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A multi-country study of power ARCH models and national stock market returns

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Abstract

Ding et al. (1993) [Ding, Z., Granger, C.W.J., Engle, R.F., 1993. A long memory property of stock market returns and a new model. *Journal of Empirical Finance* **1**, 83–106] suggested a model which extends the ARCH family of models for analyzing a wider class of power transformations than simply taking the absolute value or squaring the data as in the conventional conditional heteroscedastic models. This paper analyzes the applicability of these power ARCH (PARCH) models to national stock market returns for 10 countries plus a world index. We find the PARCH model to be generally applicable once GARCH and leverage effects are taken into consideration. In addition, we also find that the optimal power transformation is remarkably similar across countries. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The use of conditionally heteroscedastic models to analyze time-varying volatility in high frequency financial data has become so widespread that there now exists a number of survey papers which document the properties and empirical applications of the ARCH class of models (see inter alia, Bollerslev et al., 1992, 1994; and Bera and Higgins, 1993). As these survey articles show, the ARCH family of models has

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been extended well beyond the simple specification of the initial ARCH model of Engle (1982) and GARCH model of Bollerslev (1986). These additions to the family have attempted to refine both the mean and variance equations to better capture the stylized features of high frequency data.

A common feature of the standard class of ARCH models is that they relate the conditional variance to lagged squared residuals and past variances. However, it is not necessary to impose a squared power term in the second moment equation. For example, the Taylor (1986) class of ARCH models specifies a power term of unity in that they relate the conditional standard deviation of a series to lagged absolute residuals and past standard deviations. In fact, it is possible to specify any positive value as the power term in the second moment equation. This is so because the absolute changes in an asset's price will exhibit volatility clustering and the inclusion of a power term acts so as to emphasize the periods of relative tranquillity and volatility by magnifying the outliers in that series. The common use of a squared term in this role is most likely a reflection of the normality assumption traditionally invoked (or at least seriously entertained) regarding the data. As is well known, if a data series is normally distributed then we are able to completely characterize its distribution by its first two moments. As such, it is appropriate to focus on a squared term and, hence, a measure of the variance.

However, if we accept that (high frequency) data are very likely to have a non-normal error distribution, then one must transcend into the realm of the higher moments of skewness, kurtosis and beyond to adequately describe the data. In this instance, the superiority of a squared term is lost and other power transformations may be more appropriate. Indeed, for non-normal data, by squaring the returns one effectively imposes a structure on the data which may potentially furnish sub-optimal modelling and forecasting performance relative to other power terms. An important contribution of the current paper is to augment our understanding of whether and to what extent these types of more flexible models are superior to their less sophisticated counterparts.

The other major advantage of having a general power term is that it nests the two major classes of ARCH models, namely, those which directly model the variance (for example, Engle, 1982, and Bollerslev, 1986) and those that directly model the standard deviation (for example, Taylor, 1986) as special cases of the general model. This provides an encompassing framework which facilitates comparison between these two broad classes of model.

The preceding discussion thus provides the key intuition underlying the role of parameterizing the power term and, more generally, the interest in developing a specification of the GARCH model that allows such a parameterization. Accordingly, recognizing the possibility that a squared power term may not necessarily be optimal, Ding et al. (1993) proposed a class of model which allows an optimal power transformation term to be estimated. This addition to the ARCH family is known as the power ARCH (PARCH) model and Ding et al. (1993) applied it to US stock returns data and concluded that a power transformation of 1.43 was optimal. Subsequent to this, Hentschel (1995) proposed a more general class of power ARCH model and also applied it to U.S. stock return data. In this paper, the optimal value for the power term was found to be 1.52. An added feature of these PARCH models is that they are closely related to the long memory ARCH model introduced by Ding and Granger (1996) and the fractionally integrated GARCH model introduced by Baillie et al. (1996).

The evidence provided by Ding et al. (1993) and Hentschel (1995) suggests that PARCH models are applicable to US stock market data. An interesting research issue is to explore how generally applicable the class of power ARCH models are to a wider range of financial data. In this paper we attempt to address this issue by estimating an asymmetric power ARCH (APARCH) model for 10 series of national stock market index returns plus a world index. These countries are Australia, Canada, France, Germany, Hong Kong, Japan, New Zealand, Singapore, the United Kingdom and the United States. As the asymmetric power generalized ARCH model adopted in this paper nests the standard ARCH and GARCH models as well as a number of other more 'exotic' variants, the relative ranking of each of these models can be considered using the standard likelihood ratio testing procedure. As a further analysis, a ranking of the models can be produced using standard measures such as Akiake Information Criterion (AIC) and the Schwarz Bayesian Criterion (SBC).

The rest of this paper is structured as follows. In Section 2 we detail the general model and discuss how various ARCH models are nested within the asymmetric power ARCH structure. Section 3 describes the national and world index data to be used in this study and presents the empirical results. Parameter estimates for the APARCH model are presented, as are the results of the likelihood ratio testing procedure. The robustness of these findings is assessed using the AIC and the SBC. Section 4 contains some concluding remarks.

2. The power ARCH model

While not to suggest the mean equation is without interest, the purpose of this paper is to consider the impact of alternative variance equation specifications.[1] As such, the mean equation for each model shall be specified in one of two ways. Where a national stock market return series exhibits significant first order autocorrelation, an AR(1) specification is adopted as the mean equation for that series, that is:

$$r_t=\phi_0+\phi_1 r_{t-1}+\epsilon_t, \tag{1}$$

where r_t is the returns to the stock market index and ϵ_t is the error term in period t. Where the t-statistic associated with ϕ_1 in Eq. (1) is insignificant, a naïve no-change mean equation is specified[2]:

$$r_t=\phi_0+\epsilon_t. \tag{2}$$

[1] As shown in Nelson (1991, 1992) the effect of mis-specification of the conditional mean equation tends to wash out of the conditional variance in high frequency data.

