difficult!

Determining the nature of a particular state is again quite another story ...

$\boxed{\text{Thank you for your attention!}}$