Homework6

Question 5:

a.

In order to show $5n^3 + 2n^2 + 3n = \theta(n^3)$, we need to find c_1, c_2 and n_0 such that for all $n \geq n_0$,

$c_1 \cdot n^3 \leq 5n^3 + 2n^2 + 3n \leq c_2 \cdot n^3$

If we let $c_1 = 5$, $c_2 = 10$ and $n_0 = 1$, we can get,

$5n^3 \leq 5n^3 + 2n^2 + 3n \leq 10n^3$ for all $n \geq 1$

Therefore, $5n^3 + 2n^2 + 3n = \theta(n^3)$.

B.

In order to show $\sqrt{7n^2 + 2n - 8} = \theta(n)$, *we need to find* $c_{1,}$ c_2 *and* n_0, *such that for all* $n \geq n_0$,

$c_1 \cdot n \leq \sqrt{7n^2 + 2n - 8} \leq c_2 \cdot n$

If we let $n_0 = 4$, *which leads to* $2n - 8 \geq 0$ *and* $2n < 2n^2$ *for all* $n \geq 4$, *we can square the inequalities to get*:

$c_1 \cdot n^2 \leq 7n^2 + 2n - 8 \leq c_2 \cdot n^2$

$7n^2 \leq 7n^2 + 2n - 8 \leq 7n^2 + 2n^2$

$7n^2 \leq 7n^2 + 2n - 8 \leq 9n^2$ *for all* $n \geq 4$

By taking the square root on the the inequalities, we can get,

$\sqrt{7}n \leq \sqrt{7n^2 + 2n - 8} \leq 3n$ *for all* $n \geq 4$

Therefore, $\sqrt{7n^2 + 2n - 8} = \theta(n)$.