[2] It should be noted that no significant higher order serial correlations were found in the series.

The error term in the mean equation, whether it is specified as Eq. (1) or Eq. (2), may be decomposed as $\epsilon_t = \sigma_t e_t$ where $e_t \approx N(0,1)$. The general asymmetric power ARCH model introduced by Ding et al. (1993) specifies σ_t as of the form:

$$\sigma_t^d = \alpha_0 + \sum_{i=1}^{\rho} \alpha_1 (|\epsilon_{t-1}| + \gamma_1 \epsilon_{t-1})^d + \sum_{i=1}^{q} \beta_1 \sigma_{t-1}^d, \tag{3}$$

where the α_i and β_i are the standard ARCH and GARCH parameters, the γ_i is the leverage parameter and d is the parameter for the power term. To operationalize Eq. (3) we need to specify the lag structure and in this paper a first order lag structure is adopted for both the ARCH and GARCH terms:

$$\sigma_t^d = \alpha_0 + \alpha_1 (|\epsilon_{t-1}| + \gamma_1 \epsilon_{t-1})^d + \beta_1 \sigma_{t-1}^d. \tag{4}$$

Eq. (4) shall hereafter be referred to as an asymmetric power GARCH (APGARCH) model to reflect the inclusion of the β term. Thus, we are able to distinguish this model from a version in which $\beta_1=0$, that we shall refer to as an asymmetric power ARCH (APARCH) model.

Following the approach of Ding et al. (1993) and Hentschel (1995), it is possible to nest a number of the more standard ARCH and GARCH formulations within the APGARCH model by specifying permissible values for α, β, γ, and d in Eq. (4). Table 1 summarizes the restrictions required to produce each of the models nested within this APGARCH model.

Table 1
ARCH/GARCH model specifications[a]

Model	d	α_i	β_i	γ_i
ARCH	2	free	0	0
GARCH	2	free	free	0
Leverage ARCH	2	free	0	$\lvert\gamma_i\rvert \leq 1$
Leverage GARCH	2	free	free	$\lvert\gamma_i\rvert \leq 1$
GJR–ARCH	2	$\alpha_i(1+\gamma_i)^2$	0	$-4\alpha_i\gamma_i$
GJR–GARCH	2	$\alpha_i(1+\gamma_i)^2$	free	$-4\alpha_i\gamma_i$
Taylor ARCH	1	free	0	0
Taylor GARCH	1	free	free	0
TARCH	1	free	0	$\lvert\gamma_i\rvert \leq 1$
Generalized TARCH	1	free	free	$\lvert\gamma_i\rvert \leq 1$
NARCH	free	free	0	0
Power GARCH	free	free	free	0
Asymmetric PARCH	free	free	0	$\lvert\gamma_i\rvert \leq 1$
Asymmetric PGARCH	free	free	free	$\lvert\gamma_i\rvert \leq 1$

[a] The restrictions required on the asymmetric power GARCH model of Ding et al. (1993) to nest particular special cases of other ARCH/GARCH models.

$$\sigma_t^d = \alpha_0 + \sum_{i=1}^{\rho} \alpha_i (|\epsilon_{t-i}| + \gamma_i \epsilon_{t-i})^d + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^d.$$

From Table 1, where α_i is free, d=2 and β=γ=0, this model reduces to Engle's (1982) ARCH model. Further, when we extend this model to allow both α_i and β_i to take on any value, we get Bollerslev's (1986) GARCH model. Taylor (1986) and Schwert (1989) have suggested that it is the conditional standard deviation which should be the focus of an ARCH type model. This class of model is also nested in the APGARCH specification. Specifically, a Taylor ARCH model is specified where α_i is free, d=1 and β_i=γ=0 and a Taylor GARCH model is specified where α_i and β_i are free, d=1 and γ=0. Beyond the standard ARCH and GARCH models, it is also possible to nest in Eq. (4) other ARCH models which have been proposed in the literature. For example, the non-linear ARCH model (NARCH) of Higgins and Bera (1992) is obtained where d and α_i are free and β=γ=0. If we extend this NARCH model to allow β_i to also be free, then a Power GARCH specification is the result.

The models nested so far have assumed a symmetrical response of volatility to innovations in the market. However, empirical evidence suggests that positive and negative returns to the market of equal magnitude will not generate the same response in volatility (See inter alia Black (1976) and Nelson (1990)). Glosten et al. (1993) provided one of the first attempts to model asymmetric or leverage effects with a model which utilizes a GARCH type conditional variance specification. The GJR–GARCH model includes an indicator (threshold) function based on the sign of the residuals. It can be shown that this model is equivalent to the standard leverage model, with $\alpha_{i,GJR}=\alpha_i(1+\gamma_i)^2$ and $\gamma_{i,GJR}=-4\alpha_i\gamma_i$. In addition, a pure ARCH version of this specification (GJR-ARCH) can be achieved by imposing the restriction β_i=0. The leverage GARCH (ARCH) model is obtained by extending Bollerslev's original GARCH model (Engle's original ARCH model) to allow γ_i to take on positive values (i.e. $|\gamma_i|\leq 1$) in the σ_t^d equation. The TARCH model of Zakoian (1994) is defined where α_i is free, d=1, $|\gamma_i|\leq 1$ and we restrict β_i=0. A generalized version of this TARCH (GTARCH) model is derived by allowing β_i to be free.[3]

3. Empirical results

3.1. Data

Daily stock price index data for ten countries plus the Morgan Stanley Capital International (MSCI) world index were sourced from the Equinet database for the period February 1989 to December 1996 giving a total of 2062 observations. The ten countries and their respective price indices are: Australia (All Ordinaries Accumulation Index (AOAI)), Canada (Toronto Stock Exchange 300 (TSE 300)), France (CAC 40), Germany (German Aktien Index (DAX)), Hong Kong (Hang Seng), Japan (Nikkei Dow), New Zealand (NZSE 40), Singapore (Straits-Times), United Kingdom (FTSE) and the United States (S and P 500). Each of these indices

[3] For full details and proofs of this nesting process see Ding et al. (1993) and Hentschel (1995).

are provided in local currency terms and were converted to US dollars using the appropriate daily bilateral exchange rate.

For each national index expressed in US dollars, the continuously compounded return was estimated as $r_t = \log(p_t) - \log(p_{t-1})$ where p_t is the closing price on day t. The correlogram for each returns series was examined and where first order autocorrelation was found, AR(1) models were fitted. Details of the mean equation adopted for each index are reported in Table 2. The error term in this mean equation, particularly when using daily data, may contain outliers which cause the distribution of the returns to exhibit excess kurtosis. As such, the use of a Gaussian distribution is inappropriate. To accommodate the presence of such leptokurtosis, the models in this paper were estimated assuming the errors were drawn from a conditional *t*-distribution (see Bollerslev, 1987).

3.2. Basic estimation results

Using the Berndt-Hall–Hall-Hausman algorithm, the asymmetric power GARCH model was estimated for each national stock market return series. The model was then subsequently re-estimated, accommodating various sets of parameter restrictions according to Table 1, to produce estimates for each of the nested variants. To conserve space, Table 2 presents details of the variance equation parameters for the most general APGARCH model only.[4]

As seen in the table, generally for all of the models estimated the *t*-statistics on the coefficients were largely significant at the 5% level. The Jarque–Bera statistic is highly significant for each of the models indicating non-normality of the data and the estimated degrees of freedom parameter for the conditional *t*-distribution was significant at the 5% level in each case. The power term estimated for the APGARCH model fitted to each of these 10 national indices and the world market index is also presented in the fifth column of Table 2. The maximum power term was 2.489 for Singapore and the minimum was 0.912 for the German index. Beyond these two extreme cases, the remainder of the estimated power terms were between 1.0 and 1.5, with most observed values close to the mean value of 1.36. The estimated power term for the S & P500 US index was 1.211 which is somewhat lower than the 1.524 estimated by Hentschel (1995) and 1.43 estimated by Ding et al. (1993). The power terms provided by the APGARCH model are typical of those generated by the other three models in which d is free, namely, the NARCH, PGARCH and APARCH models.

[4] Detailed results for the other models estimated are not reported to conserve space. Similarly, only selected key results are reported for the power term tests presented in Section 3.3, for the log likelihood ratio test results of Sections 3.4 and 3.5, for the local currency analysis of Section 3.6 and for the AIC/SBC analysis of Section 3.7. The complete set of analysis is available from the authors on request.

Table 2
Asymmetric power GARCH model estimation: US dollar returns[a]

Index	Mean Equation	α_1	β_1	d	γ	t-distribution parameter (S.E.)	t-stat H_0: d=1	t-stat H_0: d=2	Jarque–Bera
S & P500	Naïve	0.032 (3.23)	0.951 (87.1)	1.211 (3.96)	−0.502 (2.56)	4.88 (0.53)	0.69	−2.58	3775.0
FTSE	Naïve	0.025 (3.02)	0.962 (101.3)	1.437 (4.03)	−0.343 (1.82)	8.15 (1.13)	1.23	−1.58	352.8
Nikkei Dow	Naïve	0.064 (5.40)	0.917 (74.7)	1.225 (4.91)	−0.546 (4.88)	6.07 (0.78)	0.90	−3.11	487.6
Hang Seng	AR(1)	0.093 (5.32)	0.813 (29.9)	1.359 (5.51)	−0.294 (3.08)	4.56 (0.41)	1.46	−2.60	5891.0
NZSE40	AR(1)	0.098 (5.67)	0.832 (27.5)	1.372 (4.98)	−0.153 (2.04)	6.43 (0.68)	1.35	−2.28	2940.0
DAX	AR(1)	0.057 (5.43)	0.926 (69.0)	0.912 (4.79)	−0.159 (1.36)	5.54 (0.46)	−0.46	−5.71	41,698
CAC40	Naïve	0.049 (3.45)	0.906 (43.4)	1.170 (5.08)	−0.603 (2.89)	7.65 (0.86)	0.74	−3.60	3679.0
Straits-Times	AR(1)	0.073 (2.63)	0.628 (9.00)	2.489 (4.07)	−0.263 (3.38)	5.44 (0.51)	2.43	0.80	18,018
TSE300	AR(1)	0.072 (3.65)	0.795 (18.7)	1.452 (4.01)	−0.395 (2.83)	6.37 (0.81)	1.25	−1.51	707.5
AOAI	AR(1)	0.062 (3.31)	0.820 (14.6)	1.011 (2.72)	−0.353 (1.83)	7.98 (1.08)	0.03	−2.66	701.2
MSCI world	AR(1)	0.086 (4.90)	0.849 (34.4)	1.370 (4.69)	−0.483 (4.42)	6.20 (0.79)	1.27	−2.16	540.1

[a] This table reports the results of the estimation of an asymmetric power GARCH model with t-distributed errors fitted to 10 national stock market indices and a world index quoted in US dollars sampled at a daily frequency over the period Feb. 1989 to Dec. 1996. The ARCH and GARCH coefficients are presented as are the power and leverage effect term (t-statistics in parentheses). The penultimate column presents the estimated parameter and standard error for the t-distribution assumed for the data and the final column is the standard Jarque–Bera test for normality.

3.3. Tests of power term parameters in APGARCH model

Recall that the two common values of the power term imposed throughout much of the GARCH literature are the values of two (the Engle and Bollerslev family of models) and unity (the Taylor family of models). As argued in Section 1, the invalid imposition of a particular value for the power term may lead to sub-optimal modeling and forecasting performance. If a value of two is appropriate then imposing a unity power is inferior; alternatively if a value of unity is appropriate then imposing a value of two is inferior; or indeed it may be the case that neither of these values are optimal. Since this issue cannot be resolved analytically, it is an empirical question which can be readily addressed in the context of the APGARCH (and related) models. Accordingly, we test whether the estimated power terms are significantly different from unity or two by comparing them to their standard error.

As reported in Table 2, with the exception of the Singapore-Straits Times index (d=2.48), none of the estimated power coefficients are significantly different from unity. Further, with the exception of the FTSE, Straits Times and the TSE300, each of the power terms are significantly different from 2. Hence, on the basis of these results, in the majority of cases support is found for the Taylor family of GARCH models in which a unity power term is imposed. It may also be interesting to test whether the power terms are significantly different to the mid-point value of 1.5. In this case, none of the estimated d coefficients are significantly different from 1.5 with the sole exception of Germany (d=0.912).

With the exception of the power term estimated for the Singapore index, the optimal power term estimated for each of the remaining 10 country indices appear remarkably similar.[5] A formal test of this hypothesis may be undertaken by conducting a pairwise test of estimated power terms of the following form:

$$t=\frac{d_1-d_2}{\sqrt{SE_{d_1}^2+SE_{d_2}^2}} \tag{5}$$

where d_1 (d_2) is the power term from the APGARCH model estimated for the national stock market index for country 1 (country 2) and SE is the standard error associated with the APGARCH model estimated for each country. This estimated test statistic is t-distributed and may be compared to its critical values to establish whether the power terms are significantly different from one another.

The results of this pairwise testing procedure generally provide support for the null hypothesis that the estimated power terms are not significantly different from one another. For example, indices which generated very similar power terms such as the NZSE40 (d=1.372) and the world MSCI (d=1.370) were, as expected, not significantly different (t=0.003). Even for indices which generated power terms that

[5] Indeed, the estimated power term for the world return is very similar to the average of the power terms for our sample of 10 countries. At first glance this may appear to be an obvious result, however, given the non-linear nature of this phenomenon, the close relationship between the world and individual power terms is more likely to be coincidence.

were prima facie somewhat dissimilar, such as the AOAI (d=1.011) and the TSE300 (d=1.452), were still not statistically different to one another in a formal statistical test (t=0.849). The major exception to this general finding were pairwise tests involving the Singapore index (d=2.49) whose APGARCH model estimated power term was significantly greater than that of the DAX, CAC40 and AOAI at the 5% level and the S and P500, Nikkei Dow, Hang Seng, NZSE40 and MSCI at the 10% level. The power term estimated for the Singapore index was not, however, significantly different to the FTSE, and TSE300 at conventional significance levels. Generally, these findings re-enforce the conclusion that the Taylor family of models is preferred.

In summary, the evidence reported in Sections 3.2 and 3.3 provides support for the belief that a power ARCH type model has quite a general empirical validity across many different markets. However, it may be the case that a more restricted version of this power ARCH model (namely, the standard deviation version) is preferred and it is to this question that we now turn our attention.

3.4. Standard GARCH model likelihood ratio tests

As discussed in Section 2, the APGARCH model of Ding et al. (1993) nests a number of other ARCH models which may be derived by simple parameter restriction. We may however, extend this concept further. Many of these nested models themselves nest other ARCH models which may be derived by further restriction. For example, from Table 1 we can see that the simple ARCH model of Engle (1982) is nested within not only the APGARCH model, but also within Bollerslev's (1986) standard GARCH model, the leverage ARCH and GARCH models, the GJR–GARCH and GJR–ARCH models, the NARCH model, the power GARCH model and the APARCH model. We can test the significance of the restrictions necessary to derive the nested ARCH model from each of these alternatives using the standard likelihood ratio testing procedure. Given the possible complex nesting permutations, we discuss the results in terms of natural groupings to assist the reader in understanding the analysis.

3.4.1. Likelihood ratio tests: Engle (1982) ARCH model as the restricted case

First, the likelihood ratio test was applied to each nested pair that involves Engle's (1982) ARCH model as the alternative model. An insignificant test statistic (P-value $>$0.05) indicates a preference for this nested model. The basic ARCH–GARCH model comparison test results presented in Panel A of Table 3 clearly reject the nested ARCH model in favour of the GARCH model and this was true (in unreported results) of all the other estimated tests which compared the ARCH model to the leverage GARCH, GJR–GARCH, power GARCH and APGARCH models. This result holds for all 10 countries and the global market index and the rejections are very strong. Thus, the inclusion of a GARCH term in the conditional variance equation would appear to augment the model in a worthwhile fashion.

The ARCH model is also nested within a number of the more exotic ARCH specifications such as the NARCH, GJR–ARCH, leverage ARCH and APARCH models. In unreported tests we find that for seven countries and the global index we are

Table 3
Selected likelihood ratio tests for nested ARCH models fitted to national indices: US dollar returns

Index	S&P500	FTSE	Nikkei Dow	Hang Seng	NZSE40	DAX	CAC40	Straits-Times	TSE300	AOAI	MSCI world
Panel A											
ARCH vs GARCH	113.0848	61.4632	160.6933	104.1157	100.1064	76.0218	54.0718	55.5058	40.8249	90.9277	13.1430
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Panel B											
GARCH vs Leverage GARCH	5.0221	1.4592	42.1495	12.8420	4.6131	0.2474	13.1235	16.0700	13.0637	32.4729	7.5294
	0.025	0.227	0.000	0.000	0.031	0.618	0.000	0.000	0.000	0.000	0.006
GARCH vs GJR–GARCH	4.9772	1.4552	42.1460	12.8850	4.6109	0.2463	13.0550	16.1360	13.0400	32.5170	7.5611
	0.025	0.227	0.000	0.000	0.031	0.619	0.000	0.000	0.000	0.000	0.005
GARCH vs Power GARCH	0.8587	1.1362	5.4143	1.8640	4.5347	23.5286	4.2411	0.9065	0.3113	1.6882	3.3854
	0.354	0.286	0.019	0.172	0.033	0.000	0.039	0.341	0.576	0.193	0.065
GARCH vs APGARCH	9.7945	3.6043	47.8174	15.8274	8.2563	25.4234	22.1676	17.6408	13.9847	34.7926	9.3383
	0.007	0.164	0.000	0.000	0.016	0.000	0.000	0.000	0.000	0.000	0.009
Leverage GARCH vs APGARCH	4.7724	2.1451	5.6678	2.9854	3.6432	25.4331	9.0440	1.5707	0.9209	2.3197	1.8089
	0.028	0.143	0.017	0.084	0.056	0.000	0.002	0.210	0.337	0.127	0.178
GJR–GARCH vs APGARCH	5.0005	2.2262	5.6188	2.9419	3.6454	25.4367	9.0837	1.5045	0.9447	2.2751	1.7772
	0.025	0.135	0.017	0.086	0.056	0.000	0.002	0.219	0.331	0.131	0.182
Panel C											
Power GARCH vs APGARCH	4.4679	2.3702	21.2015	6.9817	1.8608	0.9474	8.9632	8.0630	7.0171	2.9764	16.5521
	0.002	0.029	0.000	0.000	0.053	0.168	0.000	0.000	0.000	0.014	0.000
APARCH vs APGARCH	121.5328	62.9610	203.9201	109.8272	106.8055	99.3188	74.2159	65.1309	49.7737	17.7197	121.9082
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Panel D											
Taylor ARCH vs Taylor GARCH	113.7225	65.9641	169.9223	117.1382	106.6910	93.4632	62.3116	52.0558	41.2554	20.4064	99.9389
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

unable to reject the ARCH model in favor of the NARCH model at conventional significance levels. The exceptions are rejections at the 10% significance level for Hong Kong and New Zealand and a rejection at the 5% significance level for France (CAC40). Similarly, for seven (four) countries we are unable to reject the ARCH model in favour of the APARCH model (GJR–ARCH and leverage ARCH models) at conventional significance levels. This suggests that the addition of a power term independent of the existence of a GARCH and leverage effect term adds little to the model.

3.4.2. Likelihood ratio tests: asymmetric ARCH model as the restricted case

The two asymmetric ARCH models considered in this paper, namely, the Leverage ARCH and the GJR-ARCH models, are also nested in other more complex specifications. Specifically, the leverage ARCH model is nested within the APARCH model, the leverage GARCH model and the APGARCH model. The outcome of the likelihood ratio tests (unreported) provides a clear rejection of the leverage ARCH model against the latter two models in all cases. Further, the restrictions of the leverage ARCH model can only be rejected in favor of the asymmetric power ARCH model in the case of the US.

These findings qualitatively mimic the results when the GJR–ARCH model is the restricted model. Specifically, the GJR–ARCH model is nested within the APARCH model, the GJR–GARCH model and the APGARCH model. The unreported likelihood ratio test results indicate that the restrictions to obtain the GJR–ARCH model cannot be rejected relative to the APARCH model (except for the US index). However, the test results clearly reject the GJR–ARCH model in preference to the two alternatives which contain a GARCH term. A comparison of these results suggest that empirically there is very little practical difference between how the leverage ARCH and GJR–ARCH models capture the leverage effects in conditional volatility. In addition, the earlier conclusion of the importance of GARCH terms is again reinforced by these results.

3.4.3. Likelihood ratio tests: GARCH models as the restricted case

The standard GARCH model of Bollerslev (1986) is nested within the leverage GARCH model, the GJR–GARCH model, the Power GARCH model and the APGARCH model. Panel B of Table 3 presents the test results for these nested comparisons and although the results vary greatly across countries, one may observe strong leverage effects in the data. For six countries plus the world index, the standard GARCH model is strongly rejected in favor of the leverage and GJR–GARCH models. Further, a weaker rejection of the standard GARCH model is obtained for the US and New Zealand. In stark contrast, the standard GARCH model cannot be rejected against the leverage GARCH or GJR–GARCH models for both the United Kingdom (FTSE) and Germany (DAX). There is moderate evidence supporting the need for power effects in the absence of leverage effects. Specifically, in the case of six countries plus the world index, the standard GARCH model cannot be rejected in favor of the power GARCH model. For Japan, New Zealand and France there is a mild rejection and for Germany a strong rejection. There is however, much stronger evidence against the standard GARCH model in favor of a combination of leverage and power effects. Specifically, for nine countries and the global index, the GARCH model is strongly rejected in favour of the APGARCH model which exhibits a combination of leverage and power effects. The exception is the United Kingdom (FTSE) where the standard GARCH model cannot be rejected.

The leverage GARCH model is nested within the APGARCH model. The results of this test, shown in Panel B of Table 3, are mixed in terms of supporting the presence of power effects after leverage effects have been taken into consideration. For two countries (Germany and France) there is strong evidence of power effects. For a further two countries (Japan and US) there is mild evidence of power effects, and finally for two countries (Hong Kong and New Zealand) there is only weak evidence of power effects. In contrast, the remaining four countries (United Kingdom, Australia, Singapore and Canada) and the global index show no evidence of power effects after leverage effects have been taken into account, as the leverage GARCH model cannot be rejected in favor of the APGARCH model. The GJR–GARCH model is also nested within the APGARCH model. The log likelihood test results of this case, also shown in Panel B of Table 3, are very similar which is to be expected given the strong theoretical similarities between the leverage GARCH model and the GJR–GARCH model. Specifically, the results exhibit only mixed evidence of power effects after leverage effects have been taken into consideration. In a qualitative sense, the results comparing the GJR–GARCH model to the asymmetric power GARCH model are identical to the results comparing the leverage GARCH model to the asymmetric power GARCH model.

3.4.4. Likelihood ratio tests: power ARCH/GARCH models as the restricted case

A series of tests in which the restricted case is a model belonging to the Power ARCH/GARCH grouping was also performed. To begin with, the NARCH model is nested within the power GARCH model and the APGARCH and APARCH models. Testing against the first two of these models the NARCH is always strongly rejected for all 10 countries plus the global index. However, testing against the APARCH model, the NARCH model cannot be rejected in the case of five countries, again suggesting that the GARCH term is important. Further, the power GARCH model is nested within the APGARCH model (see Panel C of Table 3). The log likelihood ratio tests provide strong evidence in support of the more general model as eight of the countries tested and the world index generate significant test statistics. Only the NZSE40 and the DAX generated insignificant likelihood ratio test statistics. Finally in the current grouping of tests, the APARCH model is nested within the APGARCH model and the estimated results provide unanimous support for the more general model. Generally, the preceding results provide additional support for the notion that modeling the power effects themselves is not sufficient unless the GARCH effects and leverage effects are also modelled.

3.5. Taylor-ARCH model likelihood ratio tests

3.5.1. Likelihood ratio tests: Taylor-ARCH model as the restricted case

The standard deviation version of the ARCH model (Taylor-ARCH) is nested within the TARCH, NARCH, Taylor-GARCH, power GARCH, generalized TARCH, APARCH and APGARCH models. The results for the basic Taylor-ARCH–Taylor-GARCH comparison are presented in Panel D of Table 3. In general, as exemplified by this comparison, we find that the restrictions of the Taylor-ARCH model are rejected, particularly, relative to those models containing GARCH terms.

The standard deviation version of the GARCH model suggested by Taylor (1986) is nested with the generalized TARCH model, the power GARCH model and the APGARCH model. As with Bollerslev's GARCH model, the Taylor-GARCH model provides similar results (unreported here to conserve space) when compared to the power GARCH and asymmetric PARCH models. Specifically, there is a lack of evidence to suggest the need for power effects in the absence of leverage effects as the likelihood ratio test produces insignificant calculated values indicating an inability to reject the Taylor-GARCH model over the power-GARCH model for seven of the national indices tested, as well as the world index. However, the modeling of leverage effects in the generalized TARCH and the asymmetric power GARCH model is preferred to the standard specification of the Taylor-GARCH model. These results again re-enforce those found for Bollerslev's GARCH model, as the inclusion of leverage effects enhances the power transformation element of the model.

The original version of the TARCH model suggested by Zakoian (1994) is nested within the generalized TARCH model, as well as the asymmetric power ARCH and GARCH models. The likelihood ratio test results reject the TARCH model in favour of the other two models which include the power term, the GARCH term and leverage effects in all instances. However, consistent with the previous results the TARCH model holds up much better against the asymmetric power ARCH model. Finally, the generalized TARCH model is nested within the asymmetric power GARCH model. For all of the countries tested with the exceptions of Hong Kong and Singapore, the test statistics indicate a preference for the generalized TARCH model over the asymmetric power GARCH model. This result is most interesting and suggests that, consistent with earlier results, allowing the power term to take on values other than unity, does not significantly enhance the model.

3.6. Local currency analysis

As a measure of the robustness of the previous results, it is possible to test the relative superiority of these nested ARCH models using the national stock market indices expressed in their home currency rather than in US dollars. Research by Ding and Granger (1996) has found that while the long memory property of stock returns is greatest when d=1. For exchange rate returns however, this property was found to be greatest when d=0.25. Thus, it is possible that the use of a common currency to express returns may have an influence on the results given the unique characteristics of exchange rate data. As such, the asymmetric power GARCH model was fitted and Table 4 presents estimates of the variance equation parameters for this model fitted to the local currency denominated stock market returns.

From Table 4, the average estimated power term for these robust APGARCH models is 1.34 and the individual power terms estimated for each series are generally similar in value to those estimated for the US dollar denominated returns. Interestingly, the correlation between the power terms for the US and local currency APGARCH models is 0.931.[6,7] Pairwise comparisons may be conducted between the nested ARCH models fitted to these local currency national stock market returns series, similar to those for the US dollar returns series reported and discussed earlier. To conserve space, a selection of these results is presented in Table 5.

In general, the results of the likelihood ratio testing on the local currency denominated national indices are similar to those generated by the common currency analysis, only the conclusions are strengthened in each instance. For example, the simple ARCH model is rejected against all other models except the NARCH model and, to some extent, the leverage ARCH and asymmetric power ARCH models. Bollerslev's GARCH model is inferior to all nested models and the evidence of the need for a power term without the presence of a leverage effect term is strengthened.

This trend is further evidenced when one considers the tests of the leverage GARCH model and the GJR-GARCH model to the asymmetric power GARCH model. Previously the evidence was mixed in these two comparisons although it tended to indicate a lack of a need for a power term once the leverage effects had been taken into account. For the local currency results however, while the results when comparing the GJR–GARCH model to the APGARCH are still fairly mixed, the less restrictive leverage GARCH model is now rejected for six indices in preference to the APGARCH model. Previously, it was rejected in only three instances. The NARCH model is again rejected in all instances and the APGARCH model retains its superiority over the power GARCH model. A preference is found for Taylor's GARCH model in only the case of the power GARCH model as seven of 10 countries and the world index generated an insignificant test statistic. The calcu-

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[6] Empirical evidence suggests exchange rates exhibit distinct power characteristics compared to stock returns. Ding and Granger (1996) found that the long memory property of the Deutschmark/US exchange rate is greatest when a power term of 0.25 is used. McKenzie (1998) considered the optimal power transformation in the case of forecasting exchange rate volatility for a wide range of Australian bilateral rates. He found that the optimal exponent varied considerably between each exchange rate series, taking values in the range 0.25 to 1.50.

[7] McKenzie and Mitchell (1999) applied the power GARCH class of models to exchange rates and found the optimal power term to be 1.37, on average. When compared to the average optimal power term for the US dollar national stock market indices (1.36) one may be tempted to infer a relationship. However, this is not the case as the power term of an APGARCH model fitted to US dollar quoted national stock market index returns has little correlation (ρ=0.38) to the estimated power term of an APGARCH model fitted to the bilateral exchange rate series used to convert local currency returns to a US dollar equivalent (see McKenzie and Mitchell (1999) for details of these models). Further, the power term for an APGARCH model fitted to the local currency stock market returns series also bears little in common with the power term estimated for the US–local currency bilateral exchange (ρ=0.37).

Table 4
Asymmetric power GARCH model estimation: local currency returns[a]

Index	Mean equation	α_1	β_1	d	γ	t-distribution parameter(S.E.)	t-stat H_0: d=1	t-stat H_0: d=2	Jarque–Bera
FTSE	Naïve	0.037 (3.54)	0.948 (70.55)	1.017 (3.21)	−0.601 (3.45)	11.48 (1.960)	0.05	−3.10	175.9
Nikkei Dow	Naïve	0.065 (5.59)	0.916 (81.41)	1.277 (5.25)	−0.639 (5.33)	5.953 (0.709)	1.14	−2.97	741.1
Hang Seng	Naïve	0.089 (5.28)	0.824 (31.20)	1.273 (5.34)	−0.292 (3.04)	4.273 (0.390)	1.15	−3.05	6083.0
NZSE40	AR(1)	0.112 (6.35)	0.816 (28.69)	1.360 (5.09)	−0.183 (2.64)	5.768 (0.605)	1.35	−2.40	1864.0
DAX	Naïve	0.058 (5.31)	0.923 (71.82)	1.014 (5.64)	−0.322 (2.71)	5.313 (0.423)	0.08	−5.48	62,099.0
CAC40	Naïve	0.045 (3.53)	0.926 (60.96)	1.036 (5.11)	−0.840 (3.76)	7.380 (0.807)	0.18	−4.75	5318.0
Straits-Times	AR(1)	0.055 (1.84)	0.546 (6.56)	3.267 (3.65)	−2.358 (3.84)	4.756 (0.433)	2.53	1.42	38,897.0
TSE300	AR(1)	0.063 (3.24)	0.787 (15.11)	1.320 (3.61)	−0.363 (2.26)	5.290 (0.611)	0.88	−1.86	1153.0
AOAI	AR(1)	0.071 (3.95)	0.817 (17.90)	1.122 (3.86)	−0.397 (2.48)	6.945 (0.777)	0.42	−3.02	4195.0

[a] The following table reports the results of the estimation of an asymmetric power GARCH model with t-distributed errors fitted to 10 national stock market indices and a world index quoted in local currency terms sampled at a daily frequency over the period Feb. 1989 to Dec. 1996. The ARCH and GARCH coefficients are presented as are the power and leverage effect term (t-statistics in parentheses). The penultimate column presents the estimated parameter and standard error for the t-distribution assumed for the data and the final column is the standard Jarque–Bera test for normality.

Table 5
Selected likelihood ratio tests for nested ARCH models fitted to national indices: local currency returns

Index	FTSE	Nikkei Dow	Hang Seng	NZSE40	DAX	CAC40	Straits-Times	TSE300	AOAI
Panel A									
ARCH vs GARCH	73.5695	248.9425	102.7338	126.3037	131.7042	85.4650	6.0442	2.2760	90.9472
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Panel B									
GARCH vs Leverage GARCH	18.8679	62.1043	12.8194	8.5886	2.1990	27.0286	17.3672	5.9208	32.4729
	0.000	0.000	0.000	0.003	0.138	0.000	0.000	0.014	0.000
GARCH vs GJR–GARCH	18.9033	62.1193	12.8674	8.5891	2.1903	26.8359	17.5123	5.9032	32.5175
	0.000	0.000	0.000	0.003	0.138	0.000	0.000	0.015	0.000
GARCH vs Power GARCH	5.1900	7.8688	1.5214	6.5091	17.9788	3.9907	3.7828	0.1805	1.7035
	0.022	0.005	0.217	0.010	0.000	0.045	0.051	0.670	0.191
GARCH vs APGARCH	26.1005	70.0523	15.8302	12.5133	26.8012	45.0616	22.6938	6.2067	34.7926
	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.044	0.000
Leverage GARCH vs APGARCH	7.2325	7.9479	3.0108	3.9246	24.6021	18.0329	5.3266	0.2859	2.3197
	0.007	0.004	0.082	0.047	0.000	0.000	0.021	0.592	0.127
GJR–GARCH vs APGARCH	7.1971	7.9330	2.9628	3.9241	24.6109	18.2256	5.1814	0.3034	2.2751
	0.027	0.018	0.227	0.140	0.000	0.000	0.074	0.859	0.320
Panel C									
Power GARCH vs APGARCH	10.4552	31.0917	7.1544	3.0020	4.4111	20.5354	9.4555	3.1936	4.5752
	0.000	0.000	0.000	0.014	0.002	0.000	0.000	0.011	0.002
APARCH vs APGARCH	96.8487	312.7383	108.0780	132.1824	158.4946	129.8499	76.5246	26.8989	33.4437
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Panel D									
Taylor ARCH vs Taylor GARCH	83.1658	262.6752	115.8391	130.1612	156.6915	91.8465	51.4195	27.3915	28.8393
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

lated values of the likelihood ratio test statistic were all significant when comparing Taylor's GARCH to the generalized TARCH and APGARCH models, indicating a rejection of the restrictions of the former model.

Finally, Zakoian's TARCH is found to be inferior to its more general counterpart models (generalized TARCH, APARCH APGARCH) while the generalized TARCH model is again found to be superior to the APGARCH model. In general, the results of the likelihood ratio testing procedure where the national stock market indices are expressed in local currency terms, concur with the US dollar common currency analysis conducted in this study. As such, the use of the US dollar bilateral exchange rate to convert local currency stock market returns to US dollar returns does not appear to alter the results even though exchange rates themselves have been found to exhibit different characteristics to stock market returns in the APGARCH modeling context.

3.7. AIC and SBC model selection results[8]

The application of likelihood ratio tests to resolving the general model selection question, as reported and discussed in previous sections, follows the work of others in the literature, most notably Ding et al. (1993) and Hentschel (1995). A possible alternative to the log-likelihood testing procedure is to apply either the Akiake Information Criterion (AIC) or Schwarz Bayesian Criterion (SBC) to our set of APGARCH models. As a general rule, the AIC or SBC approaches (the latter imposes a heavier penalty on the inclusion of additional terms in the model) suggest selecting the model which produces the lowest AIC or SBC value. The use of the AIC and SBC techniques for comparing models has the advantage of being relatively less onerous compared to log-likelihood ratio testing procedure which only allows formal pairwise testing of nested models. Thus, as a robustness check of the log-likelihood ratio testing results discussed in Sections 3.4 to 3.6, we compare the AIC and SBC values for each of the models estimated.

Table 6 presents a summary of the optimal ARCH model type (i.e. the specification that produced the lowest AIC or SBC) for the US dollar and local currency analysis according to each criteria. From this table it can be seen that the optimal model for the US dollar denominated index analysis most commonly came from the Taylor family of models. Specifically, according to the AIC criteria, the Taylor GARCH model was optimal for the Hang Seng and NZSE40 indices and the asymmetric version (GTARCH) was preferred for the S and P 500, Nikkei, DAX, CAC40 and AOAI indices. The SBC results largely concur with the AIC results except for the DAX, AOAI and MSCI world index in which simpler models are preferred. The use of local currency does not significantly alter these results as can be seen from the final two columns of Table 6. Only in the case of the FTSE did the optimal model differ according to both criteria where the local currency was considered.

[8] The authors would like to acknowledge, with thanks, an anonymous referee for suggesting this alternative analysis.

Table 6
Optimal ARCH model identified by the AIC and SBC model selection criteria

Index	US dollars		Local currency	
	AIC	SBC	AIC	SBC
S and P 500	Generalized TARCH	Generalized TARCH	Generalized TARCH	Generalized TARCH
FTSE	GARCH	GARCH	Generalized TARCH	Generalized TARCH
Nikkei Dow	Generalized TARCH	Generalized TARCH	Generalized TARCH	Generalized TARCH
HangSeng	Taylor GARCH	Taylor GARCH	Taylor GARCH	Taylor GARCH
NZSE40	Taylor GARCH	Taylor GARCH	Taylor GARCH	Taylor GARCH
DAX	Generalized TARCH	Taylor GARCH	Generalized TARCH	Generalized TARCH
CAC40	Generalized TARCH	Generalized TARCH	Generalized TARCH	Generalized TARCH
Straits-Times	GJR–GARCH	GJR–GARCH	APGARCH	GJR–GARCH
TSE300	Leverage GARCH	Leverage GARCH	Leverage GARCH	GARCH
AOAI	Generalized TARCH	Taylor GARCH	Generalized TARCH	Generalized TARCH
MSCI	Asymmetric PGARCH	GJR–GARCH	Asymmetric PGARCH	GJR–GARCH

Finally, comparing the pairwise testing results of the log-likelihood procedure (as reported and discussed in previous sections) to the relative model rankings provided by the AIC and SBC criteria, we observe that the findings are generally robust. For example, where the log-likelihood results provided unanimous or near unanimous support for one model such as the GARCH model over the ARCH model reported in Section 3.4.1, the AIC and SBC concurred without exception. In those cases where the results of the log-likelihood procedure provided mixed evidence on the superior model for particular comparisons, the AIC and SBC did on occasion differ in terms of their relative ranking for a particular country, but overall they were generally consistent.

4. Conclusion

A recent development in the ARCH literature has been the introduction of the power ARCH class of models which allow a free power term rather than assuming an absolute or squared term in their specification. While the properties of this type of model have been investigated for US stock market data, they have yet to be considered in a wider context.

An important question is, however, why is this of interest? Our answer to this question relies on the following intuitive argument. The common use of a squared power term is most likely a reflection of the normality assumption traditionally invoked (or at least seriously entertained) regarding financial data. However, if we accept that (high frequency) data are very likely to have a non-normal error distribution, then the presumption of an obvious superiority of a squared power term is lost. Other power transformations may be more appropriate. Indeed, for non-normal data, by squaring the returns one effectively imposes a structure on the data which may potentially furnish sub-optimal modelling and forecasting performance relative to other power terms. More generally, adopting any fixed value for the power term may only by chance, constitute a value sufficiently proximate to the true underlying value, to avoid these sub-optimal outcomes. As this issue cannot readily be resolved analytically, it remains as an empirical question which can be rigorously explored in the context of the asymmetric power GARCH (APGARCH) model.

The major advantage of using the APGARCH model is that it nests the two major classes of ARCH models, namely, those which directly model the variance, such as those of Engle (1982) and Bollerslev (1986), and those that directly model the standard deviation, such as Taylor (1986), as special cases of the general model. This provides an encompassing framework which facilitates comparison between these two broad classes of model. Thus, an important contribution of the current paper was to augment our understanding of whether and to what extent these types of more flexible model are statistically superior to their less-sophisticated counterparts.

Accordingly, the purpose of the current paper was to consider the applicability of these power ARCH models to the national stock market returns for 10 countries and a world index. The results of our analysis indicate that strong leverage effects are present in national stock market data. Further, once these leverage effects are modelled in a GARCH framework, the inclusion of a power term is a worthwhile addition to the specification of the model. Both Bollerslev's GARCH and the standard deviation version of the GARCH model proposed by Taylor (1986) were tested against the power GARCH model and the asymmetric power GARCH model amongst others. The results of this study suggest that irrespective of the choice of the Taylor or Bollerslev GARCH model, or of the choice of local or US dollar returns, the relative ranking of nested ARCH models is unchanged.

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